TRANSIENTS OF DISSIPATIVE LOW-PASS ELECTRIC WAVE FILTER WITH A TERMINATING RESISTANCE.

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Abstract

Formulas are derived for the solution of the transient currents of dissipative low-pass T-type electric wave filters. Oscillograms taken by cathode ray oscillograph for d-c. and a-c. cases are found to agree with results calculated from these formulas. From these calculations, the following conclusions are derived. When terminating resistance is gradually increased from O, the damping constants of the sine terms begin to differ from each other, ranging in decreasing magnitude from term of the lowest frequency to the last term of cut-off frequency. Hence the transient is ultimately of the cut-off frequency. At cut-off frequency, this constant is near to but greater than $R/2L$. For each increase of section, there is introduced an additional sine term with smaller damping constant. Therefore transients die out faster in filters of smaller number of sections. Since transient amplitudes are of the same order of magnitude before and after cut-off, filtering property only exists in the steady states.

Transients of non-dissipative electric wave filters were first treated by John R. Carson in 1923 in Bell System Technical Journal. In this treatment he considered the filters to be terminated by a transducer having at one end the same image impedance as that of wave filters and at the other end constant resistance image impedance within most part of transmission range. Then the formulas for currents at any section were deduced on the basis of infinite number of sections. But in wave filters, it is very important to know the behavior of receiving end current in the constant resistance termination. This has not been solved, for a transducer is a complicated network, the solution of whose indicial admittance is very difficult. Furthermore, beyond cut-off, the image impedance of a transducer is far from being a constant resistance, and the basis of infinite number of sections is not quite justified. In 1935, E. Weber and M. J. Di Toro (Electrical Engineering, June, 1935) calculated the transient current of low-pass filters of finite number of sections with a terminating resistance of non-dissipative case. Therefore it is worthwhile to solve the transients for dissipative case.

Derivation of formulas and calculation of transient currents

In figure O, let $2L$=total inductance per section, C=capacity per section, $2R$=total resistance of the series inductance $=2K_R\sqrt{\frac{L}{C}}$, r=resistance of $C=K_r\sqrt{\frac{L}{C}}$, R_o=terminating resistance $=K_{Ro}\sqrt{\frac{L}{C}}$ where K_R, K_r and K_{Ro} are any constants.

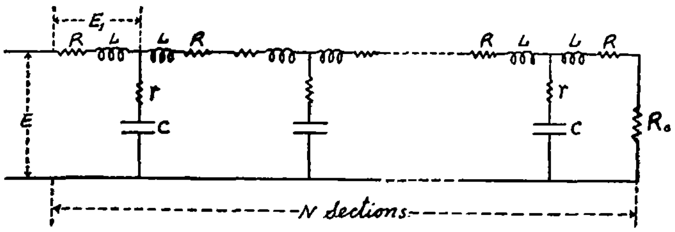


Fig. O.

The sectional angle θ and image impedance Z_o **are given** respectively by $\theta=2\sinh^{-1}\sqrt{\frac{Z_1}{2Z_2}}$, and $Z_o=\sqrt{Z_1^2+2Z_1Z_2}$. In the transient state, $Z_2=R+LP$, $Z_2=r+1/CP$, where P=generalized angular velocity. Then

$$Z_1-R=LP,$$

$$Z_2-r=\frac{1}{CP},$$

$$\frac{L}{C}=(Z_1-K_R\sqrt{L/C})(Z_2-K_2\sqrt{L/C})$$
$$=Z_1Z_2-(K_rZ_1+K_RZ_2)\sqrt{L/C}+K_RK_rL/C.$$

From this equation,

$$\sqrt{\frac{L}{C}}=[\sqrt{K_r^2Z_1^2+K_R^2Z_2^2+2K_RK_rZ_1Z_2+4Z_1Z_2(1-K_RK_r)}$$
$$-(K_RZ_1+K_rZ_2)]/2(1-K_RK_r).$$

Now $\tanh\theta'=\frac{R_o}{Z_o}=\frac{K_{Ro}\sqrt{\frac{L}{C}}}{\sqrt{Z_1^2+2Z_1Z_2}}$

$$=\frac{K_{Ro}}{2(1-K_RK_r)}\left[\sqrt{\frac{K_r^2Z_1^2+K_r^2Z_2^2+(4-2K_RK_r)Z_1Z_2}{Z_1^2+2Z_1Z_2}}-\frac{K_rZ_1+K_RZ_2}{\sqrt{Z_1^2+2Z_1Z_2}}\right],$$

where θ' is the angle included by R_o. Since

$\sinh\frac{\theta}{2}=\sqrt{\frac{Z_1}{2Z_2}}$, $\cosh\frac{\theta}{2}=\sqrt{\frac{Z_1^2+2Z_1Z_2}{2Z_1Z_2}}$, $\tanh\frac{\theta}{2}=\frac{Z_1}{\sqrt{Z_1^2+2Z_1Z_2}}$,

$\sinh\theta=\frac{\sqrt{Z_1^2+2Z_1Z_2}}{Z_2}$, $\cosh\theta=\frac{Z_1+Z_2}{Z_2}$. Therefore

$$\tanh\theta'=R_o/Z_o$$
$$=[K_{Ro}/2(1-K_RK_r)][-K_r\tanh\frac{\theta}{2}-\frac{K_R}{\sinh\theta}$$
$$+\sqrt{Kr^2\tanh^2\frac{\theta}{2}+\frac{K_R^2}{\sinh^2\theta}+\frac{2-K_RK_r}{\cosh^2\frac{\theta}{2}}}].$$

In practical cases, K_r and K_R are very small, and their squares are negligible. Then

$$\tanh\theta'=\frac{K_{Ro}}{2(1-K_RK_r)}[-K_r\tanh\frac{\theta}{2}-\frac{K_R}{\sinh\theta}+\sqrt{2-K_RK_r}/\cosh\frac{\theta}{2}].$$

Now the sending-end and receiving end currents are respectively given by $I_s=\frac{E}{Z_o\tanh(N\theta+\theta')}$ and $I_r=\frac{E\cosh\theta'}{Z_o\sinh(N\theta+\theta')}$, where E si the sending-end voltage and N is the number of sections. To separate these admittance functions into partial fractions, the denominators must be factored. Let

$$Z_o\tanh(N\theta+\theta')=0,$$

then $$\tanh N\theta=-\tanh\theta'$$

$$=\frac{K_{R_0}\,[2K_R\,\sinh^2\frac{\theta}{2}+K_R-2\,\sinh\frac{\theta}{2}\sqrt{2-K_R\,K_r}]}{[2(1-K_RK_r)\,\sinh\theta]}$$

$=\sinh N\theta/\cosh N\theta$.

Clearing of fractions and rearranging,

$$[1+\frac{K_{R_0}K_r}{2(1-K_RK_r)}]\cosh(N-1)\theta$$

$$-[1-\frac{K_{R_0}K_r}{2(1-K_RK_r)}]\cosh(N+1)\theta+\frac{K_{R_0}(K_R-K_r)}{1-K_RK_r}\cosh N\theta$$

$$=\frac{K_{R_0}\sqrt{2-K_RK_r}}{1-K_RK_r}[\sinh(N+\tfrac{1}{2})\theta-\sinh(N-\tfrac{1}{2})\theta],\cdots\cdots\cdots(1).$$

Substituting $a+jb$ for θ in (1), and separating the real and imaginery parts,

$$[1+\frac{K_{R_0}K_r}{2(1-K_RK_r)}]\cosh(N+1)a\cos(N-1)b$$

$$-\left[1-\frac{K_{R_0}K_r}{2(1-K_RK_r)}\right]\cosh(N+1)a\cos(N+1)b$$

$$+\frac{K_{R_0}(K_R-K_r)}{1-K_RK_r}\cosh Na\cos Nb$$

$$=\frac{K_{R_0}\sqrt{2-K_RK_r}}{1-K_RK_r}[\sinh(N+\tfrac{1}{2})a\cos(N+\tfrac{1}{2})a$$

$$-\sinh(N-\tfrac{1}{2})a\cos(N-\tfrac{1}{2})b],\cdots\cdots\cdots(2),$$

and $$[1+\frac{2(1-K_RK_r)}{K_{R_0}K_r}]\sinh(N-1)a\sin(N-1)b$$

$$-[1-\frac{K_{R_0}K_r}{2(1-K_RK_r)}]\sinh(N+1)a\sin(N-1)b$$

$$+\frac{K_{R_0}(K_R-K_r)}{1-K_RK_r}\sinh Na\sin Nb$$

$$=\frac{K_{R_0}\sqrt{2-K_RK_r}}{1-K_RK_r}[\cosh(N+\tfrac{1}{2})a\sin(N+\tfrac{1}{2})b$$

$$-\cosh(N-\tfrac{1}{2})a\sin(N-\tfrac{1}{2})b],\cdots\cdots\cdots(3).$$

From these three formulas, $N+1$ values of θ can be found out by cut-and-try method, of which one is a negative number [from (1)], and the rest are complex numbers with real parts negative [from (2) and (3)]. Having found θ the admittance functions of I_s and I_r can be expanded into partial fractions as follows:

$$\theta=2\sinh^{-1}\sqrt{\frac{Z_1}{2Z_2}}=2\sinh^{-1}\sqrt{\frac{R+LP}{2(r+1/CP)}}$$
$$=2\sinh^{-1}\sqrt{\frac{K_R\sqrt{LCP}+CLP^2}{2(K_r\sqrt{LCP}+1)}}.$$

Solving for P,

$$P=\frac{-1}{\sqrt{LC}}\left[\frac{K_R}{2}-K_r\sinh^2\frac{\theta}{2}\right]$$
$$-\sqrt{\frac{1}{CL}\left(\frac{K_R}{2}-K_r\sinh^2\frac{\theta}{2}\right)^2+\frac{\theta}{LC}\sinh^2\frac{\theta}{2}}=-X_k\pm jY_k.$$

Then the factors of $Z_\circ\tanh(N\theta+\theta')$ are of the type

$$(P+X_k-jY_k)(P+X_k+jY_k)=P^2+2X_kP+(X_k^2+Y_k^2).$$

$\text{Tanh}\theta'=R_\circ/Z_\circ$

$$=\frac{K_{R\circ}}{\sqrt{(\sqrt{CLP})^2+(2K_R+2K_r)\sqrt{CLP}+(K_R^2+2K_RK_r+2)+2K_R/\sqrt{CLP}}}.$$

From this,

$$\frac{d\tanh\theta'}{dp}=-K_{R\circ}\sqrt{CL}\left[\sqrt{CLP}+K_R+K_r-\frac{K_R}{(\sqrt{CLP})^2}\right]/\left[(\sqrt{CLP})^2\right.$$
$$\left.+2(K_R+K_r)\sqrt{CLP}+(K_R^2+2K_RK_r+2)+\frac{2K_R}{\sqrt{CLP}}\right]^{3/2}.$$

Now $\frac{d\theta}{dP}=\sqrt{CL}\,[K_r(\sqrt{CLP})^2+2\sqrt{CLP}+K_R]/[(\sqrt{LCP})^2+$

$$(2K_r+K_R)\sqrt{CLP}+2\sqrt{K_R\sqrt{CLP}+CLP^2}\,(K_r\sqrt{CLP}+1)]$$

hence $\frac{d\tanh N\theta}{dP}=[N\,\text{sec}\,\text{h}^2N\theta\sqrt{CL}(K_rCLP^2+2\sqrt{CLP}+K_R)]/$

$[\sqrt{(\sqrt{LCP})^2+(2K_r+K_R)\sqrt{CLP}+2\sqrt{K_R\sqrt{CLP}+CLP^2}(K_r\sqrt{CLP}}$
$+1)]$.

$$\frac{1}{Z_\circ\tanh(N\theta+\theta')}=\frac{A}{P+X_\circ}+\sum_{k=1}^{k=N}\frac{B_kP+B_k'}{P^2+2X_kP+(X_k^2+Y_k^2)}$$
$$=\frac{1+\tanh N\theta\tanh\theta'}{Z_\circ(\tanh N\theta+\tanh\theta')},$$

where $A=\left[\dfrac{1+\tanh N\theta\tanh\theta'}{Z_\circ\left(\dfrac{d\tanh N\theta}{dP}+\dfrac{d\tanh N\theta'}{dP}\right)}\right]_{(P=-X_\circ)}$

and $B_kP + B'_k = \left[\dfrac{(1+\tanh N\theta \tanh \theta')(2P+2X_k)}{Z_\circ \left[\dfrac{d\tanh N\theta}{dP} + \dfrac{d\tanh \theta'}{dP}\right]}\right]_{P=P_k}$.

Similarly, $\dfrac{\cosh \theta'}{Z_\circ \sinh (N\theta+\theta')} = \dfrac{\cosh \theta' \operatorname{sech} (N\theta+\theta')}{Z_\circ \tanh (N\theta+\theta')}$

$$= \frac{A}{P+X_\circ} + \sum_{k=1}^{k=N} \frac{B_kP+B_k'}{P^2+2X_kP+X_k^2+Y_k^2},$$

where $A = \left[\dfrac{(1+\tanh N\theta \tanh \theta') \cosh \theta'}{Z_\circ \left(\dfrac{d\tanh N\theta}{dP} + \dfrac{d \tanh \theta'}{dP}\right)}\right]_{(P=-X_\circ)}$

and $B_kP+B_k' = \left\{\dfrac{[(1+\tanh N\theta \tanh \theta')(2P+2X_k) \cosh \theta']}{[Z_\circ\left(\dfrac{d \tanh N\theta}{dP} + \dfrac{d \tanh \theta'}{dP}\right) \cos k\pi]}\right\}_{(P=P_k)}$

The voltage across the first coil,

$$E_1 = I_s (LP + K_R \sqrt{\frac{L}{C}}) = \frac{LP + K_R \sqrt{L/C}}{Z_\circ \tanh (N\theta + \theta')}$$

$$= 1 + \frac{A}{P+X_\circ} + \sum_{k=1}^{k=N} \frac{B_kP+B'_k}{P^2+2X_kP+X_k^2+Y_k^2},$$

where $A = \left[\dfrac{(1+\tanh N\theta \tanh \theta') (LP+K_R\sqrt{L/C})}{Z_\circ \left(\dfrac{d\tanh N\theta}{dP} + \dfrac{d \tanh \theta'}{dP}\right)}\right]_{(P=-X_\circ)}$,

and $B_kP+B_k' = \left\{\dfrac{[(1+\tanh N\theta \tanh\theta')2P+2X_k)(LP+K_R\sqrt{\frac{L}{C}})]}{[Z_\circ(\dfrac{d\tanh N\theta}{dP} + \dfrac{d \tanh \theta'}{dP})]}\right\}_{(P=P_k)}$

Having expanded the admittance functions into partial fractions, the indicial admittances and $a-c$ transients can be solved by the following formulas which are derived from Heaviside's expansoin theorem and superposition formulas. For the admittance functions of the fosms $\dfrac{A}{P+X_\circ}$ and $\dfrac{BP+B'}{P^2+DP+F}$, the indicial admittances are respectively given by

$$A(t) = \frac{A}{X_\circ}(1-\varepsilon^{-_\circ Xt})$$

and $A(t) = \dfrac{B'}{F}(1-\varepsilon^{-.5Dt} \cos Wt) + \dfrac{2BF-B'D}{2FW} \varepsilon^{-.5Dt} \sin Wt$,

where $W = \sqrt{F-25D^2}$.

Under an impressed sending-end voltage of the form $\sin(\omega t+\alpha)$, the $a-c$ transients are given by the following formulas. For an indicial admittance of the type

$$A(t)=B+D\varepsilon^{-Ft},$$

$$I(t)=B\sin(\omega t+\alpha)-D\varepsilon^{-Ft}\sin\theta\cos(\alpha+\theta)$$
$$+D\cos\theta\sin(\omega t+\alpha+\theta),$$

where $\tan\theta = F/\omega$. For an indicial admittance of the type

$$A(t)=G\varepsilon^{-Ht}\sin(Wt+\theta),$$

$I(t)$ is equal to j part of the following expressions:

$$G\varepsilon^{-Ht}\sin(Wt+\theta)(\cos\alpha+j\sin\alpha)$$

$$-\frac{j\omega G\cos\theta\varepsilon^{j\alpha-Ht}}{(H+j\omega)^2+W^2}[(H+j\omega)\sin Wt+W\cos Wt]$$

$$+\frac{j\omega G\sin\theta\varepsilon^{j\alpha-Ht}}{(H+j\omega)^2+W^2}[-(H+j\omega)\cos Wt+W\sin Wt]$$

$$+\frac{j\omega G\varepsilon^{j(\omega t+\alpha)}}{(H+j\omega)^2+W^2}[W\cos\theta+(H-j\omega)\sin\theta].$$

By means of these formulas, the indicial admittances and $a-c$. transients were calculated under various conditions as shown in the following tables.

Table I.

$R=r=O$, $R_o=\sqrt{2}\sqrt{\frac{L}{C}}$, $N=5$. From formulas (1), (2), and (3), $\theta_o=-.44162$, $\theta_1=-.3812+j\,26.2°$, $\theta_2=-.2665+j\,57.1°$, $\theta_3=-.1718+j\,9137°$, $\theta_4=-.0968+j\,126.62°$, and $\theta_5=-.0313+j\,162.182°$. When the impressed voltage is unit $d-c$. volt, or $\sin[(K/\sqrt{CL})+\alpha]$, the calculated current is of the form

$$A\sqrt{\frac{C}{L}},$$

or $A\sqrt{\frac{C}{L}}\sin\left(\frac{Kt}{\sqrt{CL}}+\theta\right)+A_o\sqrt{\frac{C}{L}}\varepsilon^{-\frac{.315}{\sqrt{CL}}t}$

$$+A_1\sqrt{\frac{C}{L}}\,\varepsilon^{-\frac{.2343}{\sqrt{CL}}t}\sin\left(\frac{.3264}{\sqrt{CL}}t+\theta_1\right)$$

$$+A_2\sqrt{\frac{C}{L}}\,\varepsilon^{-\frac{.166}{\sqrt{CL}}t}\sin\left(\frac{.683}{\sqrt{CL}}t+\theta_2\right)$$

$$+A_3\sqrt{\frac{C}{L}}\,\varepsilon^{-\frac{.085}{\sqrt{CL}}t}\sin\left(\frac{1.017}{\sqrt{CL}}t+\theta_3\right)$$

$$+A_4\sqrt{\frac{C}{L}}\,\varepsilon^{-\frac{.0308t}{\sqrt{CL}}}\sin\left(\frac{1.263t}{\sqrt{CL}}+\theta_4\right)$$

$$+A_5\sqrt{\frac{C}{L}}\,\varepsilon^{-\frac{.00343t}{\sqrt{CL}}}\sin\left(\frac{1.4t}{\sqrt{CL}}+\theta_5\right).$$

K and α	D−c.	D−c.	1.75,0°
I	I_r	I	I_r
A	.7114	.706	−.001325
θ			−36.074°
A_0	−1.008	−.2196	+.176
A_2	−1.2844	+.3845	+.3154
θ_1	−2.644°	−51.161°	−51.184°
A_2	+.4896	+.266	−.2314
θ_2	+46.614°	−24.3°	−24.55°
A_3	−.1872	+.1924	+.1957
θ_3	+67.6°	−12.517°	−12.8°
A_4	+.7904	+.157	−.1342
θ_4	+75.348°	−7.733°	−10.225°
A_5	−.02246	+.14224	+.04974
θ_5	+78.303°	−5.916°	−11.1°

Table II.

$r=0$, $R=.1\sqrt{L/C}$, $R_\circ=\sqrt{2}\sqrt{L/C}$, $N=5$. $\theta_0=-.8283$, $\theta_1=-.242+j\,24°$, $\theta_2=-.233+j\,58.3°$, $\theta_3=-.1655+j\,92°$, $\theta_4=-.095+j\,126.85$, $\theta_5=-.03085+j\,162.24°$. The current is of the form

$$A\sqrt{\frac{C}{L}},$$

or $A\sqrt{\frac{C}{L}}\sin\left(\frac{Kt}{\sqrt{CL}}+\theta\right)+A_\circ\sqrt{\frac{C}{L}}\,\varepsilon^{-6547t./\sqrt{CL}}$

$$+A_1\sqrt{\frac{C}{L}}\,\varepsilon^{-.22t/\sqrt{CL}}\sin\left(\frac{.2934t}{\sqrt{CL}}+\theta_1\right)$$
$$+A_2\sqrt{\frac{C}{L}}\,\varepsilon^{-.195t/\sqrt{CL}}\sin\left(\frac{.6905t}{\sqrt{CL}}+\theta_2\right)$$
$$+A_3\sqrt{\frac{C}{L}}\,\varepsilon^{-.1318t/\sqrt{CL}}\sin\left(\frac{1.02t}{\sqrt{CL}}+\theta_3\right)$$
$$+A_4\sqrt{\frac{C}{L}}\,\varepsilon^{-.0801t/\sqrt{CL}}\sin\left(\frac{1.267t}{\sqrt{CL}}+\theta_4\right)$$
$$+A_5\sqrt{\frac{C}{L}}\,\varepsilon^{-.0534t/\sqrt{CL}}\sin\left(\frac{1.397t}{\sqrt{CL}}+\theta_5\right).$$

K and α	D—c.	D—c.	1.397,0°	1.397,90°	1.7462,0°	1.7462,90°
I	I_r	I_s	I_r	I_r	I_r	I_r
A	.41708	.41237	.237	.2368	—.001266	—.001266
θ			—58.63°	+31.38°	+6.089°	+83.911°
A_0	—.0878	—.002786	+.0338	—.01576	+.02886	—.010824
A_1	—1	+.62	+.268	—.0713	+.2144	—.04624
θ_1	+30.675°	—24.76°	—18.5°	—73°	—19.76°	—68.29°
A_2	+.3926	+.2634	—.248	—.1334	—.1886	—.07896
θ_2	+47.03°	—17.33°	—17.61°	+86.63°	—21.1°	+84.03°
A_3	—.17652	+.1882	—.26386	—.195	+.156	+.0924
θ_3	+63.94°	—11.53°	+2.433°	—85.16°	—11.078°	+86.054°
A_4	+.07764	+.15544	—.33622	+.3054	—.1185	—.08688
θ_4	+72.89°	—7.617°	+16.263°	+70.083°	—5.325°	+88.326°
A_5	—.02214	+.1412	+.29	—.2902	+.04894	+.03926
θ_5	+76.73°	—5.48°	+77.83°	—10°	—3.282°	+88.89°

Table III.

$R = .046526\sqrt{L/C}$, $r = .007172\sqrt{L/C}$, $R_\circ=\sqrt{2}\sqrt{L/C}$, $N=5$. $\theta_\circ=-.66009$, $\theta_1=-.306+j\,24°$, $\theta_2=-.25+j\,57.64°$, $\theta_3=-.1687+j\,91.61°$, $\theta_4=-.096+j126.7°$, $\theta_5=-.031+j162.2°$. The current is of the form

$$A\sqrt{\frac{C}{L}}$$

or $A\sqrt{\frac{C}{L}}\sin\left(\frac{Kt}{\sqrt{CL}}+\theta\right)$

$$+A_0\sqrt{\frac{C}{L}}\,\varepsilon^{-.499t/\sqrt{CL}}$$
$$+A_1\sqrt{\frac{C}{L}}\,\varepsilon^{-.2366t/\sqrt{CL}}\sin\left(\frac{.296t}{\sqrt{CL}}+\theta_1\right)$$
$$+A_2\sqrt{\frac{C}{L}}\,\varepsilon^{-.1805t/\sqrt{CL}}\sin\left(\frac{.6875t}{\sqrt{CL}}+\theta_2\right)$$
$$+A_3\sqrt{\frac{C}{L}}\,\varepsilon^{-.1102t/\sqrt{CL}}\sin\left(\frac{1.012t}{\sqrt{CL}}+\theta_3\right)$$
$$+A_4\sqrt{\frac{C}{L}}\,\varepsilon^{-.05925t/\sqrt{CL}}\sin\left(\frac{1.263t}{\sqrt{CL}}+\theta_4\right)$$
$$+A_5\sqrt{\frac{C}{L}}\,\varepsilon^{-.0334t/\sqrt{CL}}\sin\left(\frac{1.397t}{\sqrt{CL}}+\theta_5\right).$$

K and α	D—c.	1.397,68.3°	1.747,0°	1.747,90°	D—c.	1.747,90°
I	I_r	I_r	I_r	I_r	I_s	I_s
A	.5262	.416	−.001842	−.001842	+.5357	+.973
θ		−25°	−10.9°	+78.9°		+3.112°
A_0	−.2975	+.0366	+.07324	−.01925	−.02193	−.001423
A_3	−1.277	+.19425	+.2688	−.06	+.565	−.02747
θ_1	+20.14°	−10.122°	−25°	−78.59°	−37.6°	+43.33°
$A_?$	+.429	−.1862	−.2143	−.08338	+.2633	−.1126
θ_2	+48.49°	+15.96°	−20.27°	+83.11°	−20.415°	+6.33°
A_3	−.1836	+.2202	+.16035	+.09466	+.192	−.0989
θ_3	+66.43°	+38.5°	−11°	+84.9°	−11.93°	+6.58°
A_4	+.08	−.3352	−.1632	−.0877	+.1582	−.1743
θ_4	+78.52°	+61.25°	−2.141°	+89.71°	−3.712°	+7.55°
A_5	−.0219	−.4525	+.0486	+.03895	+.1405	−.2488
θ_5	+73.88°	−55.9°	−9.86°	+81.54°	−10.81°	−3.25°

Table IV.

$R=.046526\sqrt{L/C}$, $R_0=0$, $N=5$. [Formulas of transient currents in this case, were given in an article on the same subjeit by W. Chu in Transactions of World Engineering Congress in Tokyo, 1929]. Under an impressed voltage $\sin\left(\frac{1.747}{\sqrt{CL}}t\right)$,

$$I(t)=-.002316\sqrt{\frac{C}{L}}\sin\left(\frac{1.747t}{\sqrt{CL}}+22.86^\circ\right)$$
$$-.001525\sqrt{\frac{C}{L}}\,\varepsilon^{-.0465t/\sqrt{CL}}$$

$$+\sqrt{\frac{C}{L}}\varepsilon^{-.02325t/\sqrt{CL}}[.03052\sin(\frac{4372t}{\sqrt{CL}}+6.517^\circ)$$

$$-.0706\sin(\frac{.832t}{\sqrt{CL}}+4.177^\circ)+.1317\sin(\frac{1.145t}{\sqrt{CL}}+4.0917^\circ)$$

$$-.2163\sin(\frac{1.345t}{\sqrt{CL}}+4.866^\circ)+.13435\sin(\frac{1.415t}{\sqrt{CL}}+5.462^\circ)].$$

Table V.

The filter under test has the following constants: R = 52.5 ohms = $.046526\sqrt{L/C}$, r = 8.11 ohms = $.007172\sqrt{L/C}$, $R_\circ$ = 1595.8 ohms = $\sqrt{2}\sqrt{L/C}$, $C = .50264\times 10^{-6}$ farad, L=.64 henry and N=5. The current under various conditions has the form A, or

$$A\sin(\omega t+\theta),+A_\circ\varepsilon^{-879t}+A_1\varepsilon^{-417.25t}\sin(522.5t+\theta_1)$$

$$+A_2\varepsilon^{-318t}\sin(1213t+\theta_2)+A_3\varepsilon^{-194.3t}\sin(1787.6t+\theta_3)$$

$$+A_4\varepsilon^{-104.5t}\sin(2228.6t+\theta_4)+A_5\varepsilon^{-58.9t}\sin(2463t+\theta_5)$$

Table VI.

Variation of sectional resistance R and terminating resistance $R_\circ$. For one section with terminating resistance (r=0), the indicial admittance at receiving end is of the form

$$A(t)=A\sqrt{\frac{C}{L}}+A_\circ\sqrt{\frac{C}{L}}\varepsilon^{-d_\circ t/\sqrt{CL}}$$

$$+A_1\sqrt{\frac{C}{L}}\varepsilon^{-d_1 t/\sqrt{CL}}\sin(\frac{W_1 t}{\sqrt{CL}}+\theta_1).$$

$K_{R_\circ}=\sqrt{2}, K_r=0$

K_R	A	A_0	d_0	A_1	d_1	W_1	θ_1
.1	.6194	−.4712	−1.0407	−.464	.28674		18.574°
.04653	.6637	−.5178	.9745	−.4712	.26638	1.2136	17.921°
0	.7071	−.57	.91508	−.474	.24956	1.218	16.963°

$K_R=.1, K_r=0$

$K_{R_\circ}$	A	A_0	d_0	A_1	d_1	W_1	θ_1
.7071	1.1028	−1.0875	.48323	−.392	.21194	1.3522	+2.443°
.35	1.817	−1.817	.27909	−.3634	.1354	1.395	+.343°

Receiving-end currents in milliamperes

ω	D—c.	1202	1787.6	2228.6	2463	2525.8	3079	3079
α		50.15°	68.3°	300.05°	48.93°	141.7°	90°	0°
A	.46607	—.566	+.6322	+.665	+.368	+.1517	—.00163	—.00163
Θ		—57.3°	—34.3°	+28.33°	—25°	+3.78°	+78.9°	—10.9°
A_0	—.2636	+.009867	—.00906	+.07585	+.0324	—.08185	—.01704	+.0648
A_1	—1.131	+.3223	+.1358	+.2428	+.172	—.2705	—.05312	+.238°
Θ_1	+20.14°	+28.37°	+37.14°	—43.815°	—10.122°	—35.64°	—78.59°	—25°
A_2	+.3802	+.659	—.2853	—.227°	—.16475	+.2169	—.07378	—.1897
Θ_2	+48.49°	—61.76°	+68.04°	—51.87°	+15.96°	—37.05°	+83.11°	—20.27°
A_3	—.16255	—.2377	—.7275	+.302	+.1951	—.2112	+.08378	+.142
Θ_3	+66.43°	+26.23°	—36.69°	—46.77°	+38.5°	—32.91°	+84.9°	—11°
A_4	+.0708	+.08223	+.1845	—.7685	—.2968	+.262	—.07762	—.14444
Θ_4	+78.52°	+51.73°	+51.16°	+21.85°	+61.25°	—24.58°	+89.71°	—2.141°
A_5	—.0194	—.02097	—.03938	+.1026	—.4013	—.2896	+.0345	+.043
Θ_5	+73.88°	+50.8°	+54.6°	+89.07°	—55.9°	—8.4°	+81.54°	—9.86°

Sending-end currents in milliamperes

ω	D—c.	1202	1787.6	2228.6	2463	2525.8	3079	D—c. (E_1)
α		50.15°	68.3°	300.05°	48.93°	141.7°	90°	
A	+.4738	+7744	+1.098	+1.943	+3.32°	+2.856°	+.8612	.02467
θ		+49.6°	+63.68°	−65.38°	+20.26°	+78.4°	+3.112°	
A_0	−.01943	+.00073	−.00067	+.0056	+.00239	−.00604	−.00126	.00991
A_2	+.5005	−.1427	−.0602	+.10275	−.08315	−.1203	−.02433	.1995
θ_1	−37.6°	−29.2°	−20.17°	+78.61°	−58°	+86.614°	+43.33°	85.164°
A_2	+.233	−.403	−.1752	+.1382	−.10135	−.133	−.09976	.1845
θ_2	−20.415°	+49.4°	−.81°	+59.36°	−52.79°	+74.1°	+6.33°	80.675°
A_3	+.17	+2485	−.758	+.315	−.2035	−.221	−.08755	.19465
θ_3	−11.93°	−52.12°	+64.95°	+54.9°	−39.76°	+68.59°	+6.58°	81.634°
A_4	+.14	+.1629	+.6565	−1.53	−.588	−.52	−.1543	.2006
θ_4	−3.712°	−30.59°	−16.753°	−60.4°	−21°	+73.11°	+7.55°	86.88°
A_5	+.12438	+.1343	+.249	−.657	−2.576	−1.847	−.2203	.1967
θ_5	−10.811°	−33.94°	−30.54°	+4.37°	+39.7°	+86.93°	−3.25°	78.72°

For 2 sections with $K_R=.1$, $K_R=\sqrt{2}$ and $K_r=0$, the indicial admittance has the form

$$A(t)=A\sqrt{\frac{C}{L}}+A_o\sqrt{\frac{C}{L}}\varepsilon^{-.7616t/\sqrt{CL}}$$
$$+A_1\sqrt{\frac{C}{L}}\varepsilon^{-.38037t/\sqrt{CL}}\sin\left(\frac{.7282t}{\sqrt{CL}}+\theta_1\right)$$
$$+A_2\sqrt{\frac{C}{L}}\varepsilon^{-.09579t/\sqrt{CL}}\sin\left(\frac{1.8255t}{\sqrt{CL}}+\theta_2\right).$$

Experiments for checking some of the formulas derived

The transients were taken by a cathode-ray oscillograph with a moving film camera. As it was not advisable to leave the spot of the cathode ray oscillograph stationary at high intensity, so during operation, there was applied to plates an A. C. voltage of 300 or 400 cycles whose photograph might serve as the time scale of the transients. The connection diagram was shown in figure I, where P, P represented the horizontal plates of the oscillograph, and the starting and stopping switches were those for controlling the motion of the motor of the camera. These two switches together with switches A, B and C were all firmly fastened to a rigid and vertical frame and were controlled by means of a drop weight.

The procedures for taking the picture of *d-c.* transient of the filter were as follows: Switches A & B were thrown to the side *a a′*, starting switch and switch *C* were opened, and stopping switch was closed; the oscillograph was started and *A. C.* voltage was put on. The dropped weight were let fall from a definite height along a rigid and smooth rod; it closed the starting switch, threw switch *A* to the side *b b′* (changing the connection of the deflecting plates from *A. C.* voltage to the filter circuit), closed switch *C* (starting the *D. C.* voltage), threw the switch *B* to the side *b b′* (replacing the *D. C.* voltage on the deflecting plates by the A. C. one) and finally stopped the motor. The photograph was a *D. C.* transient with *A. C.* stationary voltage on each side.

The process for taking the photograph of the *A. C.* transients of the wave filter was more complicated than that for

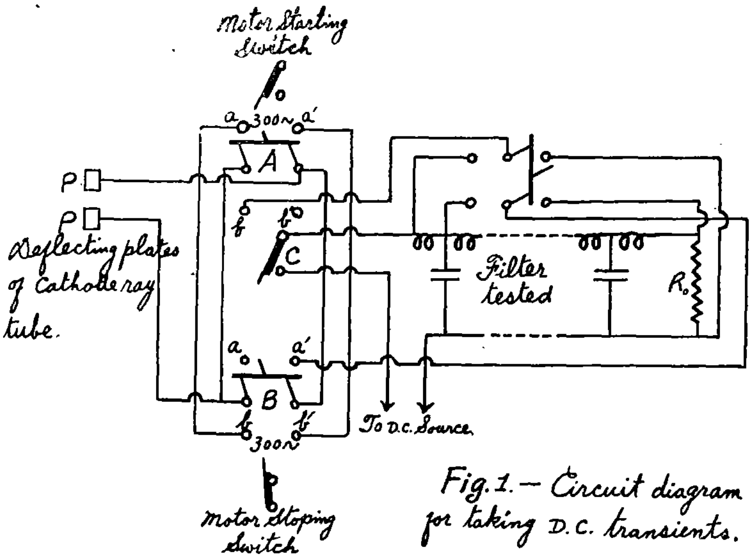


Fig. 1.—Circuit diagram for taking D.C. transients.

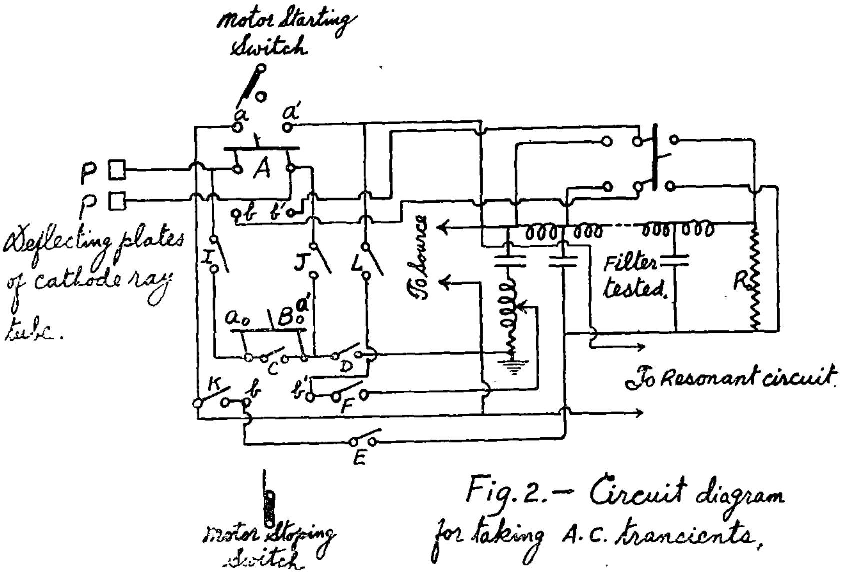


Fig. 2.—Circuit diagram for taking A.C. trancients.

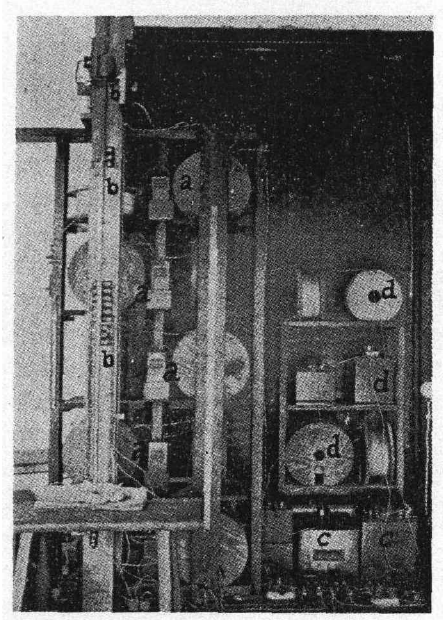


Fig. 1a

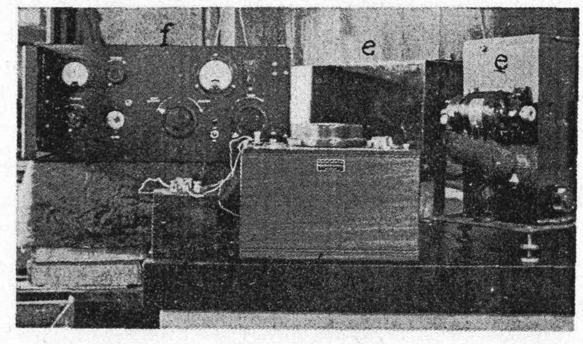


Fig. 2a

taking the *D. C.* ones. The power capacity of the usual low frequency oscillator was found inadequate and the voltage dropped considerably as the load was suddenly taken by the filter. To eliminate this voltage transient, there was interposed between the oscillator and the filter an audio frequency amplifier and a resonant circuit whose inductance might be adjusted and short-circuited for any convenient portion. A switch was arranged so that it connected the filter to the source and short-circuited a portion of the inductance simultaneously. The source was first loaded with the resonant circuit, and by short-circuiting a portion of the inductance, the circuit was made not resonant and the load was thus decreased. When this decrease of load was equal to the increase of load due to the parallel insertion of filter, the voltage of oscillator remained unchanged. Therefore in operation the portion of the inductance to be short-circuited was adjusted until the voltage of the source remained unchanged as the switch to the filter was suddenly closed. To minimize the effect of harmonics in the output wave of the source, a low pass filter was connected to the output terminals.

The characteristic of the transient depended upon the phase angle of impressed *A. C.* voltage. To evaluate this angle, the connections of the deflecting plates of the oscillograph were changed from oscillator voltage to receiving end voltage of filter by a dropped weight which then closed the sending-end switch *B* of the filter. The phase angle of the oscillator voltage when switch *A* was closed was shown on the photograph, and the time required for dropped weight to travel from switch *A* to switch *B*, can be easily measured, and the phase angle of oscillator voltage when sending end switch *B* was closed can be calculated. The connection diagram was shown in figure 2. Switches *A* and *B* and those of the motor were arranged in the same manner as before. The procedure for taking the picture was as follows: Switches *C, D, E* and *F* were closed, and switches *I, J, K* and *L* were opened. The switches *A, B* were thrown to the side *a a′*. The stopping switch was closed and the starting switch was opened. The oscillograph was started and the *A. C.* source was put on. The weight was let fall from a definite height, it started the motor, threw switches *A* and *B* from the side $a\ a'$ to the side $b\ b'$ and stopped the motor finally. The voltages of the sending and receiving ends of the filter were measured by a vacuum tube voltmeter. The picture was a stationary *A. C.* voltage and the *A. C.* transients of the filter with a long dash between them. To determine the time for the dropped weight to travel from *A* to *B*, switches *A*, *B* and those of the motor were arranged as before but switches *C, D, E, F* were opened and switches *I, J, K, L* were now closed. The weight was let fall from the same position as before, a second picture was taken. This was two stationary *A. C.* voltages with a long dash in the middle.

From the second picture, the phase angles at which the first voltage ended and that at which the second voltage started were known. From the first picture, the phase angle at which the stationary *A. C.* voltage ended was known. From these three angles, the phase angle of the impressed *A. C.* voltage under which the transient of the filter was taken can be evaluated. A photograph of the whole apparatus was shown in Figs. 1*a* and 2*a*, where a was the filter tested; *b*, the frame containing dropped weight and switches; *c*, the resonant circuit; *d*, the low pass filter to eliminate the harmonics; *e*, the oscillograph tube and camera; and *f*, the power supply unit of the oscillograph. The filter tested was a 5 section lowpass *T*-type filter with the following constants: $R=52.5$ ohms$=.046526\sqrt{\frac{C}{L}}$, $L=.64$ henry, $C=.50264\times10^{-6}$ farad, $r=8.11$ ohms$=.007172\sqrt{\frac{C}{L}}$, $R=\sqrt{2}\sqrt{\frac{L}{C}}$ $=1595.8$ ohms. The $a-c$. transients were taken mostly at the resonant frequencies of the filter. The picture taken were shown in Figs. 3 to 10. The calculated and experimestal results were poltted side by side in Figs. 11 to 16 & Fig. 21, where the discrepancy was found to be small.

Conclusions

The formulas gave results which checked pretty well with experimental values, and the formulas were correct. From calculated results as shown in tables III and IV, the effect of terminating resistance is to make the damping constants of the damped terms differ from each other. Without termination resistance, the damping constant is R/L for the constant term and $R/2L$ for all the sine terms; and as a result, the transient die out very slowly and in transient state there is no definite frequency. As terminating resistance is gradually increased, the damping constants all increase, and differ from each other, ranging in decreasing magnitude from constant term of 0 frequency to the last term of cut-off frequency. No matter what the external frequency is, the transient is ultimately of the cut-off frequency, because it has the lowest damping constant. But this constant is near to but greater than $R/2L$. Hence the transients now die out faster than the case where termination is absent. As terminating resistance is increased, the deviation of damping constant from $R/2L$ is increased as shown in Table VI, and transients die out faster.

The effect of sectional resistance is very large on the damping constants, as their deviation due to termination is all based on $R/2L$. Thus with $R=0$, the smallest damping constant (table I) is only $\frac{.00343}{\sqrt{CL}}$, and transients die out very slowly.

The effect of number of section can be seen from data in Table VI. For same sectional resistance and terminating resistance, the damped terms increase with number of section. For one section, there are 2 damped terms with very large damping constants. With each increase of section, there is an increase of one transient term of smaller damping constant. Hence transients die out faster in a filter of smaller number of section. Thus in Fig. 4, the *d-c.* transients of I section die out very fast. While sectional resistance, terminal resistance and section effect damping constants considerably, they have no large effects on the transient amplitudes.

From data in Tables I-IV, the transient amplitudes after cut-off were of the same order as those inside the transmission band and the amplitudes are enormous compared with the steady-state terms. Hence filter property only exists in the steady state.

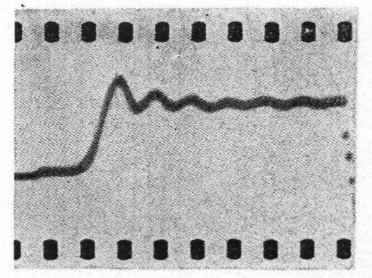


Fig. 3

Indicial admittance for the receiving-end of 5 section Filter

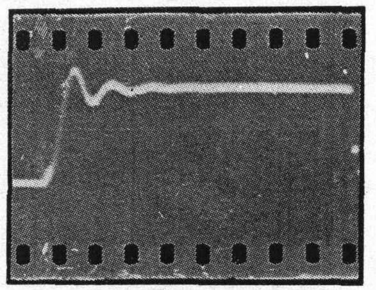


Fig. 4

Indicial admittance for the receiving-end of 1 section Filter

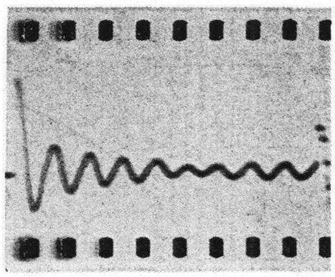


Fig. 5

Voltage across the first coil of 5 section Filter under unit d-c. impressed voltage

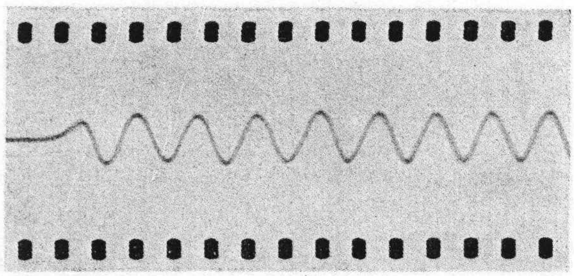
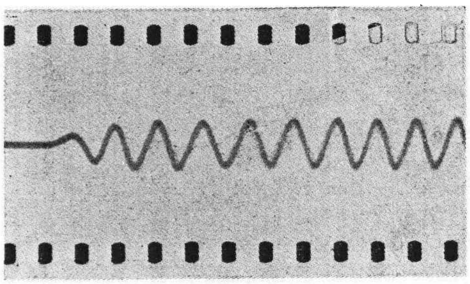


Fig. 6

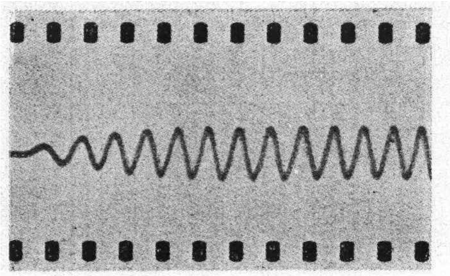
$I_R(t)$ of 5 section Filter under an impressed voltage sin $(1202\,t+50.15°)$

Fig. 7



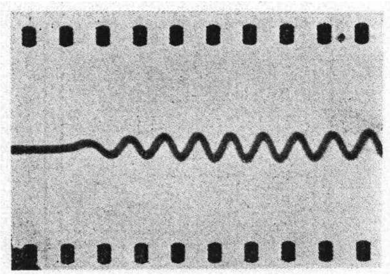
$I_R(t)$ of 5 section Filter under an impressed voltage sin $(1787.6t+68.3°)$

Fig. 8



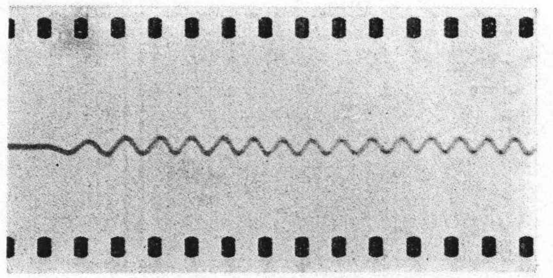
I_R of 5 section Filter under an impressed voltage sin (2228.6t+300.05°)

Fig. 9



$I_R(t)$ of 5 section Filter under an impressed voltage sin (2463t+48.93°)

Fig. 10



$I_R(t)$ of 5 section Filter under an impressed voltage sin (2525.8t+141.7°)

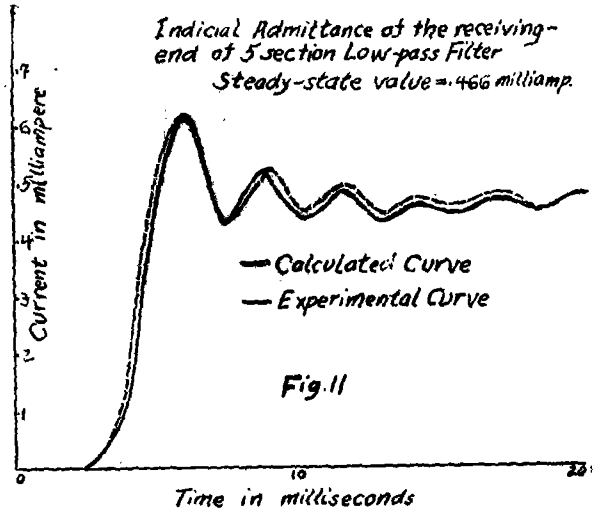

Indicial Admittance of the receiving-end of 5 section Low-pass Filter
Steady-state value = .466 milliamp.
Current in milliampere
Calculated Curve
Experimental Curve
Time in milliseconds

Fig. 11

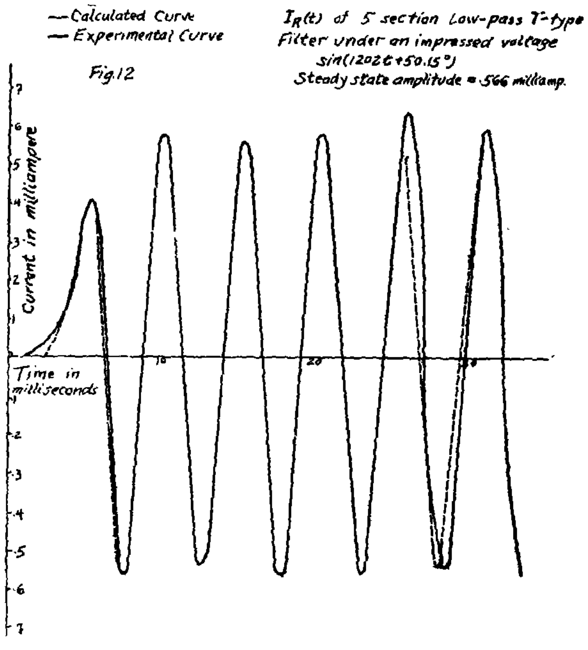
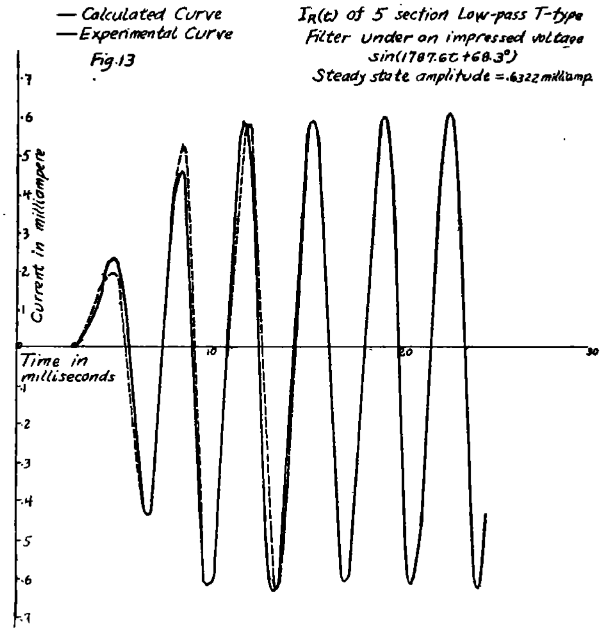
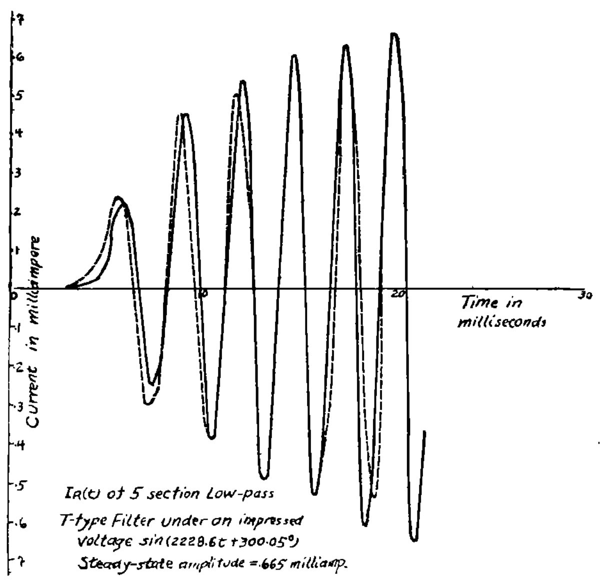

Calculated Curve
Experimental Curve
$I_R(t)$ of 5 section Low-pass T-type Filter under an impressed voltage sin(1202t+50.15°)
Steady state amplitude = .566 milliamp.
Current in milliampere
Time in milliseconds

Fig. 12



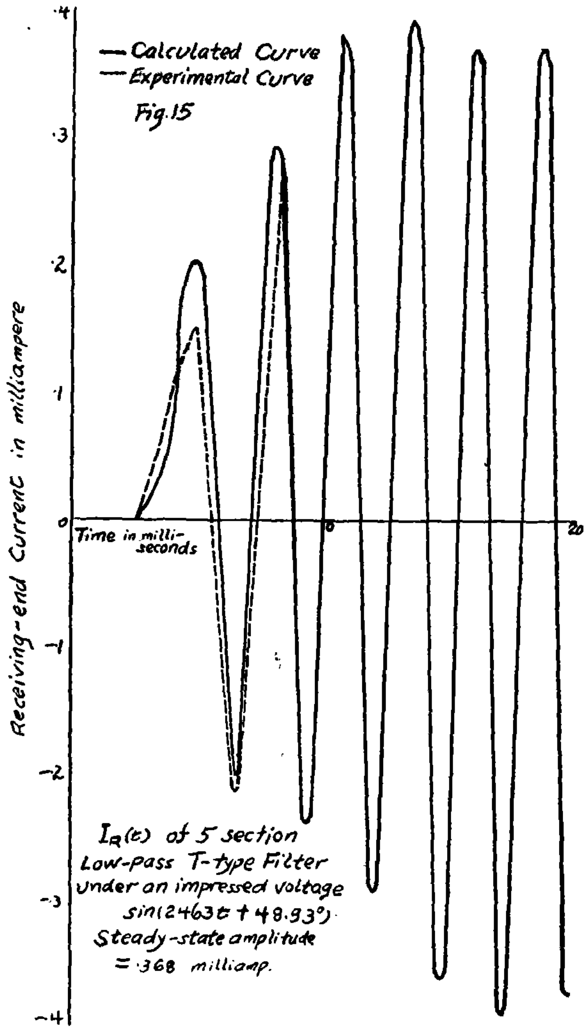
— Calculated Curve
— Experimental Curve
$I_R(t)$ of 5 section Low-pass T-type Filter under an impressed voltage sin(1787.6t +68.3°)
Steady state amplitude =.6322 milliamp.
Current in milliampere
Time in milliseconds
·7
·6
·5
·4
·3
·2
·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
10
20
30

Fig.13

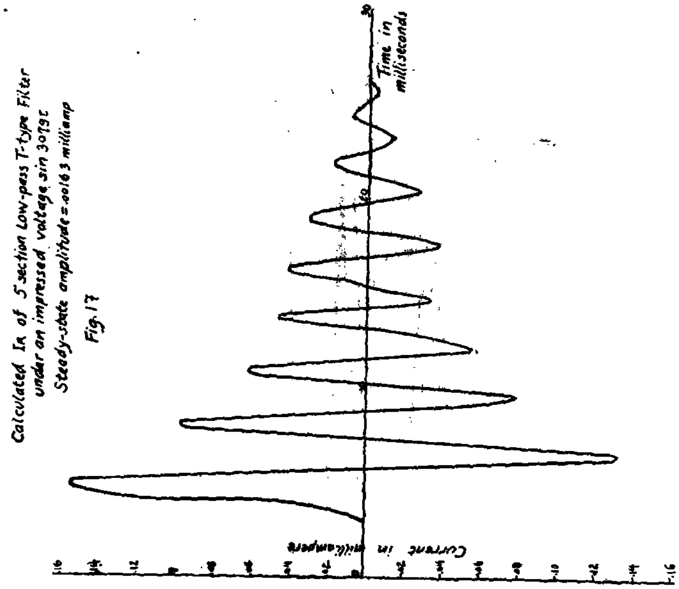


— Calculated Curve
— Experimental Curve
Current in milliampere
Time in milliseconds
$I_R(t)$ of 5 section Low-pass T-type Filter under an impressed voltage sin(2228.6t +300.05°)
Steady-state amplitude =.665 milliamp.
·7
·6
·5
·4
·3
·2
·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
10
20
30

Fig.14

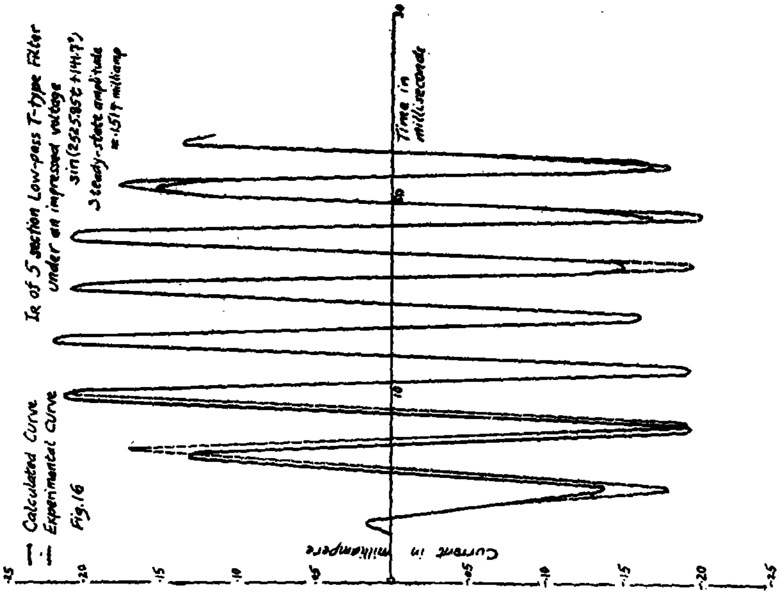


— Calculated Curve
--- Experimental Curve
Fig. 15
·4
·3
·2
·1
0
-·1
-·2
-·3
-·4
Receiving-end Current in milliampere
Time in milli-seconds
10
20
$I_R(t)$ of 5 section
Low-pass T-type Filter
under an impressed voltage
sin(2463t + 48.93°)
Steady-state amplitude
= ·368 milliamp.



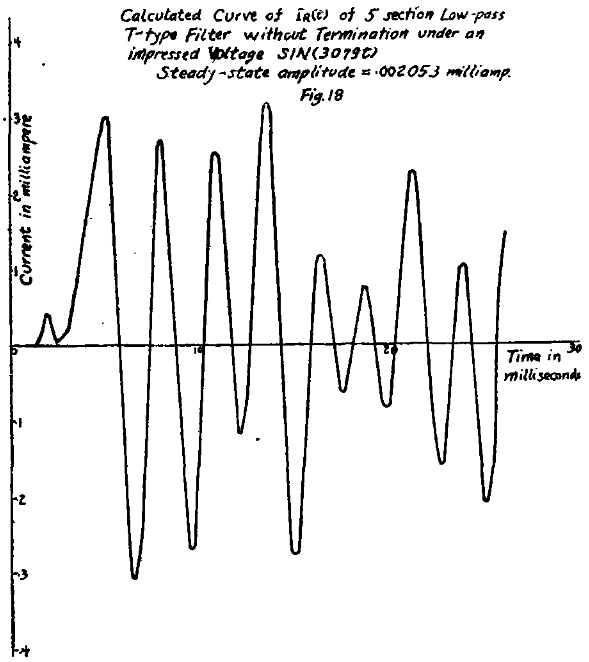
Calculated I_R of 5 section Low-pass T-type Filter under an impressed voltage sin 3079t Steady-state amplitude = .00163 milliamp

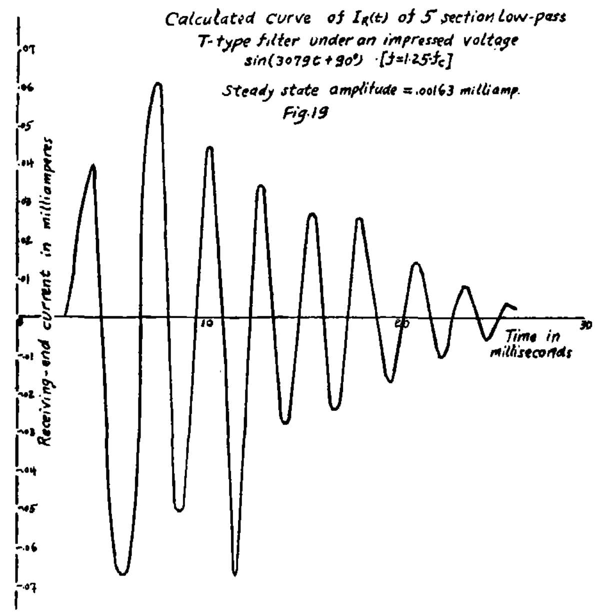
Fig. 17

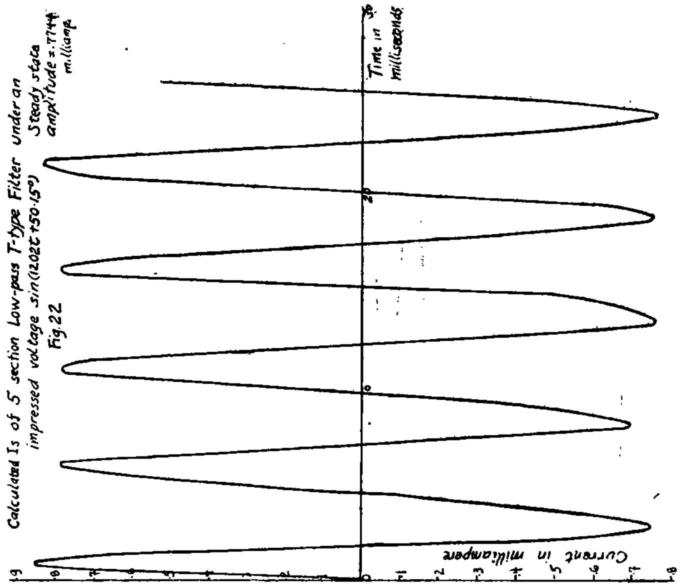


I_R of 5 section Low-pass T-type Filter under an impressed voltage sin(2525.85t+141.7°) Steady-state amplitude = .1519 milliamp

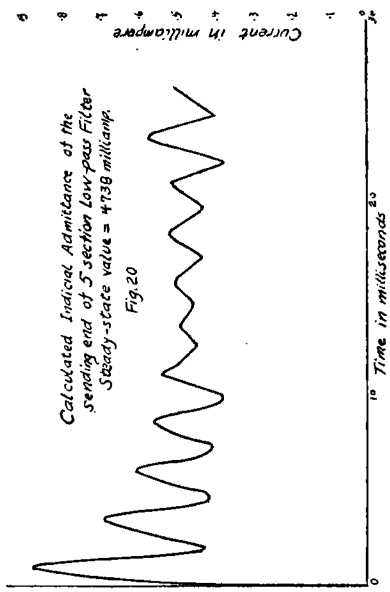
Fig. 16


Calculated Curve of IR(t) of 5 section Low-pass
T-type Filter without Termination under an
impressed Voltage SIN(3079t)
Steady-state amplitude = ·002053 milliamp.
Fig.18
Current in milliampere
Time in milliseconds


Calculated curve of IR(t) of 5 section Low-pass
T-type filter under an impressed voltage
sin(3079t+90°) ·[f=1·25·fc]
Steady state amplitude =.00163 milliamp.
Fig.19
Receiving-end current in milliamperes
Time in milliseconds



Calculated Is of 5 section Low-pass T-type Filter under an impressed voltage sin(1202t+50.15°)
Fig. 22



Calculated Indicial Admittance at the sending end of 5 section Low-pass Filter
Steady-state value = 4.738 milliamp.
Fig. 20

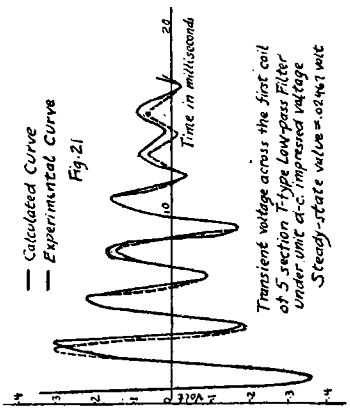
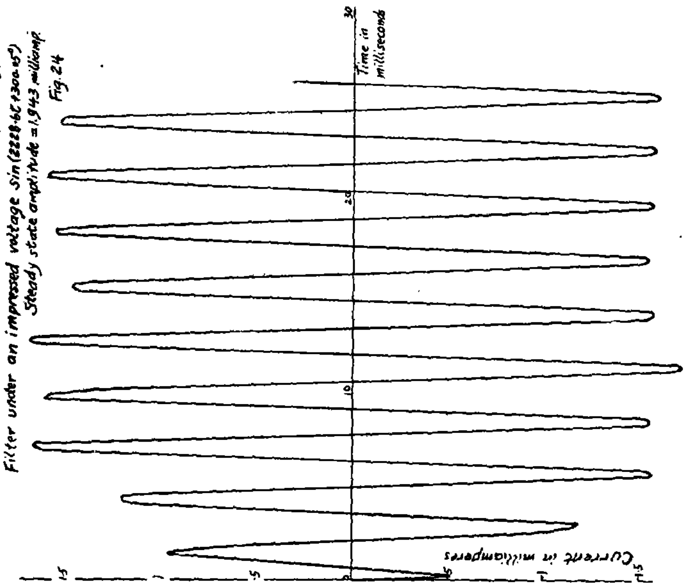
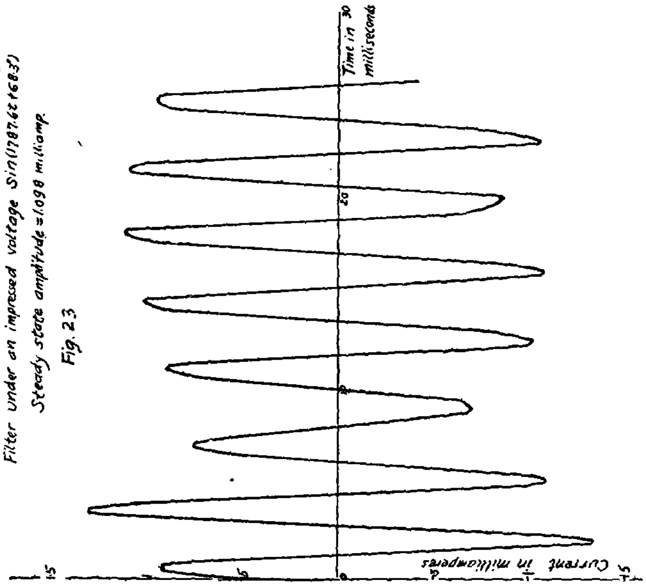


Fig. 21
Transient voltage across the first coil of 5 section T-type Low-pass Filter under unit d-c. impressed voltage
Steady-state value = .02461 volt



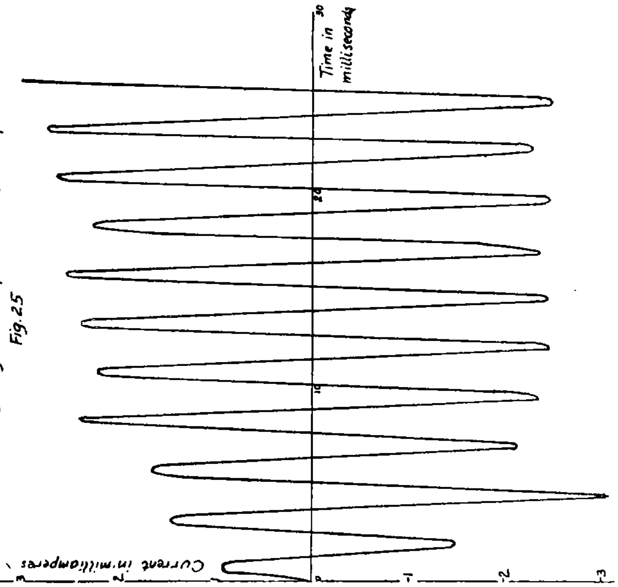
Calculated Curve of $I_5(t)$ of 5 section Low-pass T-type Filter under an impressed voltage Sin(2228.6t+300.5°) Steady state amplitude =1.943 milliamp.

Fig. 24

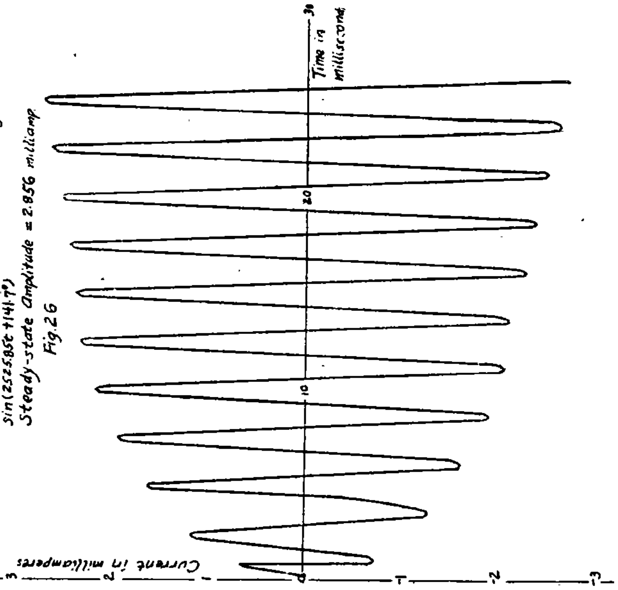


Calculated Curve of $I_5(t)$ of 5 section Low-pass T-type Filter under an impressed voltage Sin(1787.6t+168.3°) Steady state amplitude =1.098 milliamp.

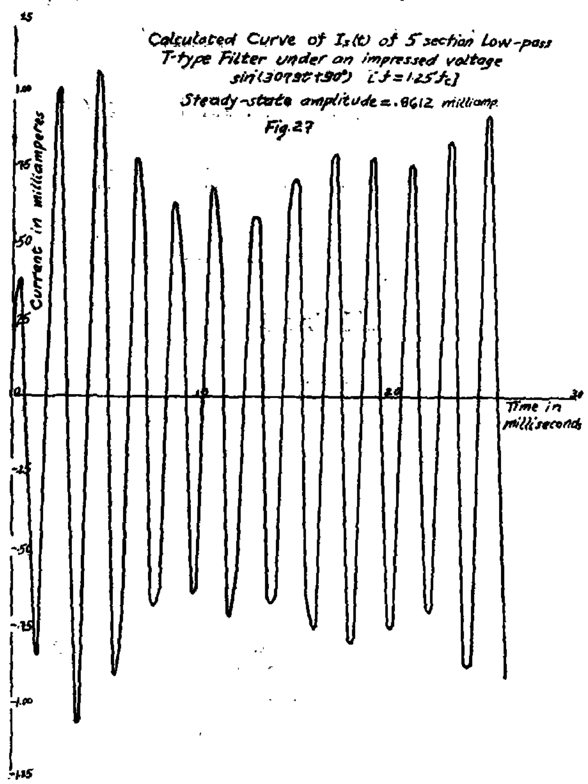
Fig. 23



Calculated Curve of $I_5(t)$ of 5 section Low-pass T-type Filter under an impressed voltage sin(2463t + 48.93°)
Steady state amplitude = 3.32 milliamp.
Fig. 25



Calculated curve of $I_5(t)$ of 5 section Low-pass T-type Filter under an impressed Voltage sin(2525.85t + 141.7°)
Steady-state Amplitude = 2.856 milliamp.
Fig. 26


Calculated Curve of $I_5(t)$ of 5 section Low-pass
T-type Filter under an impressed voltage
sin(3079.8t+90°) [f=1.25fc]
Steady-state amplitude=.8612 milliamp.
Fig.27
Current in milliamperes
1.00
.75
.50
.25
0
-.25
-.50
-.75
-1.00
-1.25
10
20
30
Time in milliseconds

文用二單變函數之較,以表驟上驟下之脈壓;而按重疊原理;以研究此種電壓在各電路上所發之脈流。茲所研究者,有十五不同之電路,計刮二十類三十六種。其間插有許多有趣之圖示,所得各脈流之特性,以一有秩序之方法,皆用算式表出之。

低頻濾波器之瞬流

朱物華　張仲桂

此篇先推求收端加電阻時,低頻濾波器瞬流之公式.依此公式算出之圖與用陰極光示波器映出之曲線相符合.自推算之結果,可得下列結論:

(一) 在濾波器收端電阻漸加時,瞬流各項之挫率漸互異.其數量由低頻項至隔阻頻之項順序漸減;其最小數仍比收端無電阻時之挫率 (R/2L) 爲大.故瞬流終必變爲隔阻頻之電流;而較收端無電阻時易于消滅.

(二) 當濾波器增加一段時,瞬流之項數亦加一.所加項之挫率皆比前有者爲小.故少段濾波器之瞬流易于消滅.

(三) 在隔阻頻後瞬流之數量與在其前者相彷恆較隔阻頻後之安定數量大數十倍.故濾波之特性僅能見之于安定狀態.

北平泉水與自流井水所含射氣量之測定

徐允貴　謝玉銘

應用斯密特之"搖動法,"測定北平鄰近之泉水,如湯山溫泉,溫泉,玉泉山水及清華,燕京,協和醫諸校內自流井所含之射氣量。諸泉源或僅含鐳射氣,釷射氣,或二種射氣皆有之。民國二十一年與二十四年,曾測驗諸泉水各一次,所含之射氣量,絕少改變。