DIFFERENTIAL INDICIAL ADMITTANCES:

Currents Produced by Unit Differential Pulse Voltage

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Abstract.

The difference of two unit functions is used to represent a pulse voltage of abrupt rise and abrupt fall. By the principle of superposition, the currents produced by such a voltage on fifteen different circuits, covering twenty cases and thirty-six types are analyzed. Many interesting graphs are included. The main characteristics of the pulse currents are given by formulas with a system of notations.

1. Introduction

A quantity, which is zero before the time $t=0$, and is unity after $t=0$, is known as the unit function. Let it be denoted by $1_0(t)$. The indicial admittance of a circuit[1] is defined as the current in the circuit produced by the unit function of voltage applied to it. In other words, it is the transient current produced by the application of a constant voltage of unit magnitude at the time $t=0$. A unit differential pulse voltage[2] of

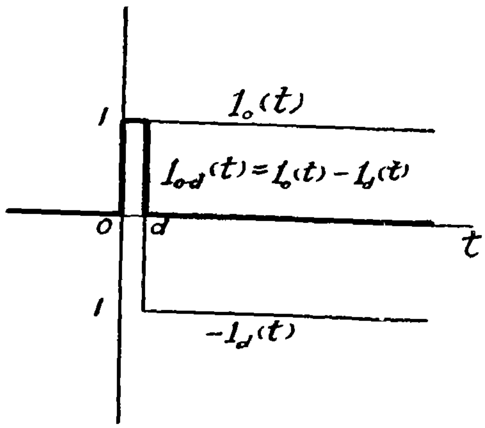


Fig. 1. Differential unit function.

1. See, for instance, Bush's Operational Circuit Analysis, Chapter IV.

2. For generation of differential pulse voltage, see Chinese Journal of Physics, Vol. I, No. 3, P. 68.

duration d can evidently be expressed by the difference of two unit functions, namely, $1_0(t)-1_d(t)$, the latter being the retarded unit function, which is zero before $t=d$ and unity after $t=d$. This is permissible by the principle of superposition. It is interesting and useful to know what currents, of course mainly transient, will be produced in various circuits by such a voltage. These currents will be termed the differential indicial admittances of the circuits. The method of representation of such a voltage immediately points out a way of finding them.

2. Characteristics of Differential Indicial Admittances and their Notations

Any differential indicial admittance consists of one or more pulses. The fundamental characteristics are (1) the number of the pulses (2) the durations of the pulses (3) the maximum (positive or negative) amplitudes of the pulses and (4) the times at which the maximum amplitudes occur. For convenience, throughout this paper, the first will be denoted by N, the second by d_1, d_2, etc., the third by A_1, A_2, etc., and the fourth by t_1, t_2, etc., the subscripts referring to the order of the pulses. The letter d without any subscript will always be used to denote the duration of the applied differential pulse voltage, whose amplitude is uniformly unity. When a pulse is multi-peaked, a single prime will be used to denote the first peak, a double prime the second, and so on. The indicial admittance for the sudden application at $t=0$ will be denoted by $A_0(t)$, and that for the sudden application

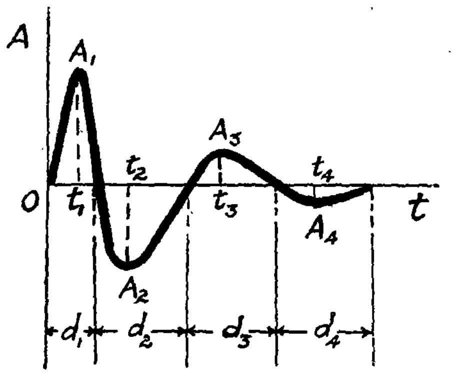


Fig. 2. Characteristics of differential indicial admittance and their notations.

or removal at the time $t=d$ by $\pm A_d(t)$. To get the latter from the former, one must replace t by $t-d$ and remember $A_d(t)=0$ before $t=d$. To emphasize the validity of superposition, $A_{0-d}(t)$ will be written for $A_0(t)-A_d(t)$.

3. Plan of Investigation

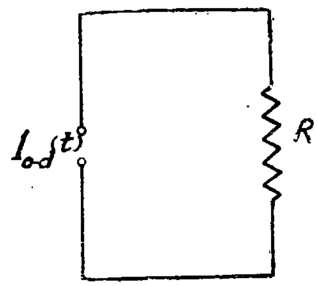
Various elementary, series, coupled, and parallel, and series-parallel circuits, subjected to a unit differential pulse voltage of duration d will be investigated by the principle of superposition of the indicial admittance for the sudden change at $t=0$ and the negative value of the retarded indicial admittance for the sudden change at $t=d$, or by the combination of such superpositions. Although somewhat monotonous, this method is the simplest and most effective for obtaining practical and useful results, the graphical representations of which being especially interesting and fascinating. For each case or each type of a case, four parts will be given: The part (a) illustrates the circuit; the part (b) shows the graphs of the component and the resulted differential indicial admittances; the part (c) gives the characteristics of the differential indicial admittance as the results of investigation; and finally the part (d) furnishes some explanatory notes.

4. Investigation of Elementary Circuits

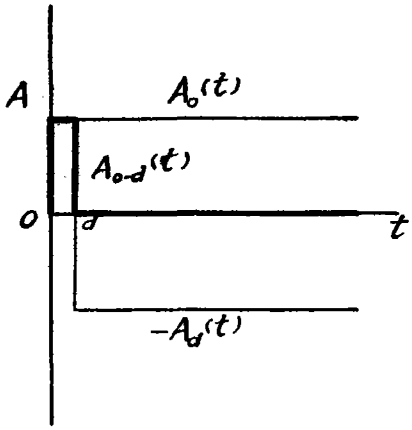
The elementary circuits are not difficult to analyse, but one must be very careful in treating them, for they are sometimes so simple as to be apt to hide mistakes. Their indicial admittances and differential indicial admittances are extremely fundamental, and must be thoroughly understood.

Case 1. *Resistance Alone.*

(a) Circuit.



(b) Indicial admittances.



(c) Characteristics.

$N = 1.$

$d_1 = \mathrm{d}.$

$A_1 = 1/\mathrm{R}.$

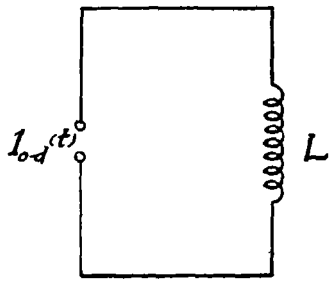
$t_1 = 0$ to d.

(d) Notes.

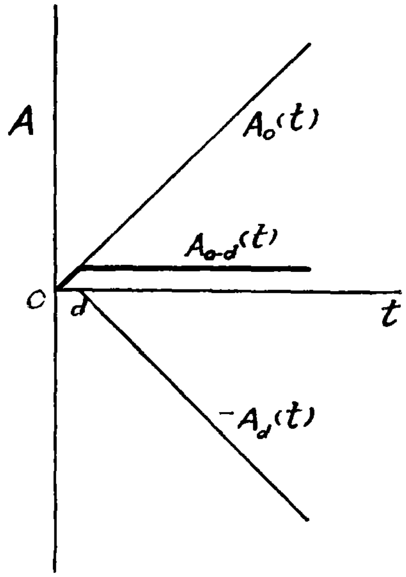
This is the simplest case, which involves no transient. The indicial admittance is $1/R$, which is independent of t, hence the retarded indicial admittance is also $1/R$. So the differential indicial admittance is one rectangular pulse, similar to that of the applied voltage.

Case 2. *Inductance Alone.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=1.$

$d_1=\infty.$

$A_1=d/L.$

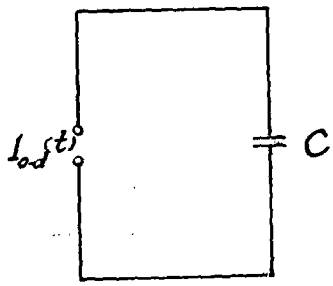
$t_1=d$ to ∞.

(d) Notes.

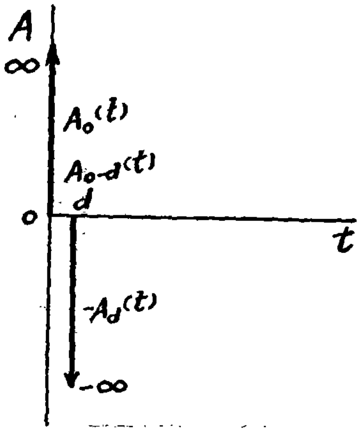
It is well known that the indicial admittance of a series circuit of an inductance L and resistance R is $(1-e^{-Rt/L})/R$, or, $\frac{1}{R}\left[1-\sum_{n=o}^{n=\infty}(1/n!)(-Rt/L)^n\right]$, which becomes t/L when $R=O$. So we have for inductance alone, $A_o(t)=t/L$. Of course this case can only be approximated by highly inductive circuits. It is important to note that a voltage of very short duration may cause a current to flow for a very long time.

Case 3. *Capacitance Alone.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=2$

$d_1=0.\ d_2=0.$

$A_1=\infty.\ A_2=-\infty.$

$t_1=0.\ t_2=d.$

(d) Notes.

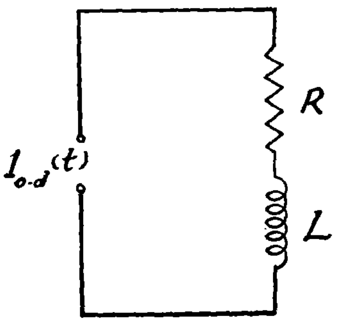
A circuit of a resistance R and a capacitance C in series has an indicial admittance given by $\frac{1}{R}\,e^{-t/(RC)}$ which, for R=0, approaches infinity at $t=0$, but zero at $t>0$. Of course this is an ideal case that can not be realized. It is even not easy to approximate it, for the pulse voltage generator may not be able to supply the sudden great currents without great changes in its own characteristics.

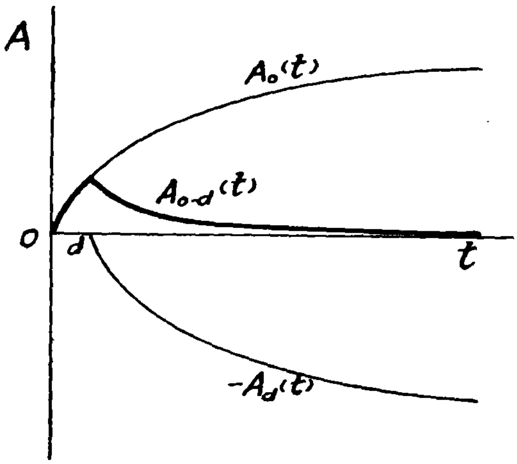
5. Investigation of Series Circuits

Two out of the three elementary constants, R, L and C, in series form three kinds of circuits, while all the three in series makes the fourth. Their indicial admittances and differential indicial admittances are fundamentally characteristic. The oscillatory circuits especially involve many interesting features.

Case 4. *Resistance and Inductance in Series.*

(a) Circuit. (b) Indicial Admittances.





(c) Characteristics.

$N = 1.$

$d_1 = \infty.$

$A_1 = \frac{1}{R}(1 - e^{-Rd/L}).$

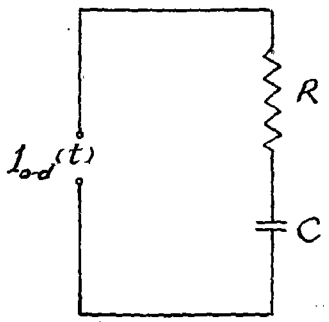
$t_1 = d.$

(d) Notes.

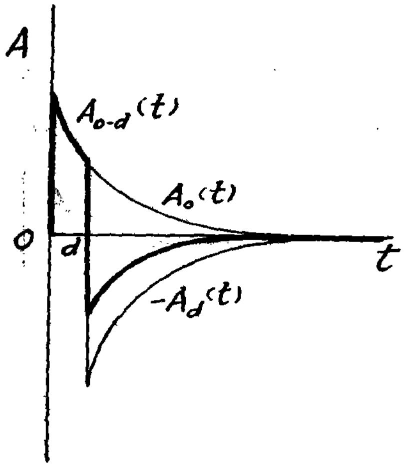
In this case, $A_o(t) = \left.\frac{1}{R}(1-e^{-Rt/L})\right]_{t \geqq 0}$, $A_d(t) = \left.\frac{1}{R}(1-e^{-R(t-d)/L})\right]_{t \geqq d}$. The difference $A_o(t) - A_d(t)$ gives $A_{o-d}(t)$, which has a point of discontinuity at $t=d$.

Case 5. *Resistance and Capacitance in Series.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=2.$

$d_1=d$ $\qquad$ $d_2=\infty.$

$A_1=1/R.$ $\qquad$ $A_2=-\frac{1}{R}(1-e^{-d/(RC)}).$

$t_1=0.$ $\qquad$ $t_2=d.$

(d) Notes.

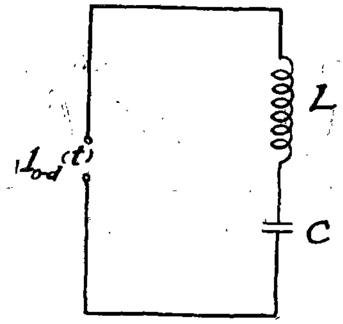
$$\text{Here, } A_o(t)=\frac{1}{R}e^{-t/(RC)}\Big]_{t\geqq 0},$$

$$A_d(t)=\frac{1}{R}e^{-(t-d)/(RC)}\Big]_{t\geqq d}.$$

It is worthwhile to note that the maximum amplitude of the first pulse A_1 is independent of C, while that of the second pulse A_2 is a function of it. $A_{o-d}(t)$ at $t=0$ minus that just before $t=d$ numerically equals that just after $t=d$.

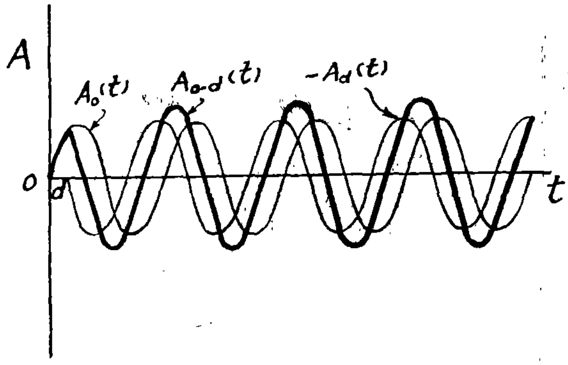
Case 6a. *Inductance and Capacitance in Series.*

(a) Circuit.

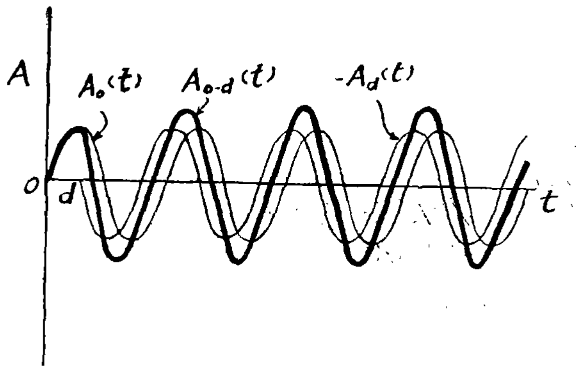


(b) Indicial Admittances.

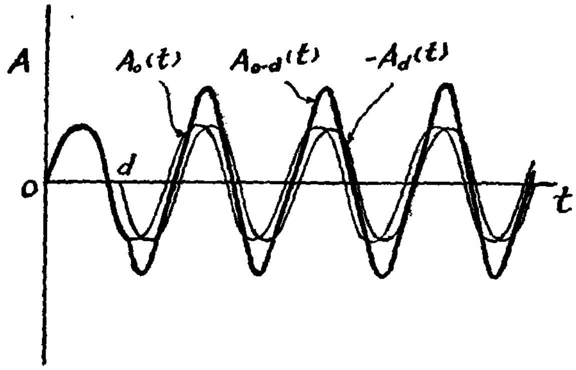
Type (1). $0 < d < \frac{\pi}{2}\sqrt{LC}.$



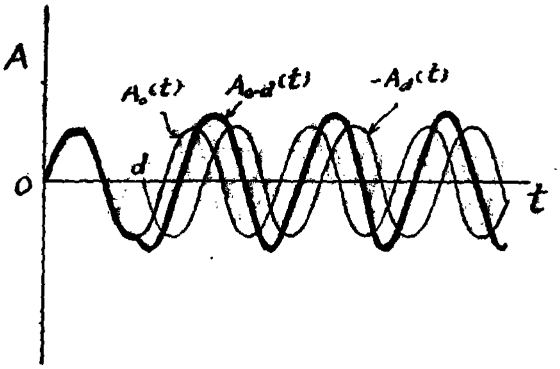
Type (2). $\frac{\pi}{2}\sqrt{LC} < d < \pi\sqrt{LC}.$



Type (3). $\pi\sqrt{LC} < d < 3\frac{\pi}{2}\sqrt{LC}$.



Type (4). $\frac{3\pi}{2}\sqrt{LC} < d < 2\pi\sqrt{LC}$.



(c) Characteristics.

General:

$$\left.\begin{aligned} &N = \infty. \\ &d_n = \pi\sqrt{LC}. \\ &A_n = 2(-1)^{n-1}\sqrt{C/L}\sin(d/2\sqrt{LC}). \\ &t_n = (n-l)\pi\sqrt{LC} + d/2. \\ &d_1 = \frac{\pi}{2}\sqrt{LC} + d/2. \end{aligned}\right\} n = 2, 3, 4, \ldots\ldots$$

Typical:

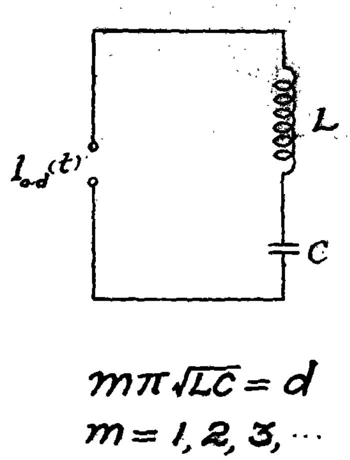
Type	N_-	d_{-n}	A_{-n}	A_1	t_1
(1)	0	—	—	$\sqrt{C/L}\sin(d/\sqrt{LC})$	d
(2)	0	—	—	$\sqrt{C/L}$	$\frac{\pi}{2}\sqrt{LC}$
(3)	1	$\pi\sqrt{LC}$	$\sqrt{C/L}$	$-2\sqrt{C/L}\sin[d/(2\sqrt{LC})]$	$\pi\sqrt{LC}+d/2$
(4)	1	$\pi\sqrt{LC}$	$\sqrt{C/L}$	$A'_1=-\sqrt{C/L}$	$t'_1=\frac{3\pi}{2}\sqrt{LC}$
				$A''_1=-2\sqrt{C/L}\sin[d/(2\sqrt{LC})]$	$t''_1=\pi\sqrt{LC}+d/2.$

(d) Notes.

This is an oscillatory circuit without any damping resistance, which has an indicial admittance $\sqrt{C/L}\sin(t/\sqrt{LC})$. There are many pulses in the differential indicial admittances. Some characteristics of them are different for different relative values of d and $\sqrt{LC}$. For convenience in expressing the general characteristics, the subscript 1 is assigned to the pulse which includes the time $t \gtreqless d$, instead of the physically first pulse. The mathematically first pulse is the first superposed pulse, and the pulses, if any, before which are un-superposed pulses. The number of un-superposed pulses are denoted by N_-, the other of which being denoted by minus numeral subscripts, with -1 for the pulse adjacent to the first superposed pulse. In this way, not only N, d_n, A_n and t_n have common expressions for all the four types, but so has also d_1. For the type (4) the first superposed pulse is double-peaked, either the earlier or the later peak being able to be the greater. So the characteristic expressions of both are given, with a single prime to denote the earlier and a double prime to denote the later peak.

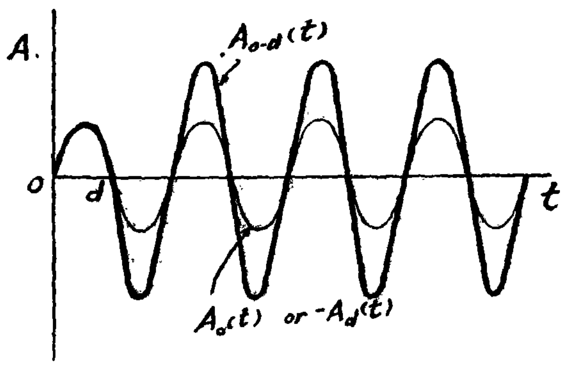
Case 6b. *Inductance and Capacitance in Series, Tuned to* $m\pi\sqrt{LC}=d$.

(a) Circuit.

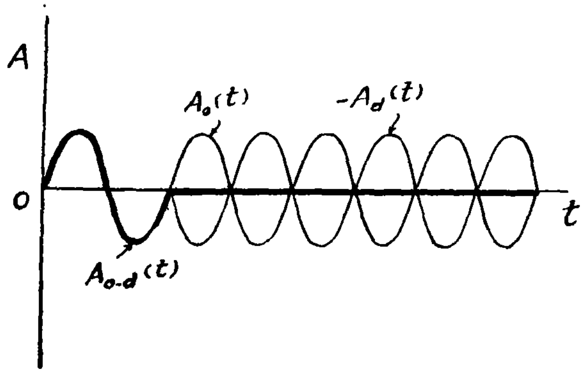


(b) Indicial admittances.

Type (5). m is odd.



Type (6). m is even.



(c) Characteristics.

General:

$N_{-}=m.$
$d_{-n}=\pi\sqrt{LC}.$
$A_{-n}=(-1)^{m-(-n)}\sqrt{C/L}$
$t_{-n}=d+[(-n)+1/2]\,\pi\sqrt{LC}.$

Typical:

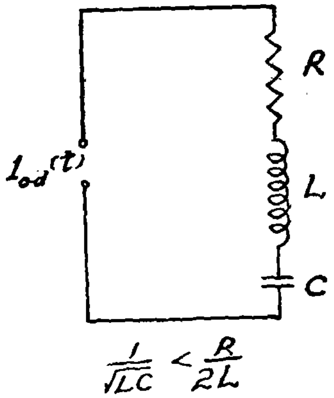
Type	N	d_n	A_n	t_n
(5)	∞	$\pi\sqrt{LC}$	$2\sqrt{C/L}$	$d+(n-1/2)\pi\sqrt{LC}$
(6)	0	—	—	—

(d) Notes.

These are the interesting types, which may find useful applications. For the type (6), it may also be thought mathematically as if there were an infinite number of superposed pulses of zero amplitude.

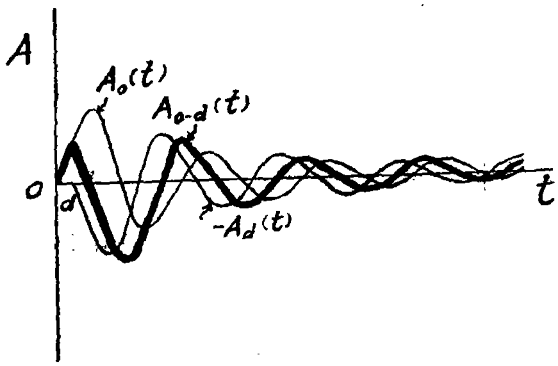
Case 7a. *Resistance, Inductance and Capacitance in Series with Oscillatory Damping.*

(a) Circuit.



(b) Indicial admittances.

Type (1). $0<d<\pi/2\beta$.



(c) Characteristics.

General:

$N=\infty$.

$$\left.\begin{aligned}
d_n &= \frac{\pi}{\sqrt{1/(LC)-R^2/(4L^2)}}.\\
A_n &= \frac{1}{L\beta}\left[e^{-\alpha t_n}\sin\beta t_n - e^{-\alpha(t_n-d)}\sin\beta(t_n-d)\right].\\
t_n &= \frac{1}{\beta}\left[\tan^{-1}\frac{\beta e^{-\alpha d}-\alpha\sin\beta d-\beta\cos\beta d}{\alpha e^{-\alpha d}-\alpha\cos\beta d+\beta\sin\beta d}+(n-1)\pi\right].\\
d_1 &= \frac{1}{\beta}\tan^{-1}\frac{\sin\beta d}{\cos\beta d-e^{-\alpha d}}. \qquad n=2,3,4\cdots
\end{aligned}\right\}$$

Typical:

Type	N_-	d_{-n}	A_{-n}	A_1	t_1
(1)	0	—	—	$\frac{1}{L\beta}e^{-\alpha d}\sin\beta d$	d
(2)	0	—	—	A_m	$\pi/(2\beta)$
(3)	1	π/β	A_m	$A_{n=1}$	$t_{n=1}$
(4)	1	π/β	A_m	$A'_1=-A_m$	$t'_1=3\pi/(2\beta)$
				$A''_1=A_{n=1}$	$t''_1=t_{n=1}$

$$A_m=\frac{1}{L\beta}e^{-\frac{\alpha}{\beta}\tan^{-1}\frac{\beta}{\alpha}}\sin\left[\tan^{-1}\frac{\beta}{\alpha}\right].$$

The subscript $n=1$ means to use the general formula with $n=1$.

(d) Notes.

For this case the indicial admittance is $\frac{1}{L\beta}e^{-\alpha t}\sin\beta t$, where $\alpha=R/2L$ and $\beta=\sqrt{1/(LC)-R^2/(4L^2)}$. This deviates from that of case 6 only in two points, namely, (a) replacement of $1/\sqrt{LC}$ by β, (b) introduction of the damping factor $e^{-\alpha t}$. Here are also four types according to the relative values of d and β, but they are so similar to those of case 6, that only one type is here graphed.

The general formulas for d_1 and d_n are obtained by equating the expressions of $A_o(t)$ and $A_d(t)$, which gives

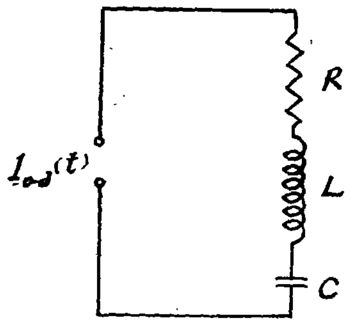
$$\frac{1}{\beta}\tan^{-1}\frac{\sin\beta d}{\cos\beta d - e^{-\alpha d}}$$

for the values of t for zero amplitudes.

The general formulas for t_n and A_n are found by equating the derivative of the expression of $A_o(t)-A_d(t)$ to zero to determine the critical values of t.

Case 7b. *Resistance, Inductance and Capacitance in Series with Critical Damping.*

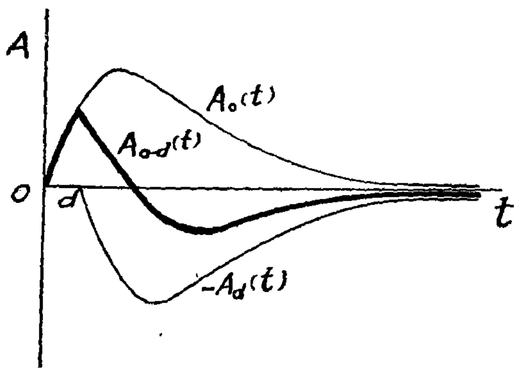
(a) Circuit.



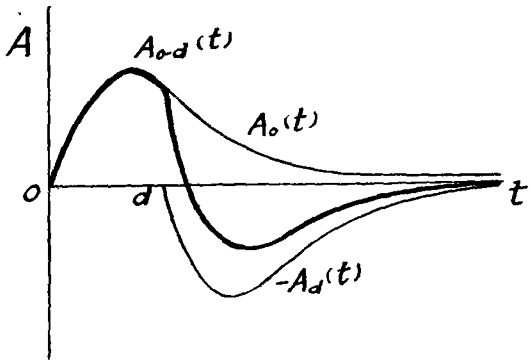
$$\frac{1}{\sqrt{LC}}=\frac{R}{2L}$$

(b) Indicial admittances.

Type (5). $d<2L/R$.



Type (6). $d>2L/R$.



(c) Characteristics.

General:

$$N=2.$$

$$d_1=\frac{d\,e^{Rd/(2L)}}{e^{Rd/(2L)}-1}.$$

$$d_2=\infty.$$

$$A_2=\left(\frac{2}{R}+\frac{d_1}{L}\right)e^{-(1+\alpha d_1)}-\left(\frac{2}{R}+\frac{d_1-d}{L}\right)e^{-(1+\alpha d_1-d)}.$$

$$t_2=\frac{2L}{R}+d_1.$$

Typical:

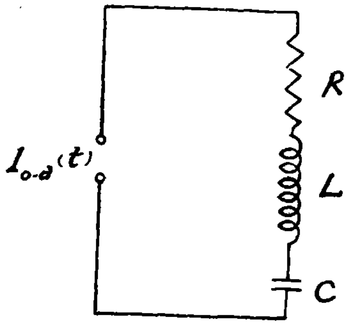
Type	A_1	t_1
(5)	$\frac{d}{L}e^{-Rd/(2L)}$	d
(6)	$2/(eR)$	$2L/R$

(d) Notes.

For this case the indicial admittance is $\frac{t}{L} e^{-Rt/(2L)}$, which has a maximum amplitude at the time $2L/R$. According to the relative values of $2L/R$ and d, some characteristics of the first pulse are different.

Case 7c. *Resistance, Inductance and Capacitance in Series with Logarithmic Damping.*

(a) Circuit.



$$\frac{1}{\sqrt{LC}} > \frac{R}{2L}$$

(b) Indicial admittances.

Type (7). $d < \frac{1}{2\beta} \log \frac{\alpha+\beta}{\alpha-\beta}$.

Type (8). $d > \frac{1}{2\beta} \log \frac{\alpha+\beta}{\alpha-\beta}$.

The graphs have similar features as those of case 7b.

(c) Characteristics.

General:

$N=2$.

$$d_1 = \frac{1}{2\beta} \log \frac{e^{(\alpha+\beta)d}-1}{e^{(\alpha-\beta)d}-1}.$$

$d_2 = \infty$.

$$A_2 = K\left\{ e^{-(\alpha-\beta)t_2} - e^{-(\alpha+\beta)t_2} \right\}.$$

$$t_2 = \frac{1}{2\beta} \log \frac{(\alpha+\beta)(e^{(\alpha+\beta)d}-1)}{(\alpha-\beta)(e^{(\alpha-\beta)d}-1)}.$$

Typical:

Type	A_1	t_1
(7)	$K\left\{e^{-(\alpha-\beta)d}-e^{-(\alpha+\beta)d}\right\}$	d
(8)	$K\left\{e^{-(\alpha-\beta)t_o}-e^{-(\alpha+\beta)t_o}\right\}$	t_o

$$t_o=\frac{1}{2\beta}\log\frac{\alpha+\beta}{\alpha-\beta}.$$

(d) Notes.

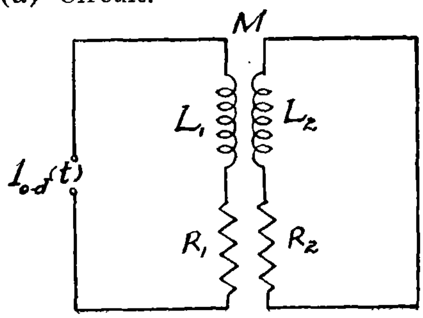
The indicial admittance for this case is $K\left[e^{-(\alpha-\beta)t}-e^{-(\alpha+\beta)t}\right]$ where $K=1/(2\beta L)$, $\alpha=R/(2L)$ and $\beta=\sqrt{R^2/(4L^2)-1/(LC)}$. The critical value of t, t_c, for its maximum amplitude is $(1/2\beta)\log\left[(\alpha+\beta)/(\alpha-\beta)\right]$.

6. Investigation of Coupled Circuit

Only one kind of coupled circuits, namely, mutual-inductively coupled, will be here investigated, for the other kinds may be treated as series-parallel combinations.

Case. 8. *Transformer Circuit.*

(a) Circuit.



(b) Indicial Admittances.

Type (1) $d<\frac{1}{2\beta}\log\frac{\alpha+\beta}{\alpha-\beta}.$

Type (2) $d>\frac{1}{2\beta}\log\frac{\alpha+\beta}{\alpha-\beta}.$

The graphs have features similar to those of case 7*b*.

Both general and typical characteristics are expressed by the same formulas as those in case 7c, the values of K, α, and β being given in the following notes.

(d) Notes.

The indicial admittance of the transformer circuit[3] is

$$K\left[e^{-(\alpha-\beta)t}-e^{-(\alpha+\beta)t}\right],$$

where

$$K=\frac{M}{2\beta(L_1L_2-M^2)},\quad \alpha=\frac{R_1L_2+R_2L_1}{2(L_1L_2-M^2)},$$

and

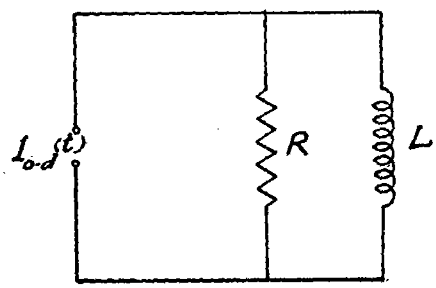
$$\beta=\frac{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}{2(L_1L_2-M^2)}.$$

7. Investigation of Parallel Circuits

All the four possible combinations of simple parallel circuits will here be investigated. To get the differential indicial admittance of a parallel circuit, the order of superposing the indicial admittances may be done in two ways, namely, (a) by taking the difference of the total admittance for $t=0$ and that for $t=d$, or (b) by adding the differential indicial admittances of the branches of the circuit. The latter method will be used, not only for the sake of breaking monotony, but also for its advantageous convenience.

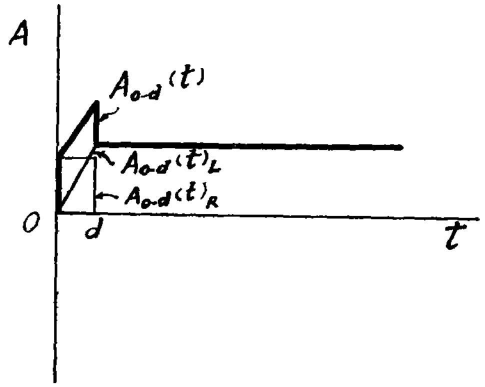
Case 9. *Resistance and Inductance in Parallel.*

(a) Circuit.



3. See Chinese Journal of Physics, Vol. I, No. 3, P. 70 for this formula and for a fuller treatment of this circuit.

(b) Indicial Admittances.



(c) Characteristics.

$N=1.$

$d_1=\infty.$

$A_1=1/R+d/L.$

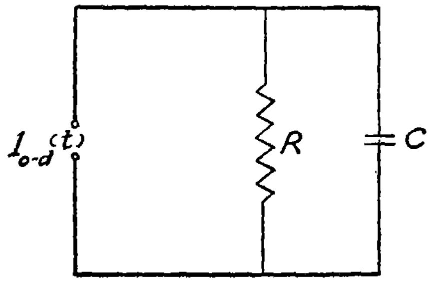
$t_1=d.$

(d) Notes.

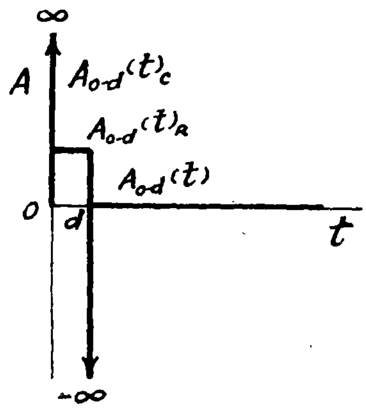
The notation of the differential indicial admittance with a subscript indicating the nature of a branch of a circuit stands for the differential indicial admittance of that branch, thus, $A_{o-d}\ (t)_L$ for the differential indicial admittance of the inductance branch, etc.

Case 10. *Resistance and Capacitance in Parallel.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=2.$

$d_1=d.$ $d_2=0.$

$A_1=\infty.$ $A=-\infty.$

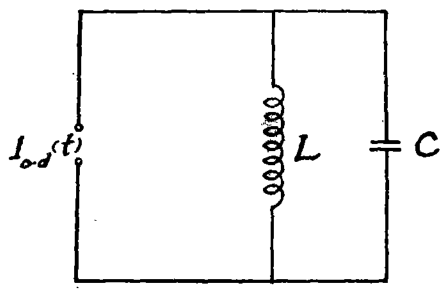
$t_1=0.$ $t_2=d.$

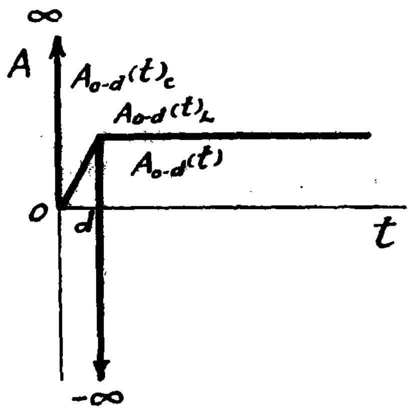
(d) Notes.

As noted before, infinite admittances can not be realized.

Case 11. *Inductance and Capacitance in Parallel.*

(a) Circuit. (b) Indicial Admittances.





(c) Characteristics.

$N=4.$

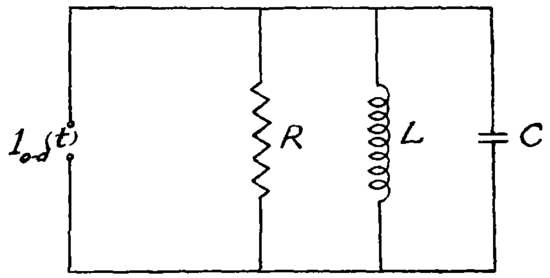
$d_1=0.$	$d_2=d.$	$d_3=0.$	$d_4=\infty.$
$A_1=\infty.$	$A_2=d/L$	$A_3=-\infty.$	$A_4=d/L.$
$t_1=0.$	$t_2=d.$	$t_3=d$	$t_4=d$ to ∞

(d) Notes.

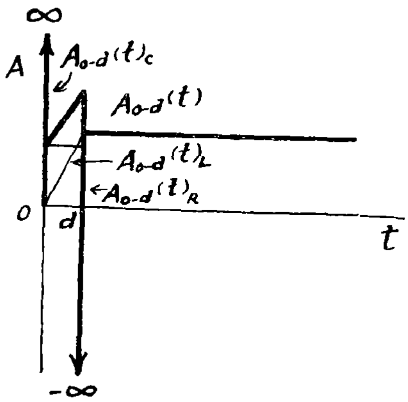
This differential indicial admittance, though only ideal, is very interesting.

Case. 12. *Resistance, Inductance and Capacitance in Parallel.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=3.$		
$d_1=d.$	$d_2=0.$	$d_3=\infty.$
$A_1'=\infty.$	$A_2=-\infty.$	$A_3=d/L.$
$t_1'=0.$	$t_2=d.$	$t_3=d$ to ∞.
$A_1''=1/R+d/L.$		
$t_1''=d$		

(d) Notes.

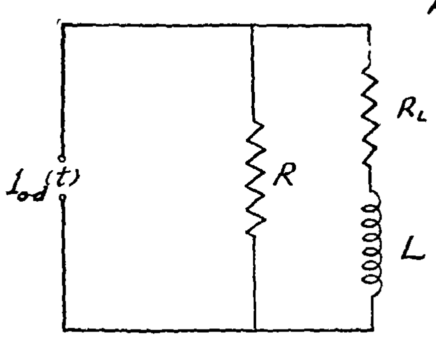
This differential indicial admittance differs from that of L and C in parallel only by increasing the amplitude between 0 and d by the amount of 1/R. However, this modification reduces the first two pulses to one double-peaked pulse.

8. Investigation of Series-Parallel Circuits

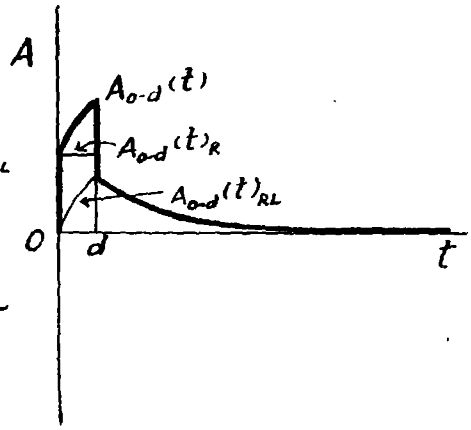
Series-parallel circuits are the practical parallel circuits, for a practical inductance or capacitance must always be treated as to contain some resistance in series. Three two-branch combinations will be here investigated. The three-branch combination is left out, for it differs from the case of resistive inductance and resistive capacitance in parallel only by a simple factor of $1/R$ for the period of 0 to d.

Case 13. *Resistance and Resistive Inductance in Parallel*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=1.$ $\qquad A_1=\frac{1}{R}+\frac{1}{R_L}\left(1-e^{-R_L d/L}\right).$

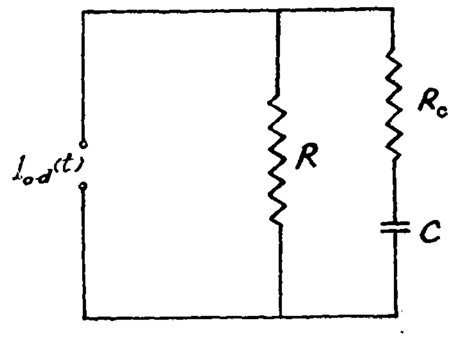
$d_1=\infty.$ $\qquad t_1=d.$

(d) Notes.

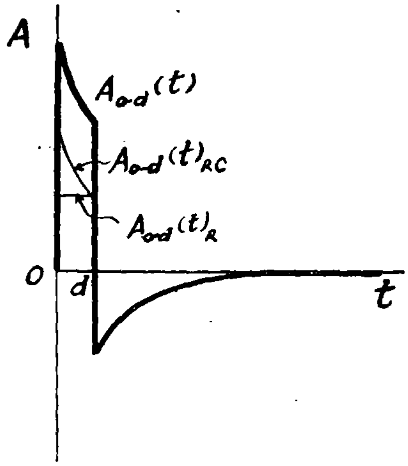
Inductive damping is characterized by the fact that $A_{o-d}(t)$ approaches zero at $t=\infty$ on the positive side.

Case 14. *Resistance and Resitsive Capacitance in Parallel.*

(a) Circuit.



(b) Indicial Admittances.



(c) Characteristics.

$N=2.$

$d_1=d. \qquad d_2=\infty.$

$A_1=1/R+1/R_c. \qquad A_2=-\frac{1}{R_c}\left[1-e^{-d/(R_cC)}\right].$

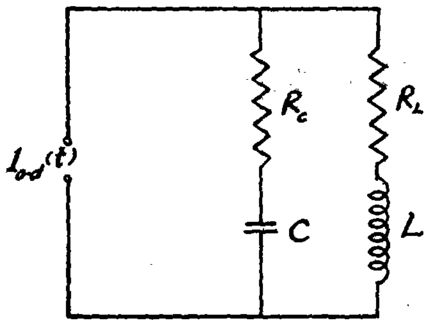
$t_1=0. \qquad t_2=d.$

(d) Notes.

Capacitive damping is characterized by the fact that $A_{o-d}(t)$ approaches zero at $t=\infty$ on the negative side.

Case 15a. *Resistive Inductance and Resistive Capacitance in Parallel, with Predominate Inductive Damping Constant.*

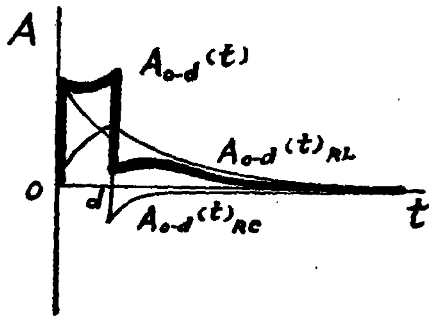
(a) Circuit.



$$\frac{R_L}{L} < \frac{1}{R_c C}$$

(b) Indicial Admittances.

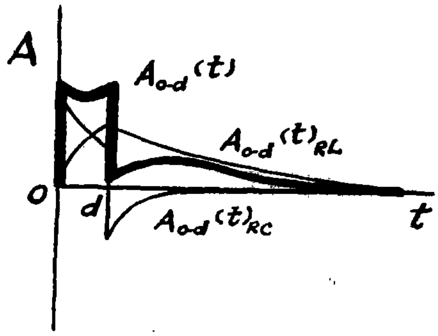
Type (1). $\frac{1}{R_L}\left(1-e^{-R_L d/L}\right) > \frac{1}{R_c}\left(1-e^{-d/(R_c C)}\right).$



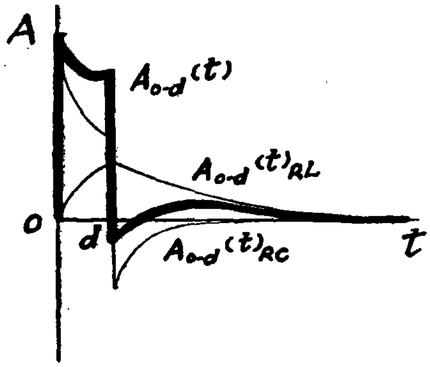
Type (2). $\frac{1}{R_L}\left(1-e^{-R_L d/L}\right) = \frac{1}{R_c}\left(1-e^{-d/(R_c C)}\right).$

Type (3). $\frac{1}{R_L}\left(1-e^{-R_L d/L}\right) < \frac{1}{R_c}\left(1-e^{-d/(R_c C)}\right).$

Type (2)



Type (3)



(c) Characteristics.

Typical:

Type	N	d_1	A_1	t_1	d_2	A_2	t_2	d_3	A_3	t_3
(1)	1	∞	$A'_1=1/R_o$ $A''_1=A_o(d)$* $A'''_1=A_{o-d}(t_o)$	$t'_1=0$ $t''_1=d$ $t'''_1=t_o$	—	—	—	—	—	—
(2)	2	d	$A'_1=1/R_o$ $A''_1=A_o(d)$	$t'_1=0$ $t''_1=d$	∞	$A_{o-d}(t_o)$	t_o	—	—	—
(3)	3	d	$A'_1=1/R_o$ $A''_1=A_o(d)$*	$t'_1=0$ $t''_1=d$	t_o-d	$A_{o-d}(d)$	d	∞	$A_{o-d}(t_o)$	t_o

*A''_1 is absent when $d \lesssim \frac{1}{1/(R_oC)-R_L/L}\log\frac{L}{R^2_oC}$, the latter denoting the time for the minimum value of $A_o(t)$.

$$A_o(d)=\frac{1}{R_L}(1-e^{-R_Ld/L})+\frac{1}{R_c}e^{-d/(R_cC)}.$$

$$A_{o-d}(t_c)=\frac{1}{R_L}(e^{R_L d/L}-1)\,e^{-R_L t_c/L}-\frac{1}{R_c}(e^{d/(R_cC)}-1)\,e^{-t_c/(R_cC)}$$

$$t_c=\frac{1}{1/(R_cC)-R_L/L}\log\frac{L(1-e^{d/(R_cC)})}{R^2_cC(1-e^{R_L d/L})}$$

$$t_o = \frac{1}{1/(R_cC) - R_L/L} \log \frac{R_L(1-e^{d/(R_cC)})}{R_c(1-e^{R_L d/L})}$$

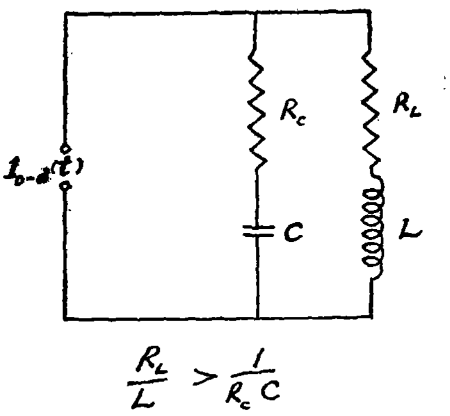
$$A_{o-d}(d) = \frac{1}{R_L}(1-e^{-R_L d/L}) - \frac{1}{R_c}(1-e^{-d/(R_cC)}).$$

(d) Notes.

For this and the following two cases, the indicial admittance is $\frac{1}{R_L}(1-e^{-R_L t/L}) + \frac{1}{R_c}e^{-t/(R_cC)}$, and the types are divided according to the relative values of $A_{o-d}(t)_{RL}$ and $A_{o-d}(t)_{RC}$ at the time just after $t=d$. t_c is the time for the critical value of $A_{o-d}(t)$, and t_o the time at which $A_{o-d}(t)$ equals zero when $t>d$.

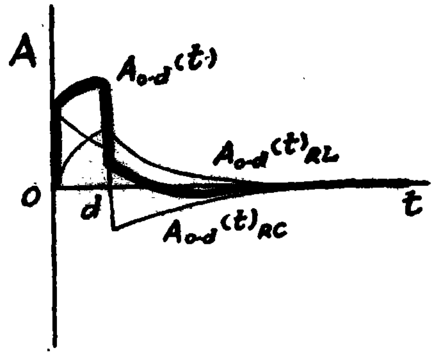
Case 15b. *Resistive Inductance and Resistive Capacitance in Parallel, with Predominate Capacitive Damping Constant.*

(a) Circuit.

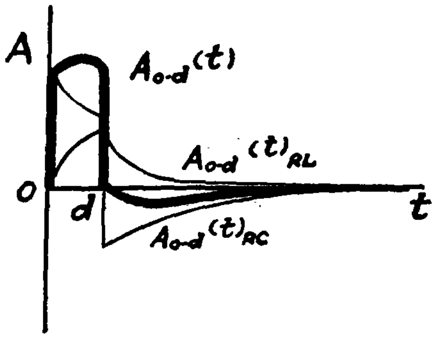


(b) Indicial Admittances.

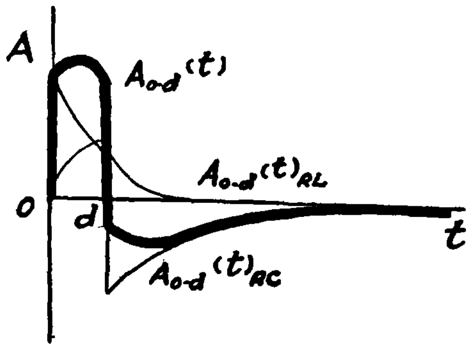
Type (4). $\frac{1}{R_L}(1-e^{-R_L d/L}) > \frac{1}{R_c}(1-e^{-d/(R_c C)})$.



Type (5). $\frac{1}{R_L}(1-e^{-R_L d/L}) = \frac{1}{R_c}(1-e^{-d/(R_c C)})$.



Type (6). $\frac{1}{R_L}(1-e^{-R_L d/L}) < \frac{1}{R_c}(1-e^{-d/(R_c C)})$.



(c) Characteristics.

General:

$N=2.$ $d_2=\infty$ $A_2=A_{o-d}(t_c),$

$$t_c=\frac{1}{R_L/L-1(R_cC)}\log\frac{R^2{}_cC(1-e^{R_L d/L})}{L(1-e^{d/(R_cC)})}.$$

$t_2=t_c.$

Typical:

Type	d_1	A_1	t_1
(4)	t_o	$A_o(d)$, when $d\lesseqgtr t_{oc}$; $A_o(t_{oc})$, when $d\gtreqless t_{oc}$.	d t_{oc}
(5)	d	$A_o(t_{oc})$	t_{oc}
(6)	d	$A_o(d)$, when $d\lesseqgtr t_{oc}$; $A_o(t_{oc})$, when $d\gtreqless t_{oc}$.	d t_{oc}

$$t_o=\frac{1}{R_L/L-1/(R_cC)}\log\frac{R_c(1-e^{R_L d/L})}{R_L(1-e^{d/(R_cC)})}.$$

$$t_{oc}=\frac{1}{R_L/L-1/(R_cC)}\log\frac{R^2{}_cC}{L}.$$

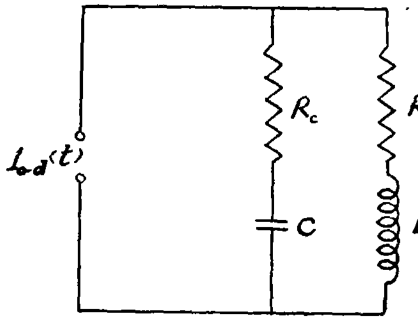
Notes.

Remembering $A_o(t)=\frac{1}{R_L}(1-e^{-R_L t/L})+\frac{1}{R_c}e^{-t/(R_cC)},$

the expressions given above can be easily derived. t_{oc} is the time for the maximum value of $A_o(t)$.

Case 15c. *Resistive Inductance and Resistive Capacitance in Parallel, with Equal Damping Constant.*

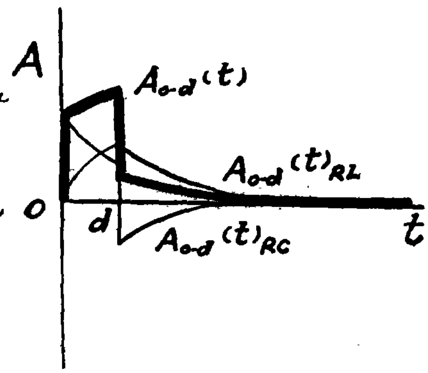
(a) Circuit.



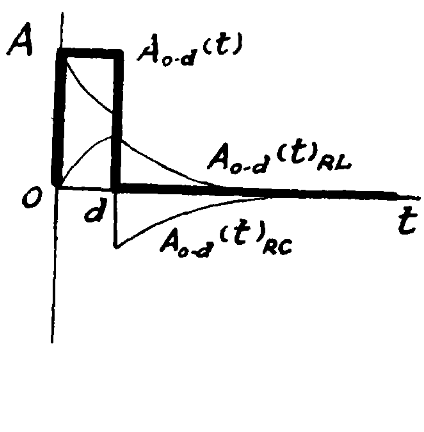
$$\frac{R_L}{L} = \frac{1}{R_c C}$$

(b) Indicial Admittances.

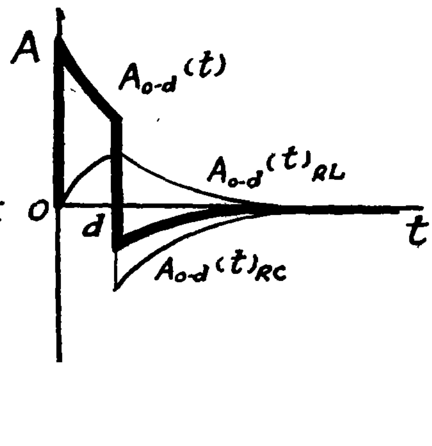
Type (7). $1/R_L > 1/R_c$.



Type (8). $1/R_L = 1/R_c$.



Type (9). $1/R_L < 1/R_c$.



(c) Characteristics.

Type	N	d_1	A_1	t_1	d_2	A_2	t_2
(7)	1	∞	$A_o(d)$	d	—	—	—
(8)	1	d	$1/R_o$	o to d	—	—	—
(9)	2	d	$1/R_o$	0	∞	$A_{o-d}(d)$	d

$$A_o(d)=\frac{1}{R_L}(1-e^{-R_L d/L})+\frac{1}{R_c}e^{-d/(R_cC)}.$$

$$A_{o-d}(d)=\frac{1}{R_L}(1-e^{-R_L d/L})-\frac{1}{R_c}(1-e^{-d/(R_cC)}).$$

(d) Notes.

In this case, the damping constants though being equal, $A_{o-d}(t)$ may have apparent inductive or capacitive damping characteristic due to the predominate amplitude in the respective branch.

As $R_L/L=1/R_oC$, the criterion for types is reduced to a comparison of the values of $1/R_L$ and $1/R_c$, and becomes independent of d. Specially interesting is the type (8), which represents an equivalent resistance circuit.

9. Concluding Remarks

As the application of indicial admittance is not limited to electric circuits, but also to heat, air, hydraulic, mechanical circuits, etc., the results listed above may also be applied to them, for instance, the case 7 of resistance, inductance and capacitance in series brings out the principle of isochronism of a pendulum.

While the investigations have been carried out for most of the important circuits, fifteen in number, covering twenty cases and thirty-six types, it is impossible to exhaust the combinations. However, understanding the cases already investigated, one can easily extend this method of investigation to any particular complicated combination or net-work that one wishes to investigate.

On one hand this paper may be criticized as exercise-like, while on the other hand the importance and variety of the results make it look like some sort of hand-book and mathematical table. The writer is confident that the time spent in this investigation is not wasted. It may not even be too optimistic to think that some day something like the content of this paper may become a chapter, or at least an appendix, of some advanced text-book.

Most of the interesting types of graphs are experimentally checked. When time permits, one experimental graph for each type will be prepared and presented.

10. Acknowledgment

The writer wishes to thank Mr. Ngaisi H. Chang (張煦) and Mr. Pe-Hsien Liang (梁百先) for carefully checking of the numerous mathematical expressions.

移動,與電場所成之直線關係,$m^2S—n^2S$ 系線在 σ 偏極面之出現,及嚴翁二氏視察得 Na 之 $3\,^2S—n\,^2D$ 線之分為 n—2 線等現象,從量子力學之微擾理論,加以討論及解釋。

水銀分子光譜的光強和在 2482 A.U. 的光帶之成因

周同慶 趙廣增

水銀分子在 2482 A.U. 左右有組光帶,是水銀分子伊洪的還是水銀分子的這問題,我們用光譜强度的測量法解决了。將供作光源的通電管內的電流或水銀氣壓依次的改變,我們發現 2482 A.U. 光帶的强度的改變和旁的已知的分子光帶絕然不同。那些不同之點只要引用『分子伊洪是 2482 光帶的原主』這說法,就都明白了我們更進一步問放出 2482 光帶的是那個高能力階位理論和實驗的結果指示出一個在最低能力階位的原子伊洪 (Hg^+) 和一個在 $3P_1$ 能力階位的原子所結合成的分子伊洪是能放 2482 光帶的高能力階位。

H^- 之吸收係數

任之恭

本文根據波動力學計算 H^- 之吸收係數若 H^- 吸收較短於 $\lambda=17254$ Å 光線時,吾人應得一極寬連續光譜。依本文極簡單計算,其係數大都在 10^{-17} cm^2 附近。

單個較差脈壓所發之脈流

陳茂康

代表過渡現象之算式,有一式可名曰單變函數,本

文用二單變函數之較,以表驟上驟下之脈壓;而按重疊原理;以研究此種電壓在各電路上所發之脈流。玆所研究者,有十五不同之電路,計刮二十類三十六種。其間插有許多有趣之圖示,所得各脈流之特性,以一有秩序之方法,皆用算式表出之。

低頻濾波器之瞬流

朱物華　張仲桂

此篇先推求收端加電阻時,低頻濾波器瞬流之公式.依此公式算出之圖與用陰極光示波器映出之曲線相符合.自推算之結果,可得下列結論:

(一) 在濾波器收端電阻漸加時,瞬流各項之挫率漸互異.其數量由低頻項至隔阻頻之項順序漸減;其最小數仍比收端無電阻時之挫率 (R/2L) 爲大.故瞬流終必變爲隔阻頻之電流;而較收端無電阻時易于消滅.

(二) 當濾波器增加一段時,瞬流之項數亦加一.所加項之挫率皆比前有者爲小.故少段濾波器之瞬流易于消滅.

(三) 在隔阻頻後瞬流之數量與在其前者相彷恆較隔阻頻後之安定數量大數十倍.故濾波之特性僅能見之于安定狀態.

北平泉水與自流井水所含射氣量之測定

徐允貴　謝玉銘

應用斯密特之"搖動法,"測定北平鄰近之泉水,如湯山溫泉,溫泉,玉泉山水及清華,燕京,協和醫諸校內自流井所含之射氣量。諸泉源或僅含鐳射氣,釷射氣,或二種射氣皆有之。民國二十一年與二十四年,曾測驗諸泉水各一次,所含之射氣量,絕少改變。