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評輻射由於吸收體的理論*

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§1. Wheeler 與 Feymann 曾提出運動電荷的輻射由於吸收作用體的理論[1]；除了採用一半超前一半推後配合的勢場外，他們將輻射的原因歸於有吸收體的存在. 既然波動方程的超前解與推後解已被證明爲完全相等，[2] 卽1)

$$\Psi(P_0, t_0) = \int_V \frac{\{\sigma\}}{r} dV + \frac{1}{4\pi} \iint_S \left(\frac{1}{r} \left\{ \frac{\partial}{\partial n} \Psi \right\} - \frac{1}{rc} \frac{\partial r}{\partial n} \left\{ \frac{\partial}{\partial t} \Psi \right\} - \right.$$

$$\left. - \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \{\Psi\} \right) dS \tag{1}$$

$$= \int_V \frac{[\sigma]}{r} dV + \frac{1}{4\pi} \iint_S \left(\frac{1}{r} \left[\frac{\partial}{\partial n} \Psi \right] + \frac{1}{cr} \frac{\partial r}{\partial n} \left[\frac{\partial}{\partial t} \Psi \right] - \right.$$

$$\left. - \frac{\partial}{\partial n} \left(\frac{1}{r} \right) [\Psi] \right) dS.$$

我們可以研討一下他們的吸收體理論的價值. 結論是，如同以前已經指出[2]，他們的吸收體並無實際的意義.

§2. 考慮以下的面積分

$$S_A(P_0, t_0) = \frac{1}{4\pi} \oiint_S \left(\frac{1}{r} \left\{ \frac{\partial}{\partial n} \Psi \right\} - \frac{1}{cr} \frac{\partial r}{\partial n} \left\{ \frac{\partial}{\partial t} \Psi \right\} - \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \{\Psi\} \right) dS, \tag{2}$$

$$S_R(P_0, t_0) = \frac{1}{4\pi} \oiint_S \left(\frac{1}{r} \left[\frac{\partial}{\partial n} \Psi \right] + \frac{1}{cr} \frac{\partial r}{\partial n} \left[\frac{\partial}{\partial t} \Psi \right] - \frac{\partial}{\partial n} \left(\frac{1}{r} \right) [\Psi] \right) dS. \tag{3}$$

如果 Ψ 爲超前的類型2)

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1) 花括號內各量取在時間 $t = t_0 + \frac{r}{c}$ 的值而方括號內各量取在時間 $t = t_0 - \frac{r}{c}$ 的值.

2) 如果源點在運動，我們將它的場分解爲 (4) 式類型的各項的和，也可以將 S 面，(2)，變形爲一個球心在 P_0 的無限大的球面而使 S_A 得到 (5) 的形式.

$$\Psi=\frac{1}{R}f\left(t+\frac{1}{c}R\right),\tag{4}$$

積分 S_A 卽取得如下的形式

$$\oiint_S\left(\frac{\partial r}{\partial n}\left(\frac{f}{r^2R}-\frac{f'}{c\,r\,R}\right)-\frac{\partial R}{\partial n}\left(\frac{f}{rR^2}-\frac{f'}{c\,r\,R}\right)\right)dS,\tag{5}$$

其中

$$f'(x)=\frac{d}{d\,x}f(x),$$

R 爲自 S 面內之源點指向面上一點的距離，而 r 爲自面內之場點 P_0 指向面上之點的距離．同樣如果 Ψ 爲推後的類型

$$\Psi=\frac{1}{R}f\left(t-\frac{R}{c}\right),\tag{6}$$

則 S_R 成爲

$$\oiint_S\left(\frac{\partial r}{\partial n}\left(\frac{f}{r^2R}+\frac{f'}{c\,r\,R}\right)-\frac{\partial R}{\partial n}\left(\frac{f}{rR^2}+\frac{f'}{c\,r\,R}\right)\right)dS.\tag{7}$$

但是 (5) 式或 (7) 式的被積函數可以分別寫成

$$\vec{n}\cdot(\mathrm{rot}\,\vec{\Gamma}_A),\quad \vec{\Gamma}_A=\frac{1}{rR}f\left(t+\frac{r}{c}+\frac{R}{c}\right)\frac{\vec{R}\times\vec{r}}{rR+\vec{r}\cdot\vec{R}},$$

或

$$\vec{n}\cdot(\mathrm{rot}\,\vec{\Gamma}_R),\quad \vec{\Gamma}_R=\frac{1}{rR}f\left(t-\frac{r}{c}-\frac{R}{c}\right)\frac{\vec{R}\times\vec{r}}{rR+\vec{r}\cdot\vec{R}}.$$

形式．既然 $\vec{\Gamma}_A$ 及 $\vec{\Gamma}_R$ 在 S 面上處處存在，由於一矢量的旋度在封閉面上的積分爲零，我們肯定面積分 (5) 及 (7) 必須恆爲零．這樣我們證明了只要源點是被包含在積分面以內，超前場的超前面積分和推後場的推後面積分都等於零．

然而一個推後場的超前面積分和一個超前場的推後面積分一般不會消失．我們現在計算一個加速運動電子的推後場在電子本身所在位置一點的超前面積分．根據 (2) 式這是

$$\vec{E}\,(P_0, t_0) = \frac{1}{4\pi}\oiint_s \left(\frac{1}{r}\left\{\frac{\partial}{\partial n}\vec{E}\right\} - \frac{1}{c\,r}\frac{\partial r}{\partial n}\left\{\frac{\partial}{\partial t}\vec{E}\right\} - \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\left\{\vec{E}\right\}\right) ds, \quad (8)$$

$$\vec{H}\,(P_0, t_0) = \frac{1}{4\pi}\oiint_s \left(\frac{1}{r}\left\{\frac{\partial}{\partial n}\vec{H}\right\} - \frac{1}{c\,r}\frac{\partial r}{\partial n}\left\{\frac{\partial}{\partial t}\vec{H}\right\} - \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\left\{\vec{H}\right\}\right) ds, \quad (9)$$

式中 P_0 是電子的位置而 $\vec{E}$ 及 $\vec{H}$ 卽爲熟知的一個點電荷的推後場,

$$\begin{aligned}\vec{E} &= e\Big[\frac{(1-\beta^2)}{r^2(1-\beta_r)^3}\,(\vec{r_0}-\vec{\beta}\,) + \\ &+ \frac{1}{c^2 r(r-\beta_r)^3}\,(\vec{r_0}\cdot\vec{\alpha})(\vec{r_0}-\vec{\beta}\,) - \frac{1}{c^2 r(1-\beta_r)^2}\,\vec{\alpha}\Big], \qquad (10)\\ \vec{H} &= \vec{r_0}\times\vec{E} = e\Big[\frac{(1-\beta^2)}{r^2(1-\beta_r)^3}\,\vec{\beta}\times\vec{r_0} + \\ &+ \frac{1}{c^2 r(1-\beta_r)^3}\,(\vec{r_0}\cdot\vec{\alpha})\;\vec{\beta}\times\vec{r_0} + \frac{\vec{\alpha}\times\vec{r_0}}{c^2 r(1-\beta_r)^2}\Big],\end{aligned}$$

其中 $\vec{\alpha}$ 是電子的加速度, $\vec{\beta}$ 是電子的速度被除以光速 c, $\vec{r_0}$ 是單位矢量 $\frac{\vec{r}}{r}$ 自點電荷指向面上一點而 β_r 爲 $\vec{\beta}$ 沿 $\vec{r_0}$ 的分量,按照前面的規定方括號內各量取在時間 $t_0 = t - \frac{r}{c}$ 的值. 令 S 爲以 P_0 作球心的無限大的球面,則[3)]

$$\frac{\partial}{\partial n} = \frac{\partial}{\partial r}, \qquad dS = r^2 \sin\theta\, d\theta\, d\phi,$$

$$\frac{\partial}{\partial t}\,[F] = \left[\frac{\partial F}{\partial t_0}\right]\frac{\partial t_0}{\partial t} = \frac{1}{[1-\beta_r]}\left[\frac{\partial}{\partial t_0}\,F\right],$$

$$\begin{aligned}\frac{\partial}{\partial r}\,[F] &= \left[\frac{\partial F}{\partial r}\right] + \left[\frac{\partial F}{\partial t_0}\right]\cdot\frac{\partial t_0}{\partial r} = \left[\frac{\partial}{\partial r}\,F\right] + \left[\frac{\partial F}{\partial t_0}\right]\cdot\,(\vec{r_0}\cdot\mathrm{grad}\,t_0)\\ &= \left[\frac{\partial}{\partial r}\,F\right] - \frac{1}{[c\,(1-\beta_r)]}\left[\frac{\partial}{\partial t_0}\,F\right],\end{aligned}$$

因此把 $\frac{d}{dt_0}\vec{\alpha}$ 寫成 $\dot{\vec{\alpha}}$ 則

$$\begin{aligned}\frac{1}{r}\left\{\frac{\partial}{\partial r}\,\vec{E}\right\} - \frac{1}{c\,r}\left\{\vec{E}\right\} = \frac{2e}{c^3 r^2}\Big(&\frac{1}{(1-\beta_r)^3}\,\dot{\vec{\alpha}} + 3\,\frac{1}{c\,(1-\beta_r)^4}\,(\vec{\alpha}\cdot\vec{\beta}\,)\,\vec{\alpha}\\ &-\Big(\frac{1}{(1-\beta_r)^4}\,(\vec{r_0}\cdot\dot{\vec{\alpha}}) + 3\,\frac{1}{c(1-\beta_r)^5}\,(\vec{r_0}\cdot\vec{\alpha})^2\Big)\,(\vec{r_0}-\vec{\beta}\,)\Big),\end{aligned}$$

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3) 注意幾何 r 與推後 r 不同, $\frac{\partial[r]}{\partial r} = 1 - v_r\,\frac{\partial t_0}{\partial r} = \frac{1}{1-\beta_r}$.

並且

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\vec{H}\right\}-\frac{1}{cr}\left\{\frac{\partial}{\partial t}\vec{H}\right\}=\frac{2e}{c^3r^2}\vec{r}_0\times\Big(\frac{1}{(1-\beta_r)^3}\dot{\vec{\alpha}}+3\frac{1}{c(1-\beta_r)^4}(\vec{\alpha}\cdot\vec{\beta})\vec{\alpha}$$

$$-\Big(\frac{1}{(1-\beta_r)^4}(\vec{r}_0\cdot\dot{\vec{\alpha}})+3\frac{1}{c(1-\beta_r)^5}(\vec{r}_0\cdot\vec{\alpha})^2\Big)(\vec{r}_0-\vec{\beta})\Big),$$

以上只保留 r^{-2} 階的各項.

取 z 軸沿 $\vec{\beta}$ 的方向,應用球面坐標和下面的式子

$$\int_{-1}^{1}\frac{dx}{(1-\beta x)^3}=\frac{2}{(1-\beta^2)^2},\quad \int_{-1}^{1}\frac{dx}{(1-\beta x)^4}=\frac{2}{3}\frac{(3+\beta^2)}{(1-\beta^2)^3},$$

$$\int_{-1}^{1}\frac{x\,dx}{(1-\beta x)^4}=\frac{8\beta}{3(1-\beta^2)^3},\quad \int_{-1}^{1}\frac{x^2\,dx}{(1-\beta x)^4}=\frac{2}{3}\frac{(1+3\beta^2)}{(1-\beta^2)^3},$$

$$\int_{-1}^{1}\frac{dx}{(1-\beta x)^5}=2\frac{(1+\beta^2)}{(1-\beta^2)^4},\quad \int_{-1}^{1}\frac{x\,dx}{(1-\beta x)^5}=\frac{2}{3}\frac{\beta(5+\beta^2)}{(1-\beta^2)^4},$$

$$\int_{-1}^{1}\frac{x^2\,dx}{(1-\beta x)^5}=\frac{2}{3}\frac{(1+5\beta^2)}{(1-\beta^2)^4},\quad \int_{-1}^{1}\frac{x^3\,dx}{(1-\beta x)^5}=2\frac{\beta(1+\beta^2)}{(1-\beta^2)^4},$$

計算積分 (8) 及 (9) 得到, 數學手續不錄,

$$\vec{E}(P_0,t_0)=\frac{e}{c^3}\Big(\frac{4}{3}\frac{1}{(1-\beta^2)^2}\dot{\vec{\alpha}}+4\frac{1}{c(1-\beta^2)^3}(\vec{\beta}\cdot\vec{\alpha})\vec{\alpha}\Big), \tag{11}$$

$$\vec{H}(P_0,t_0)=\vec{\beta}\times\vec{E}(P_0,t_0). \tag{11'}$$

用相對論的表示法,

$$\frac{d}{ds}x_m=\frac{1}{\sqrt{1-\beta^2}}(\beta_x,\beta_y,\beta_z,i),$$

$$\frac{d^2x_m}{ds^2}=\frac{1}{c^2}\frac{1}{(1-\beta^2)}(\alpha_x,\alpha_y,\alpha_z,0)+\frac{1}{c^2(1-\beta^2)^2}(\vec{\beta}\cdot\vec{\alpha})(\beta_x,\beta_y,\beta_z,i),$$

$$\frac{d^3x_m}{ds^3}=\frac{1}{c^3(1-\beta^2)^{3/2}}(\dot{\alpha}_x,\dot{\alpha}_y,\dot{\alpha}_z,0)+\frac{3}{c^4(1-\beta^2)^{5/2}}(\vec{\beta}\cdot\vec{\alpha})(\alpha_x,\alpha_y,\alpha_z,0)$$

$$+\frac{1}{c^3(1-\beta^2)^{5/2}}\Big((\vec{\beta}\cdot\dot{\vec{\alpha}})+\frac{1}{c}\alpha^2+\frac{4}{c(1-\beta^2)}(\vec{\beta}\cdot\vec{\alpha})^2\Big)(\beta_x,\beta_y,\beta_z,i),$$

(11) 式可以被寫成

$$F_{mn} = \frac{4}{3} e \left(\dot{x}_m \ddot{x}_n - \ddot{x}_m \dot{x}_n\right). \tag{12}$$

以上字母上加點數表示 $\frac{d}{ds}$ 的次數，F_{mn} 爲電磁場張量.
上式說明在電荷自己所在處的推後場的超前面積分正好是 Wheeler 與 Feymann "吸收體" 反作用場的二倍.

§ 3. 我們考慮一個加速運動的電子. 根據 (1) 式它的電磁場可以被寫成

$$\Psi = \frac{1}{2}(A + S_A) + \frac{1}{2}(R + S_R), \tag{13}$$

式中 A = 超前波， R = 推後波，
S_A = 超前面積分， 而 S_R = 推後面積分.
如倣照 Wheeler 與 Feymann 我們要求在遠處的場純粹是推後場，於是根據對 (7) 式所下的結論，

$$S_R = 0,$$

故 (13) 成爲

$$\Psi = \left(\frac{1}{2} A + \frac{1}{2} R\right) + \frac{1}{2} S_A. \tag{14}$$

他們更假定電子固有的場爲 $\frac{1}{2}A+\frac{1}{2}R$，這個場並不"作用於它本身". 電子只受到所謂"吸收體場"的作用. 從 (12) 及 (14) 二式顯然可見他們的假設等於說電子的場的 $\frac{1}{2}A + \frac{1}{2}R$ 部份對其本身無作用，或者說電子對自己的作用只有 $\frac{1}{2}S_A$ 的一部分. 這一部分使電子受到的 Lorentz 力爲

$$\begin{aligned}
K_n &= \frac{1}{2} \frac{e}{\sqrt{1-\beta^2}} \left(\vec{E} + \vec{\beta} \times \vec{H}\right), \qquad n = 1, 2, 3. \\
&= \frac{1}{2} e (1-\beta^2)^{-1/2} \left((1-\beta^2)\vec{E} + (\vec{\beta} \cdot \vec{E})\vec{\beta}\right) \\
&= \frac{1}{2} \frac{e^2}{c^3} \Big(\frac{4}{3}(1-\beta^2)^{-3/2} \dot{\vec{\alpha}} + \frac{4}{c}(1-\beta^2)^{-5/2} (\vec{\alpha} \cdot \vec{\beta}) \vec{\alpha} + \\
&\qquad + \frac{4}{3}(1-\beta^2)^{-5/2} (\dot{\vec{\alpha}} \cdot \vec{\beta}) \vec{\beta} + \frac{4}{c}(1-\beta^2)^{-7/2} (\vec{\alpha} \cdot \vec{\beta})^2 \vec{\beta}\Big),
\end{aligned}$$

$$K_4 = \frac{1}{2} e\, c(1-\beta^2)^{-1/2} (\vec{\beta} \cdot \vec{E})$$

$$= \frac{1}{2} \frac{e^2}{c^3} \left(\frac{4}{3} (1-\beta^2)^{-5/2} (\dot{\vec{\alpha}} \cdot \vec{\beta}) + \frac{4}{c} (1-\beta^2)^{-7/2} (\vec{\alpha} \cdot \vec{\beta})^2 \right),$$

或，用 (12)，

$$K_n = \frac{2}{3} e^2 (\dot{x}_n \ddot{x}_m - \dot{x}_m \ddot{x}_n) \dot{x}^m, \quad n = 1, 2, 3, 4 \tag{15}$$

這個力正好就是所謂"吸收體的反作用"． 所以他們的"吸收體的反作用"只不過是由他們假設遠處爲推後場的電子自己推後場的超前面積分[4]；因之他們的吸收體並無任何物理的眞實性．

參考文獻

[1] Wheeler and Feymann, *Rev. Mod. Phys.*, **17** (1945), 157.

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A CRITICAL ANALYSIS OF THE SO-CALLED ABSORBER THEORY OF RADIATION

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Abstract

Wheeler and Feymann have proposed an absorber theory of radiation[1], in which besides the half and half combination of retarded and advanced potential, the mechanism of radiation is ascribed to the existence of an absorber. Since the advanced and retarded solutions of the wave equation have been proved to be identical[2], a critical examination of their absorber theory is perhaps of interest. In this paper, we first prove that the

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4) 也相當於說，他們只用了一部分電磁場的作用，那可以起先假設的。

advanced surface integral for a purely advanced field and the retarded surface integral for a purely retarded field are both zero provided the source is within the surface. But neither the advanced surface integral of a retarded field nor the retarded surface integral of an advanced field vanishes. We calculated here the advanced surface integrals of the known retarded field of an accelerating electron at the position of the electron itself. In the relativistic notation the result is

$$F_{mn} = \frac{4}{3} e \left(\dot{x}_m \ddot{x}_n - \ddot{x}_m \dot{x}_n\right),$$

that is, the advanced surface integral of the retarded field at the charge itself is just twice the value of the reaction field of the "absorber" of Wheeler and Feymann.

Let us consider an accelerating charge and write its field in the certainly correct form

$$\Psi = \frac{1}{2}(A + S_A) + \frac{1}{2}(R + S_R),$$

and if we follow Wheeler and Feymann by *imposing* that the field at a distance is a pure retarded one then, by the vanishing of the retarded surface integral,

$$\Psi = \left(\frac{1}{2} A + \frac{1}{2} R\right) + \frac{1}{2} S_A.$$

Their postulate that "the charge does not react upon itself" is simply equivalent to that the part ½(A+R) does not function on the charge itself, that is the field acting on the charge only through $\frac{1}{2} S_A$, which gives, through integration a Lorentz force

$$K_n = \frac{2}{3} e^2 \left(\dot{x}_n \dddot{x}_m - \dot{x}_m \dddot{x}_n\right) \dot{x}^m, \quad n = 1, 2, 3, 4.$$

Therefore, the "reaction of their absorbers" are nothing but the advanced surface integral of the assumed "retarded field" of the charge itself, hence their absorbers can not possess any physical reality.