

A CATHODE-RAY WAVEMETER FOR DECIMETER AND CENTIMETER WAVES.*

Part 1. Theoretical Investigation.

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Abstract.

A new wavemeter with cathode-ray for decimeter and centimeter waves is proposed, utilizing special deflections of cathode ray in a rapidly alternating field. When the wavelength of the deflecting field is short enough, the deflections can be made to have maximum and zero sensitivities or phase-shifts by adjusting the anode voltage. Formulas for computing the wavelength are derived and fully discussed. All factors concerning this type of wavemeter are thoroughly investigated from the theoretical considerations and preliminary tests.

1. Introduction.

To the writer's knowledge the cathode-ray wavemeter to be described and investigated has never been used before, although cathode-ray oscillographs have been used to compare wavelengths by the familiar Lessajous's figures or by the Dye's modification of sub-loops.¹ When the wavelengths are in the order of decimeters or centimeters, those methods become impracticable on account of the multiplicity of circuits and fineness of adjustments. Any one with some experience in using cathode-ray oscillographs on very short waves can not fail to note the two difficulties, namely, inconstancy of sensitivity and shift of phases. Notably Dr. H. E. Hollmann of the

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(1) National Phys. Lab. Collected Researches, Vol. XXI, 1929, p. 205; or Proc. Phys. Soc. (London), Vol. 37, 1925.

Heinnich-Hertz Institute^{2 3} has thoroughly investigated these difficulties from the view-point of an oscillograph in order to minimize them. The undesirable difficulties to the oscillograph are the fundamental bases of the proposed wavemeter, which measures the wavelength by the anode voltages for the special sensitivities or phase-shifts of the cathode-ray deflections.

2. Theory of Cathode-ray in Alternating Field.

A schematic diagram of a cathode-ray oscilloscope tube is shown in Fig. 1, where C stands for the cathode, A the anode, and P_y and P_z the two pairs of deflecting plates at right angles.

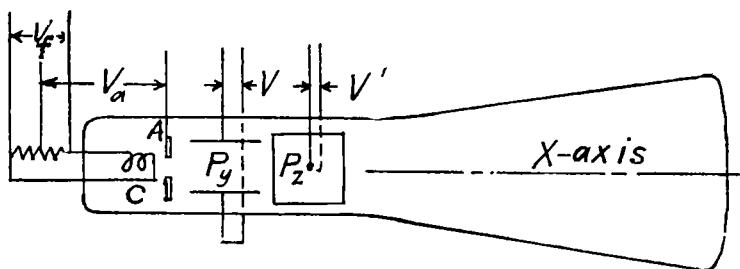


Fig. 1. Schematic diagram of cathode-ray oscillograph tube.

Let v_e be the velocity of the electron at C , which depends on the equivalent voltage of electron evaporation, V_e , usually less than four volts; and let v_o be the velocity of the electron at A . The fundamental equations of energy and motion are:

$$\frac{eV}{b} = m \frac{d^2y}{dt^2} \quad \dots \dots \dots \quad (2).$$

where V_a is the anode voltage, V the deflecting voltage on P_y , b the distance between plates, t the time, e the charge of one

(2) Experimental Wireless No. 119, Vol. 10, p. 430, Aug. 1933.

(3) Experimental Wireless No. 120, Vol. 10, p. 484, Sept. 1933.

electron, and m the mass of one electron at velocities much less than that of light.

The equation (1) may be written

$$v_o = \sqrt{\frac{2e(V_a - V_s)}{m}}, \quad (4)_t$$

and $v_o = \sqrt{\frac{2eV_a}{m}}, \quad (4')$

as V_s is usually negligible in comparison with V_a .

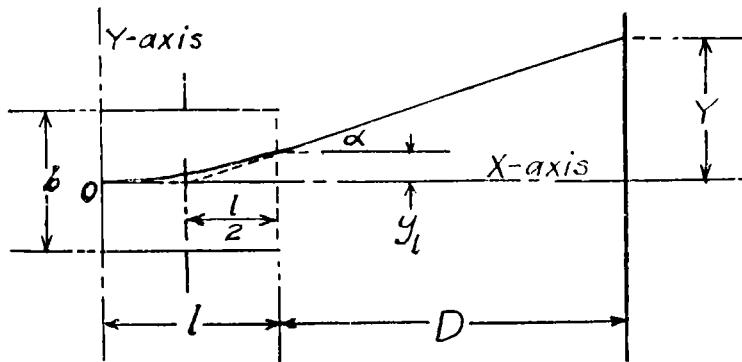


Fig. 2. Path of the electron in a steady deflecting electric field.

(a) *One pair of deflecting plates.*

First consider the case of a steady voltage V on one pair of plates and no voltage on the other pair. According to the equations (4'), (2) and (3), the path of the electron within the deflecting plates is parabolic and given by

$$y = \frac{eVx^2}{2mbv_o^2} \quad (5).$$

The rest of the path is a straight line tangent to the parabola. So the deflection of the fluorescent spot on the screen is given by

$$Y = y_l + D \tan \alpha = \left(\frac{l}{2} + D \right) \tan \alpha, \quad (6)$$

where Y is the linear deflection and α the angular deflection, and $\tan \alpha = \left(\frac{dy}{dx} \right)_{x=l} = \frac{eVl}{mbv_o^2}, \quad (7)$

The linear sensitivity is given by

and the tangent sensitivity by

which are not always proportional to each other, although they are in this case.

Now consider an alternating deflecting field,

As the electron experiences fields of delayed phases along the length of the deflecting plates, t' should be replaced by $t' + \frac{x}{v_0}$ or $t' + t$, thus we have from the equation (2)

Integrating with respect to t , we get the y -component of the velocity,

$$v_y = \frac{dy}{dt} = \frac{eV_0}{mb\omega} [\cos \omega t' - \cos \omega (t' + t)], \quad (12),$$

and the tangent sensitivity

$$\frac{\tan \alpha}{V_o} = \frac{v_o}{v_o V_o} = \frac{e}{mb\omega v_o} \left[\cos \omega t' - \cos \omega (t' + \frac{l}{v_o}) \right], \quad \dots (13).$$

Integrating (12) again respect to t , and substituting l for x , we get the linear deflection at $x=l$ as

$$y_t = \frac{eV_o}{mb\omega^2} \left[\frac{\omega l}{v_o} \cos \omega t' - \sin \omega (t' + \frac{l}{v_o}) + \sin \omega t' \right], \quad \dots \dots \quad (14).$$

As the tangents to the curved paths at $x=l$ for different values of V_0 do not meet at one point on the X-axis as they do in the case of steady deflecting field, the linear sensitivity is no longer proportional to the tangent sensitivity, but

However the value of y_t is usually negligible in comparison with that of $D \tan \alpha$.

(b) *Two pairs of deflecting plates.*

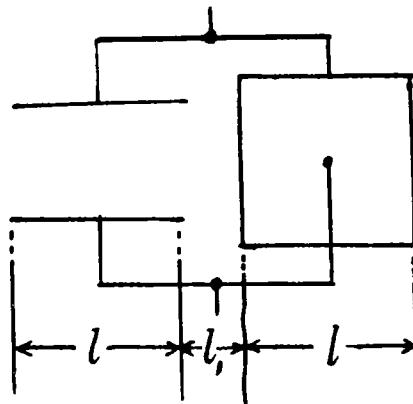


Fig. 3. Two pairs of identical plates at right angles.

When an alternating voltage of the same phase is applied to two identical pairs of deflecting plates separated by a nett distance of l between the adjacent edges, the electron of the cathode-ray which enters the first pair of plates at t , will enter the second pair at $t + \frac{l+l_1}{v_o}$. So the cathode-ray will describe a loop, unless

$$\frac{\omega(l+l_1)}{v_o} = \frac{n\pi}{2}, \quad \dots \dots \dots \quad (16),$$

n being an even integer, when the cathode-ray describes a straightline graph. If (16) is satisfied with an odd integer for the value of n , the graph will be a circle. Of course the graph will be a point in all cases if the linear sensitivity happens to be zero.

3. Principle of the Cathode-ray Wavemeter.

The operation of this wavemeter depends on the special sensitivities or phase-shifts of a specially designed cathode-ray tube under a gradually adjustable anode voltage.

The zero tangent sensitivity can be obtained from the equation (13), thus

$$O = \cos \omega t' - \cos \omega (t' + \frac{l}{v_c}),$$

n being an even integer.

The equation (13) may be re-written as

$$\frac{\tan \alpha}{V_0} = \frac{2e}{mb\omega v_0} \sin \omega (t' + \frac{l}{2v_0}) \sin \frac{\omega l}{2v_0}.$$

When only the maximum amplitude is concerned disregarding the phase, we can use

$$\frac{\tan \alpha_o}{V_o} = \frac{2e}{mb\omega_{n_o}} \sin \frac{\omega l}{2n_o}, \quad (18).$$

When the equation (18) is plotted with $(\tan \alpha_o)/V_o$ against v_o for a given value of ω , we have a curve as shown by the full line in Fig. 4, where the dotted line represents the corresponding static tangent sensitivity as given by the equation (9).

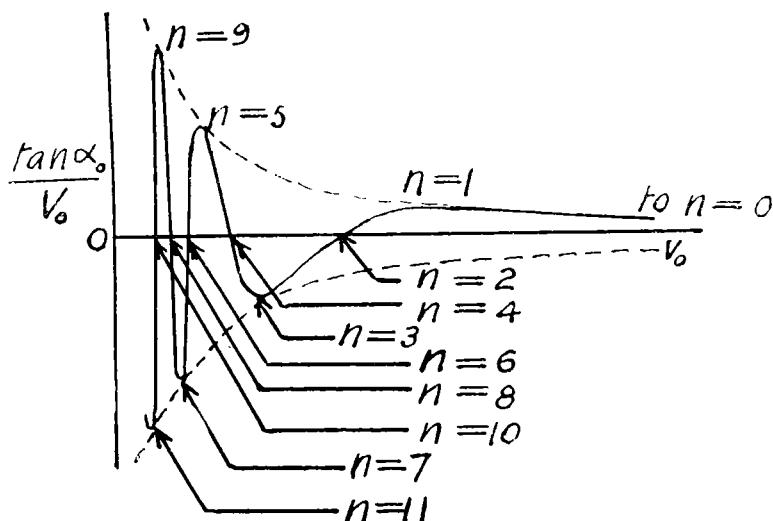


Fig. 4. Plot showing sepical tangent sensitivities.

Differentiating (18) with respect to v_o and equating the result to zero, the critical conditions are given by

It is the length of the graphs to be observed in operating this wavemeter. In the case of zero sensitivity, the initial deflection, y_1 , does not affect the shape of the graph, although it may change its position. When the sensitivity is at a maximum, y_1 becomes the most negligible in comparison with $D \tan \alpha$. So the value of y_1 can be safely neglected in both cases.

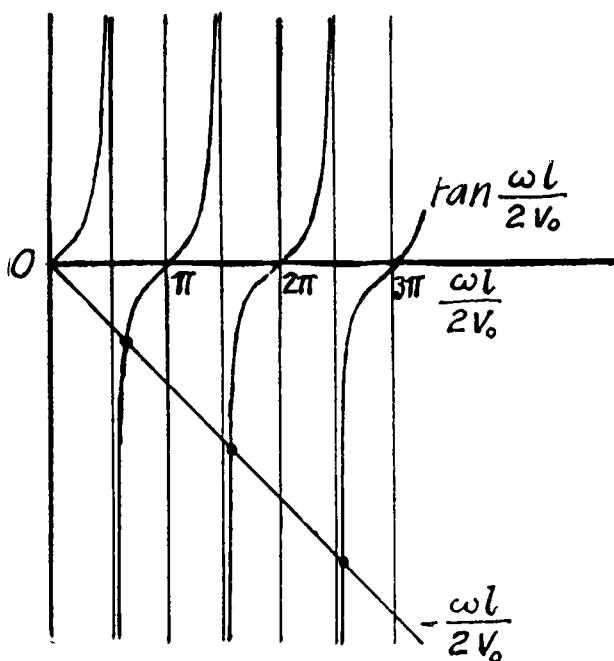


Fig. 5. Graphical solution of $\tan \frac{\omega l}{2v} = \frac{\omega l}{2v}$.

The equation (19) may either be graphically solved as shown in Fig. 5 or by a little trial calculation with the aid of a trigonometric table. The special values of $\omega l/2v_0$ are given in the table No. 1.

Table No. 1.

Special sensitivities		$\frac{\omega l}{2v_o}$	Ditto	N	k_n
Order	Kind				
1st	max.	2.027 rad.	$1.297 \times \frac{\pi}{2}$	1	1.297
2nd	zero	π	$1.000 \times \frac{2\pi}{2}$	2	1.000
3rd	max.	4.914	$1.043 \times \frac{3\pi}{2}$	3	1.043
4th	zero	2π	$1.000 \times \frac{4\pi}{2}$	4	1.000
5th	max.	7.979	$1.016 \times \frac{5\pi}{2}$	5	1.016
6th	zero	3π	$1.000 \times \frac{6\pi}{2}$	6	1.000
7th	max.	11.087	$1.001 \times \frac{7\pi}{2}$	7	1.001

The value of k_n is computed by dividing the value of $\omega l/2v_o$ by $n\pi/2$, where n denotes the order of special sensitivities. It is close to unity except for the first few maximum sensitivities. Thus we have for maximum tangent sensitivities,

$$\frac{\omega l}{2v_o} = n k_n \left(\frac{\pi}{2} \right), \text{ or, } \lambda = \frac{2cl}{n k_n v_o}, \quad (20),$$

n being an odd integer and c the velocity of light.

For zero tangent sensitivities $k=1$ and n is an even integer. [See equation (17)].

The equation (20) is the fundamental formula of this wavemeter. When the value of n is known somehow and thereby the value of k_n can be taken from the table, the wavelength of the voltage on the deflecting plates can be determined by reading the anode voltage for the zero or maximum sensitiv-

ity, assuming the effective value of the length of the deflecting plates is also known.

As an alternative, if the cathode-ray tube is provided at different sections along its axis with two identical pairs of deflecting plates at right angles, instead of the special sensitivities, the special phase-shifts, i.e., apparently quadrature ($n\pi/2$) and apparently zero ($n\pi$) phase-shifts, may be observed, providing the sensitivity is not zero. Barring the cases with an even integer for n , we can use the equation (16), which reads.

$$\frac{\omega(l+l_1)}{v_o} = \frac{n\pi}{2},$$

and may be re-written

Comparing this formula with the formula (20), it is seen that $2(l+l_1)$ is in the place of l and k_n is always unity. But it should also be noted that it is much more difficult to observe the roundness of a loop than to observe the maximum length of a straight line.

If the value of n has to be determined too, two observations either of special sensitivities or of special phase-shifts with a known interval of orders (N) between them have to be made. However one of the readings need not be very accurate, since n is an integer.

For observing two special sensitivities, from the formula (20) we get

$$\frac{\omega l}{v_o} = nk_n\pi \text{ and } \frac{\omega l}{v_o} = (n+N)k_{n+N}\pi,$$

where N is a known integer. Taking their ratio, we get

$$\text{or } n = \frac{N k_{n+N}}{k_n (v_o/v_o') - k_{n+N}}, \quad \dots \dots \dots \quad (22)$$

For observing two special phase-shifts, from the formula (21) we get

In both cases it is only the ratio v_0/v_0' or V_c/V_a that is required for determining the value of n .

4. Limiting Orders of Wavelengths.

Cathode-ray wavemeters can be designed for very different orders of wavelengths by using specially designed lengths of plates or locations of them and adopting suitable values of the anode voltages. To get some idea of the orders let us take examples from two well-known types of cathode-ray oscilloscopes, namely, the Western Electric Company's type 224B and the Leybold's No. 4476.

The former is designed to operate under an anode voltage of 300 volts. It has two pairs of deflecting plates of 1.2 cm length and a nett separation of 0.5 cm between the adjacent edges of the pairs. Here $l=1.2$ cm, $l_1=0.5$ cm $v_o = \sqrt{2eV_a/m} = 6 \times 10^7 \times V_a = 1.04 \times 10^9$ cm per sec. According to the formula (20), the first maximum sensitivity occurs at

$$\lambda = \frac{2c l}{1.297 v_0} = 53.5 \text{ cm},$$

and the first zero sensitivity occurs at

$$\lambda = \frac{c l}{v_g} = 34.6 \text{ cm},$$

According to the formula (21), the first quadrature phase-shift occurs at

$$\lambda = \frac{4C(l+l_1)}{v_o} = 196 \text{ cm},$$

and the first zero phase-shift occurs at

$$\lambda = \frac{2c(l+l_1)}{v_0} = 98.0 \text{ cm},$$

The Leybold's No. 4476 cathode-ray tube is designed to operate either at 1500 or at 3000 volts anode voltage, but it is

sufficient to take only the lower value for the present discussion, as the value of n can be multiples of one. It has $l=1.3$ cm and $l_1=1.0$ cm. With computations similar to the above, the table No. 2 is compiled.

Table No. 2.
Wavelengths in cms.

	Western Electric type 224B				Leybold No. 4476			
	Sensitivity		Phase-shift		Sensitivity		Phase-shift	
	Maximum	Zero	Quadrature	Zero	Maximum	Zero	Quadrature	Zero
First	53.5	34.6	196.0	98.0	25.9	16.8	118.8	59.4
Second	22.5	17.3	65.3	49.0	10.7	8.4	39.6	29.7
Third	13.7	11.5	39.2	32.7	6.6	5.6	23.8	19.7
Fourth	9.9	8.6	28.0	24.5	4.8	4.2	16.9	14.8

The longest wavelength shown in the table is 196 cm with the Western Electric type of tube, which operates well even below 200 volts anode voltage. The length of the plates may easily be increased many folds. Then the upper limit of the wavelength may be extended to ten meters. Theoretically there is no lower limit to the wavelength measurable by this type of wavemeter, and there should be no practical difficulties within the order of the wavelengths in centimeters.

5. Variation of the Anode Voltage and Order of the Special Sensitivities or Phase-shifts.

One difference between the cathode-ray wavemeter tube and an ordinary cathode-ray oscillograph is that the former must operate well continuously throughout a wide range of the anode voltage while the latter needs only to operate well at one value of it. The amount of variations of the anode voltage required can be computed from the formulas (22) and (23). Thus for observing special sensitivities, we have

and for observing special phase-shifts, we have

The numerical relations between the three quantities V_a / V_a' , n and N are shown in the table No. 3, which can be used both as a guide for designing the tube and as an aid for determining the value of n . The odd values of n correspond to the observation of a maximum sensitivity or a quadrature phase-shift at first and the even values of n correspond to the

Table No. 3.

Values of V_a/V_a'

n	N	For special sensitivities				For special phase-shifts			
		1	2	3	4	1	2	3	4
1		2.39	5.86	9.56	15.30	4.00	9.00	16.00	25.00
2		2.45	4.00	6.45	9.00	2.25	4.00	6.25	9.00
3		1.63	2.61	3.66	5.00	1.78	2.77	4.00	5.44
4		1.61	2.25	3.06	4.00	1.26	2.25	3.06	4.00
5		1.40	1.90	2.49	3.14	1.44	1.96	2.56	3.24
6		1.36	1.78	2.25	2.77	1.36	1.78	2.25	2.77

observation of a zero sensitivity or a zero phase-shift at first, as the anode voltage is gradually increased, while the value of N is being counted. Some ordinary cathode-ray oscillograph tubes have been found to operate properly with a variation of the anode voltage as high as seven or eight times, there should be no difficulty to construct the wavemeter tubes to meet the requirement of the variations of the anode voltage.

6. Voltage Sensitivity of the Wavemeter.

The voltage sensitivity of the wavemeter, S , is the rate of change of the anode voltage with respect to the wavelength to be measured, thus

$$S = \frac{dV_a}{d\lambda} = \frac{dV_a}{dv_o} \cdot \frac{dv_o}{d\lambda}, \quad \dots \dots \dots \quad (26).$$

As the formula (21) for the special phase-shifts can be obtained from the formula (20) for the special graph sensitivities by replacing l by $2(l+l_1)$ and k by 1, it is sufficient to take (20) only for the present discussion. Differentiating (20), we get

$$\frac{dv_o}{d\lambda} = \frac{v_o}{\lambda}.$$

Differentiating the equation (4) and neglecting v_e , we get

$$\frac{dV_e}{dv_o} = \frac{mv_o}{e}.$$

$$So, \quad S = -\frac{mv_o^2}{e\lambda} = -\frac{2V_a}{\lambda}, \quad \dots \dots \dots \quad (27),$$

which is independent of l , n and k_n , and therefore good for all cases.

The voltage sensitivity of this wavemeter is directly proportional to the anode voltage and inversely proportional to the wavelength to be measured. The minus sign indicates that V_a has an increment when λ has a decrement, but is not significant to the sensitivity.

7. Graph Sharpness of the Wavemeter.

The graph sharpness of the wavemeter is defined as the rate of change of the length or diameter of the graph with respect to the wavelength to be measured, keeping the anode voltage constant. Denoting the sharpness by P and the length or diameter of the graph by G , we have

$$P = \frac{dG}{d\lambda}, \quad \dots \dots \dots \quad (28)$$

From the equation (18) we get

$$G = \frac{2DeV_o\lambda}{\pi c m b v_o} \sin \frac{\pi cl}{\lambda v_o}, \quad \dots \dots \dots \quad (29)$$

Differentiating it with respect to λ , we get

$$P = \frac{DV_o l}{bV_a \lambda} \left[\frac{\lambda v_o}{\pi cl} \sin \frac{\pi cl}{\lambda v_o} - \cos \frac{\pi cl}{\lambda v_o} \right], \quad \dots \dots \dots \quad (30).$$

For the special sensitivities of the graph, according to the formula (20), we have

$$P_s = \frac{DV_o l}{bV_a \lambda} \left[\frac{nk_n \pi}{2} \sin \frac{nk_n \pi}{2} - \cos \frac{nk_n \pi}{2} \right], \quad (31).$$

For zero sensitivities of the graph, n is even and k_n is unity, therefore

$$P_{so} = \frac{n\pi DV_o l}{2bV_a \lambda} = \frac{n\pi G_o}{2\lambda}, \quad (32),$$

where G_o denotes the length of the graph for $2V_o$. [See the equation (8)].

For the maximum sensitivities, n is odd, and k_n is close to unity only when n is not too small. However it is close enough for the present discussion to assume $k_n=1$, thus we may take

$$P_{sm} \doteq \frac{DV_o l}{bV_a \lambda} = -\frac{G_o}{\lambda}, \quad (33),$$

which is always much smaller than P_{so} , and does not increase with n as P_{so} does.

For the special phase-shifts, according to the formula (21), we get

$$P_p = \frac{DV_o l}{bV_a} \left[\frac{n\pi}{2} \times \frac{l}{2(l+l_1)} \sin \left(\frac{n\pi}{2} \times \frac{l}{2(l+l_1)} \right) - \cos \left(\frac{n\pi}{2} \times \frac{l}{2(l+l_1)} \right) \right], \quad (34),$$

the value of which is between P_{so} and P_{sm} , and can be adjusted by choosing a value for $\frac{l}{2(l+l_1)}$.

8. Errors and Preciseness.

The preciseness is defined as $(\lambda - \Delta\lambda)/\lambda$ where $\Delta\lambda$ is the sum of the errors $(\Delta\lambda)_1$ and $(\Delta\lambda)_2$, the former being due to the error in reading the voltmeter and the latter being due to observing the graph.

Thus,

$$\Delta\lambda = (\Delta\lambda)_1 + (\Delta\lambda)_2 = \frac{\partial\lambda}{\partial V_a} \Delta V_a + \frac{\partial\lambda}{\partial G} \Delta G,$$

$$\text{or, } \Delta\lambda = \frac{\Delta V}{|S|} + \frac{\Delta G}{|P|}, \quad (35).$$

Substituting the equation (27) in (35), we get

$$\Delta\lambda = \frac{\lambda}{2} \cdot \frac{\Delta V_a}{V_a} + \frac{\Delta G}{|P|}, \quad (36),$$

which holds good for all cases.

In the case of the zero sensitivities, using (32),

$$(\Delta\lambda)_{so} = \frac{\lambda}{2} \cdot \frac{\Delta V_a}{V_a} + \frac{2\lambda}{n\pi} \cdot \frac{\Delta G}{G_o}, \quad (37),$$

and the preciseness is given by

$$\frac{\lambda - (\Delta\lambda)_{so}}{\lambda} = 1 - \frac{1}{2} \cdot \frac{\Delta V_a}{V_2} - \frac{2}{n\pi} \cdot \frac{\Delta G}{G_o}, \quad (38).$$

In the case of the maximum sensitivities, we have

$$(\Delta\lambda)_{sm} = \frac{\lambda}{2} \cdot \frac{\Delta V_a}{V_a} + \lambda \cdot \frac{\Delta G}{G_o}, \quad (39),$$

$$\text{and } \frac{\lambda - (\Delta\lambda)_{sm}}{\lambda} = 1 - \frac{1}{2} \cdot \frac{\Delta V_a}{V_a} - \frac{\Delta G}{G_o}, \quad (40).$$

For the special phase-shifts the errors and preciseness are between the two cases discussed above.

It is seen that this wavemeter can be operated with good preciseness, especially in the case of zero sensitivities. There should be no difficulty in reading a direct-current voltmeter for a few hundred or thousand volts very precisely, while the observation of a gradually changing length of a few centimeters maximum for its maximum or zero value must also be very easy. Moreover the error due to the latter is much reduced when the value of n is increased.

9. Accuracy and Tests.

The fundamental formula of this wavemeter is either

$$\lambda = \frac{nk_{ncl}}{v_o} \text{ for special sensitivities}$$

$$\text{or } \lambda = \frac{2n\pi c(l+l_1)}{v_o} \text{ for special phase-shifts,}$$

with the assumptions $v_o \ll v_e$ and l or $l+l_1 \ll D$, and $v_o = \sqrt{\frac{2eV_a}{m}}$.

Although the length of the plate, l , can be very accurately measured, but the end-effect of them may be appreciable unless they are quite long. However no end-effect needs be considered in the case of $l+l_1$, for then the distance between the centers of the pairs of plates is effective. In any case the effective value of l remains constant.

The other important factor is v_o , which has been assumed to depend on V_a only, and satisfies the relation

$$eV_a = \frac{1}{2}mv_o^2,$$

This may be tested by finding the static or low-frequency sensitivity, where

$$\text{or } V_a \left(\frac{Y}{V_o} \right) = \frac{Dl}{2b} = \text{a constant,} \quad (41).$$

When this is verified and the constant is found, the effective value of l can be found by measuring D and b with a cathetometer or a microscope.

If V_a (Y/V_o) is not constant, the tube does not function according to the assumption. Then it can be used with a calibration curve of v_o against V_a , plotted from

$$v_o = \sqrt{\frac{eDl}{mb}} \left(\frac{V_a}{Y} \right), \quad (42),$$

using the geometric value of l and the corresponding values of V_a .

The remaining factors, namely, e/m , c , π , n and k_n are all constants which are very accurate. The standard value of e/m at low velocities can be used, as the electronic mass should not have appreciably changed at velocities of the order of one tenth of that of light.

When v_o has been found to be proportional to $\sqrt{V_a}$, the formula may be re-written

$$\lambda = \sqrt{\frac{nk_n A}{V_a}}, \quad (34),$$

where A is a constant, which may be found by the direct calibration with one known or otherwise measured ultra-short wavelength.

In a more general way,

$$\lambda = n k_n F(V_a), \dots \quad (44),$$

where the function of $F(V_a)$ may be determined by extensive direct calibration with a number of very short wavelengths.

Tests should be made to see that the value of v_o is not appreciably affected by any factor except V_a . In this respect the cold-cathode tube is the best, but the hot-cathode tube is more suitable for the order of wavelengths at present interested.

Some preliminary tests have been made on the Leybold's No. 4476 and the Western Electric Company's type 224B hot-cathode tubes, both giving very favourable indications. More experimental work is in progress and special cathode-ray tubes are in the process of construction. In the second part of this paper the experimental investigation will be reported.

10. Acknowledgment.

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一量數分米或數厘米電波之陰極射線波長計

上段 理論研究

陳茂康

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著者新擬一量數分米或數厘米電波之波長計。其法用陰極射線在極短波長之電場內之特殊偏側，在相當情形下，可由調節其陽極之電壓，而使其偏側之靈敏度或相差成極大或成零。由此計算電場之波長所需各算式均已推出。且按原理及初步實驗，將凡關於此種波長計各節，亦已詳細研究。

大氣電位梯度之連續記錄

張文裕 王承書

北平燕京大學

本篇所述，為在北平燕京大學用靜電計及一連續記錄器測量大氣電位梯度之方法及結果測量方法，乃用一種均位器(equalizer)將離地面約3至6米高之電位顯示於靜電計上再用照相紙將此電位製成連續記錄。測量所得結果如下：晴天之電位梯度一日之中，有二最高值及二最低值。雨時梯度即降至零，且此值非至雨止不變。當閃電及雷颶時，梯度之方向與值，變易極速。且較晴天時為大。