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耗散介观电容耦合电路的量子化

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通过正则化变换,研究了耗散介观电容耦合电路的量子化,并讨论了系统中电荷和广义电流的量子涨落.

关键词:耗散介观电容耦合电路, 量子化, 正则化变换

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1. 引 言

随着纳米技术和纳米电子学的飞速发展[1,2],在电子器件中电路及器件的小型化和高集成度的趋势越来越显著,近年来已经达到了原子尺寸的量级[3].当电路的尺寸小到与电子的相干长度可比拟时,电路本身的量子效应就会出现,原来在研究电路时所采用的一系列经典的基本原理和方法不再成立,大量的有关纳米尺寸电路以及单电荷器件的实验结果已经充分证明了这一点[4,5].自 20 世纪 70 年代 Louisell[6] 研究了 LC 电路的量子效应以来,人们先后研究了各种介观电路的量子化问题[7—10],但是,大部分研究仅仅限于讨论无耗散介观电路.由于实际电路总是存在一定的电阻,结果必然导致电路存在一定的耗散作用,因此,对于耗散介观电路的量子效应的研究更具有普遍性和实际意义.最近,在文献[11]中,作者研究了耗散介观电容耦合电路的量子效应,但是,作者给出的哈密顿量是错误的,因为它不满足哈密顿正则方程.本文通过引入正则化变换[12],给出了正确的耗散介观电容耦合电路的哈密顿量,研究了系统中电荷和广义电流的量子涨落.

2. 耗散介观电容耦合电路的量子化

考虑一个耗散介观电容耦合电路(如图 1 所示),根据基尔霍夫定律,其经典运动方程为

$$L_1\ddot{q}_1(t)+R_1\dot{q}_1(t)+\frac{q_1(t)}{C_1}+\frac{q_1(t)-q_2(t)}{C}=\varepsilon(t), \tag{1a}$$

$$L_2\ddot{q}_2(t)+R_2\dot{q}_2(t)+\frac{q_2(t)}{C_2}-\frac{q_1(t)-q_2(t)}{C}=0, \tag{1b}$$

其中 $q_i(t)$,L_i 和 C_i($i=1,2$)分别是两个回路各自的电荷、电感和电容,C 是两个回路的耦合电容,$\varepsilon(t)$是电压.令 $p_i(t)=L_i\dot{q}_i(t)$($i=1,2$),则(1)式可表示为

$$\dot{p}_1=\varepsilon(t)-\frac{R_1}{L_1}p_1-\frac{q_1}{C'_1}+\frac{q_2}{C},\quad \dot{q}_1=\frac{1}{L_1}p_1,\quad \frac{1}{C'_1}=\frac{1}{C_1}+\frac{1}{C}, \tag{2a}$$

$$\dot{p}_2=-\frac{R_2}{L_2}p_2-\frac{q_2}{C'_2}+\frac{q_1}{C},\quad \dot{q}_2=\frac{1}{L_2}p_2,\quad \frac{1}{C'_2}=\frac{1}{C_2}+\frac{1}{C}, \tag{2b}$$

由(2)式可得

$$\frac{\partial\dot{q}_1}{\partial q_1}+\frac{\partial\dot{p}_1}{\partial p_1}=-\frac{R_1}{L_1},\quad \frac{\partial\dot{q}_2}{\partial q_2}+\frac{\partial\dot{p}_2}{\partial p_2}=-\frac{R_2}{L_2}. \tag{3}$$

(3)式表明在一个耗散系统中($R_1\neq0,R_2\neq0$),q_i 和 p_i($i=1,2$)在经典力学中不再构成正则共轭变量.

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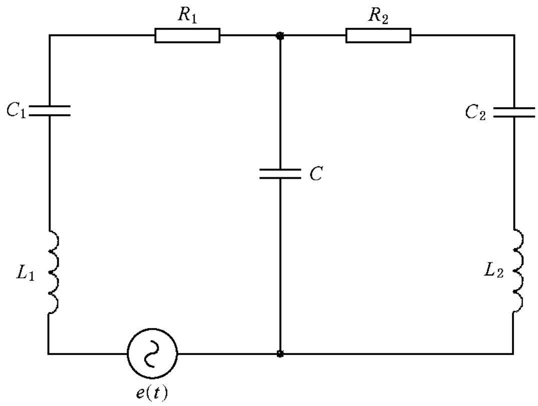

图 1　耗散介观电容耦合电路

在海森堡绘景中将(2)式量子化，即 q_i 和 p_i $(i=1,2)$ 分别表示坐标算符和动量算符，并满足一定对易关系(量子化条件). 由量子化后的(2)式可得

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}(q_1p_1 - p_1q_1)\\ &= \dot{q}_1p_1 + q_1\dot{p}_1 - \dot{p}_1q_1 - p_1\dot{q}_1\\ &= -\frac{R_1}{L_1}x_1 + \frac{1}{C}x_5,\end{aligned} \tag{4a}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = -\frac{R_2}{L_2}x_2 - \frac{1}{C}x_5, \tag{4b}$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = -\frac{R_2}{L_2}x_3 - \frac{1}{C'_2}x_5 + \frac{1}{L_1}x_6, \tag{4c}$$

$$\frac{\mathrm{d}x_4}{\mathrm{d}t} = -\frac{R_1}{L_1}x_4 + \frac{1}{C'_1}x_5 - \frac{1}{L_2}x_6, \tag{4d}$$

$$\frac{\mathrm{d}x_5}{\mathrm{d}t} = \frac{1}{L_2}x_3 - \frac{1}{L_1}x_4, \tag{4e}$$

$$\begin{aligned}\frac{\mathrm{d}x_6}{\mathrm{d}t} &= -\frac{1}{C}x_1 + \frac{1}{C}x_2 - \frac{1}{C'_1}x_3\\ &\quad + \frac{1}{C'_2}x_4 - \left(\frac{R_1}{L_1} + \frac{R_2}{L_2}\right)x_6,\end{aligned} \tag{4f}$$

其中

$$\begin{aligned}&x_1 = [q_1,p_1],\ x_2 = [q_2,p_2],\ x_3 = [q_1,p_2]\\ &x_4 = [q_2,p_1],\ x_5 = [q_1,q_2],\ x_6 = [p_1,p_2].\end{aligned} \tag{5}$$

一般说来，方程组(4)很难求解. 下面我们讨论当 $\frac{R_1}{L_1} = \frac{R_2}{L_2} = \lambda$ 时，方程组(4)的解. 把 $\frac{R_1}{L_1} = \frac{R_2}{L_2} = \lambda$ 代入方程组(4)得

$$\begin{aligned}x_1 &= [q_1,p_1] = \mathrm{i}\hbar\exp(-\lambda t),\\ x_2 &= [q_2,p_2] = \mathrm{i}\hbar\exp(-\lambda t) = x_1,\\ x_3 &= [q_1,p_2] = 0,\\ x_4 &= [q_2,p_1] = 0,\\ x_5 &= [q_1,q_2] = 0,\\ x_6 &= [p_1,p_2] = 0.\end{aligned} \tag{6}$$

如果引入如下正则化变换[12]

$$\begin{aligned}Q_1(t) &= q_1\exp\left(\frac{1}{2}\lambda t\right),\\ P_1(t) &= L_1\dot{Q}_1(t) = \left(p_1 + \frac{1}{2}R_1q_1\right)\exp\left(\frac{1}{2}\lambda t\right),\end{aligned} \tag{7a}$$

$$\begin{aligned}Q_2(t) &= q_2\exp\left(\frac{1}{2}\lambda t\right),\\ P_2(t) &= L_2\dot{Q}_2(t) = \left(p_2 + \frac{1}{2}R_2q_2\right)\exp\left(\frac{1}{2}\lambda t\right),\end{aligned} \tag{7b}$$

则(1)式可化为

$$L_1\ddot{Q}_1(t) + \lambda_1Q_1(t) - \frac{1}{C}Q_2(t) = \varepsilon'(t), \tag{8a}$$

$$L_1\ddot{Q}_2(t) + \lambda_2Q_2(t) - \frac{1}{C}Q_1(t) = 0, \tag{8b}$$

或者

$$\begin{aligned}\dot{Q}_1(t) &= \frac{1}{L_1}P_1(t),\\ \dot{P}_1(t) &= \varepsilon'(t) - \lambda_1Q_1(t) + \frac{1}{C}Q_2(t),\end{aligned} \tag{9a}$$

$$\begin{aligned}\dot{Q}_2(t) &= \frac{1}{L_2}P_2(t),\\ \dot{P}_2(t) &= -\lambda_2Q_2(t) + \frac{1}{C}Q_1(t),\end{aligned} \tag{9b}$$

其中

$$\begin{aligned}\lambda_1 &= \frac{1}{C'_1} - \frac{R_1^2}{4L_1},\\ \lambda_2 &= \frac{1}{C'_2} - \frac{R_2^2}{4L_2},\\ \varepsilon'(t) &= \varepsilon(t)\exp\left(\frac{1}{2}\lambda t\right).\end{aligned} \tag{10}$$

由(9)式可以证明 Q_i 和 P_i 构成正则共轭变量，即

$$\frac{\partial\dot{Q}_i}{\partial Q_i} + \frac{\partial\dot{P}_i}{\partial P_i} = 0,\ (i=1,2),$$

在此处及下文中将 $Q_i(t)$，$P_i(t)$ 简记为 Q_i，P_i. 利用哈密顿正则方程，由(8)式可得经典哈密顿量

$$\begin{aligned}H &= \frac{1}{2L_1}P_1^2 + \frac{1}{2L_2}P_2^2 + \frac{1}{2}\lambda Q_1^2 + \frac{1}{2}\lambda_2Q_2^2\\ &\quad - \frac{1}{C}Q_1Q_2 - \varepsilon'(t)Q_1.\end{aligned} \tag{11}$$

把(11)式量子化也就意味着将经典变量 Q_1，Q_2 和

P_1，P_2 表示为算符.由(6)式和(7)式可得如下对易关系：

$$[Q_k, P_l] = \mathrm{i}\hbar\delta_{kl}, [Q_1, Q_2] = [P_1, P_2] = 0. \tag{12}$$

为使(11)式所表达的哈密顿量 H 对角化,引入如下幺正算符 U：

$$U = \iint |AQ\rangle\langle Q| \mathrm{d}Q_1 \mathrm{d}Q_2, \tag{13}$$

其中

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \tag{14}$$

$$A_{11} = B\cos(\varphi/2), A_{12} = B\sin(\varphi/2), \tag{15}$$

$$A_{21} = -B^{-1}\sin(\varphi/2), A_{22} = B^{-1}\cos(\varphi/2), \tag{16}$$

$$B = \left(\frac{L_2}{L_1}\right)^{1/4}, \tag{17}$$

$$\mathrm{tg}\varphi = \frac{2}{C(\lambda_1 B^2 - \lambda_2 B^{-2})}, \tag{18}$$

$$|Q\rangle = \left|\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}\right\rangle = |Q_1, Q_2\rangle = |Q_1\rangle |Q_2\rangle, \tag{19}$$

其中$|Q_i\rangle (i = 1,2)$是坐标本征态[13].容易证明

$$U^{-1} Q_1 U = A_{11} Q_1 + A_{12} Q_2,$$
$$U^{-1} Q_2 U = A_{21} Q_1 + A_{22} Q_2, \tag{20}$$
$$U^{-1} P_1 U = A_{22} P_1 - A_{21} P_2,$$
$$U^{-1} P_2 U = -A_{12} P_1 + A_{11} P_2. \tag{21}$$

将(20)式和 (21)式代入(11)式可得

$$\begin{aligned} H' &= U^{-1} H U \\ &= \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{1}{2} m_1 \omega_1^2 Q_1^2 \\ &\quad + \frac{1}{2} m_2 \omega_2^2 Q_2^2 - (A_{11} Q_1 + A_{12} Q_2)\varepsilon'(t), \end{aligned} \tag{22}$$

其中

$$\frac{1}{m_1} = \frac{A_{22}^2}{L_1} + \frac{A_{12}^2}{L_2}, \frac{1}{m_2} = \frac{A_{21}^2}{L_1} + \frac{A_{11}^2}{L_2}, \tag{23}$$

$$\omega_1^2 = \left(\frac{A_{22}^2}{L_1} + \frac{A_{12}^2}{L_2}\right)\left(\lambda_1 A_{11}^2 + \lambda_2 A_{21}^2 - \frac{2}{C} A_{11} A_{21}\right), \tag{24}$$

$$\omega_2^2 = \left(\frac{A_{21}^2}{L_1} + \frac{A_{11}^2}{L_2}\right)\left(\lambda_1 A_{12}^2 + \lambda_2 A_{22}^2 - \frac{2}{C} A_{12} A_{22}\right). \tag{25}$$

由(12)式可以构造如下非厄米算符：

$$a_k = \sqrt{\frac{m_k \omega_k}{2\hbar}}\left(Q_k + \frac{\mathrm{i}}{m_k \omega_k} P_k\right), (k = 1,2), \tag{26}$$

$$a_k^+ = \sqrt{\frac{m_k \omega_k}{2\hbar}}\left(Q_k - \frac{\mathrm{i}}{m_k \omega_k} P_k\right), (k = 1,2), \tag{27}$$

此算符满足对易关系

$$[a_k, a_l^+] = \mathrm{i}\hbar\delta_{kl}, [a_k, a_l] = 0, [a_k^+, a_l^+] = 0, \tag{28}$$

于是(22) 式可写作

$$\begin{aligned} H' &= \hbar\omega_1\left(a_1^+ a_1 + \frac{1}{2}\right) + \hbar\omega_2\left(a_2^+ a_2 + \frac{1}{2}\right) \\ &\quad + V_1(t)(a_1 + a_1^+) + V_2(t)(a_2 + a_2^+), \end{aligned} \tag{29}$$

其中

$$V_1(t) = -A_{11}\varepsilon'(t)\sqrt{\frac{\hbar}{2m_1\omega_1}},$$

$$V_2(t) = -A_{12}\varepsilon'(t)\sqrt{\frac{\hbar}{2m_2\omega_2}}. \tag{30}$$

现在计算 U 的正规乘积形式,将 Fock 空间中坐标本征态$|Q_i\rangle (i = 1,2)$的表达式

$$\begin{aligned} |Q_i\rangle &= \left(\frac{m_i\omega_i}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m_i\omega_i}{2\hbar} Q_i^2 + \sqrt{\frac{2m_i\omega_i}{\hbar}} Q_i a_i^+ \right. \\ &\quad \left. - \frac{1}{2} a_i^{+^2}\right] |0\rangle_i, (i = 1,2) \end{aligned} \tag{31}$$

代入 (13) 式,并应用有序算符内的积分技术(IWOP)[7,13,14],对(13) 式进行积分,得 U 的正规乘积形式

$$\begin{aligned} U &= \iint |A_{11} Q_1 + A_{12} Q_2, A_{21} Q_1 + A_{22} Q_2\rangle \\ &\quad \times \langle Q_1, Q_2| \mathrm{d}Q_1 \mathrm{d}Q_2 \\ &= \sqrt{\frac{4m_1 m_2 \omega_1 \omega_2}{\hbar^2 \Delta}} \exp(\sigma_1 a_1^{+^2} - \sigma_1 a_2^{+^2} + \sigma_2 a_1^+ a_2^+) \\ &\quad :\exp(\Gamma_1 a_1^+ a_1 + \Gamma_1 a_2^+ a_2 + \Gamma_2 a_1^+ a_2 - \Gamma_2 a_2^+ a_1): \\ &\quad \exp(\tau_1 a_1^2 - \tau_1 a_2^2 + \tau_2 a_1 a_2), \end{aligned} \tag{32}$$

其中::为正规乘积符号,

$$\begin{aligned} \Delta &= \left(\frac{m_1\omega_1 A_{12}}{\hbar}\right)^2 + \left(\frac{m_2\omega_2 A_{21}}{\hbar}\right)^2 + \frac{m_1 m_2 \omega_1 \omega_2}{\hbar^2} \\ &\quad \times [2 + (B^2 + B^{-2})\cos^2(\varphi/2)], \end{aligned} \tag{33}$$

$$\sigma_1 = \frac{m_1\omega_1}{\Delta\hbar^2}[m_1\omega_1 A_{12}^2 + m_2\omega_2(1 + A_{11}^2)] - \frac{1}{2}, \tag{34}$$

$$\sigma_2 = \frac{\sqrt{m_1 m_2 \omega_1 \omega_2}}{\Delta\hbar^2}(m_1\omega_1 - m_2\omega_2)\sin(\varphi), \tag{35}$$

$$\Gamma_1 = \frac{2m_1 m_2 \omega_1 \omega_2}{\Delta\hbar^2}(B + B^{-1})\cos(\varphi/2) - 1, \tag{36}$$

$$\Gamma_2 = \frac{\sqrt{4m_1 m_2 \omega_1 \omega_2}}{\Delta \hbar^2}(m_1 \omega_1 B + m_2 \omega_2 B^{-1}) \sin(\varphi/2), \tag{37}$$

$$\tau_1 = \frac{m_1 \omega_1}{\Delta \hbar^2}[m_1 \omega_1 A_{12}^2 + m_2 \omega_2 (1 + A_{22}^2)] - \frac{1}{2}, \tag{38}$$

$$\tau_2 = -\frac{\sqrt{m_1 m_2 \omega_1 \omega_2}}{\Delta \hbar^2}(m_1 \omega_1 B^2 - m_2 \omega_2 B^{-2}) \sin(\varphi), \tag{39}$$

可以证明，与 H' 相应的时间演化算子 $U_s(t,0)$ 为（忽略一个相因子）[13]

$$U_s(t,0) = \exp\left[-\mathrm{i}\hbar(\omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2)t\right] \times \exp\left[-\mathrm{i}(\eta_1^* a_1^+ + \eta_1 a_1 + \eta_2^* a_2^+ + \eta_2 a_2)\right], \tag{40}$$

其中

$$\eta_k(t) = \int_0^t V_k(\tau) \exp(-\mathrm{i}\hbar\omega_k \tau) \mathrm{d}\tau. \tag{41}$$

因此,系统的波函数为

$$|\psi(t)\rangle = UU_s(t,0)|00\rangle. \tag{42}$$

这里,我们假定初始时刻系统处于双模真空态 $|00\rangle$. 如果外部电源接通无穷小时间 $t = \rho \to 0$ ($U_s(t,0) \to 1$),随即断开,此时则系统的基态为一转动的压缩真空态

$$|\psi(t = \rho)\rangle_{\rho\to 0} = U|00\rangle$$
$$= \sqrt{\frac{4m_1 m_2 \omega_1 \omega_2}{\hbar^2 \Delta}} \exp(\sigma_1 a_1^{+2} - \sigma_1 a_2^{+2} + \sigma_2 a_1^+ a_2^+)|00\rangle$$
$$= \sqrt{\frac{4m_1 m_2 \omega_1 \omega_2}{\hbar^2 \Delta}} \exp[(a_1^+ a_2 - a_2^+ a_1)\theta] \times \exp[\nu(a_2^{+2} - a_1^{+2})]|00\rangle, \tag{43}$$

其中

$$\theta = \frac{1}{2}\arctan\left(-\frac{\sigma_2}{2\sigma_1}\right), \nu = \sigma_1 + \frac{1}{2}\sigma_2 \cot(\theta), \tag{44}$$

$\exp\left[(a_1^+ a_2 - a_2^+ a_2)\theta\right]$ 是一个转动算符. 由(20)式和(21)式可得

$$\langle(\Delta Q_1)^2\rangle = \langle 00|U^+ Q_1^2 U|00\rangle - (\langle 00|U^+ Q_1 U|00\rangle)^2 = \frac{\hbar A_{11}^2}{2m_1\omega_1} + \frac{\hbar A_{12}^2}{2m_2\omega_2}, \tag{45}$$

$$\langle(\Delta Q_2)^2\rangle = \langle 00|U^+ Q_2^2 U|00\rangle - (\langle 00|U^+ Q_2 U|00\rangle)^2 = \frac{\hbar A_{21}^2}{2m_1\omega_1} + \frac{\hbar A_{22}^2}{2m_2\omega_2}, \tag{46}$$

$$\langle(\Delta P_1)^2\rangle = \langle 00|U^+ P_1^2 U|00\rangle - (\langle 00|U^+ P_1 U|00\rangle)^2 = \frac{\hbar m_1\omega_1 A_{22}^2}{2} + \frac{\hbar m_2\omega_2 A_{21}^2}{2}, \tag{47}$$

$$\langle(\Delta P_2)^2\rangle = \langle 00|U^+ P_2^2 U|00\rangle - (\langle 00|U^+ P_2 U|00\rangle)^2 = \frac{\hbar m_1\omega_1 A_{12}^2}{2} + \frac{\hbar m_2\omega_2 A_{11}^2}{2}. \tag{48}$$

由(7)式可得,在态 $U|00\rangle$ 中 q_i 和 p_i ($i = 1,2$) 的量子化涨落

$$\langle(\Delta q_1)^2\rangle = \langle q_1^2\rangle - \langle q_1\rangle^2 = [\langle 00|U^+ Q_1^2 U|00\rangle - (\langle 00|U^+ Q_1 U|00\rangle)^2] \times \exp(-\lambda t)|_{t=\rho\to 0} = \langle(\Delta Q_1)^2\rangle, \tag{49}$$

$$\langle(\Delta q_2)^2\rangle = \langle(\Delta Q_2)^2\rangle, \tag{50}$$

$$\langle(\Delta p_1)^2\rangle = \langle(\Delta P_1)^2\rangle, \tag{51}$$

$$\langle(\Delta p_2)^2\rangle = \langle(\Delta P_2)^2\rangle. \tag{52}$$

3. 结　　论

我们通过引入正则化变换,给出了正确的耗散介观电容耦合电路的哈密顿量,研究了系统的量子化问题,引入一个幺正变换把系统哈密顿量对角化,在此基础上给出了系统的态矢量随时间的演化规律,研究了系统中电荷和广义电流的量子涨落. 结果表明如果外部电源接通无穷小时间 $t = \rho \to 0$ 随即断开,则系统的基态为一转动的压缩真空态.

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Quantization of dissipative mesoscopic capacitance coupling circuit

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Abstract

By means of canonicalization transformation, we study the quantization of dissipative mesoscopic capacitance coupling circuit, and discuss the quantum fluctuations of charges and generalized currents in the system.

Keywords: dissipative mesoscopic capacitance coupling circuit, quantization, canonicalization transformation

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