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椭圓截面的无限长金属棒对任意平面电磁波在有損耗的均匀各向同性无限媒质中的散射*

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提　　要

本文利用椭圓柱形坐标系，严格地分析了具有任意偏心度的椭圓截面的无限长理想导电的金属棒，放在有損耗的均匀各向同性的无限介质中，对从任意方向入射的均匀平面波的散射問題;并将散射場表示为馬許函数[5,6].

一、問題的結构

我們知道,在无源区域內,最普遍的場量可以用两个标量函数来表示[1,2],并且这两个标量函数可以任意选择[2]. 在本文中，选取了电赫兹矢量沿 z 軸的分量 π_z 和磁赫兹矢量沿 z 軸的分量 π'_z 作为这两个标量函数.

在媒质 $(\varepsilon, \mu, \sigma)$ 中[3],

$$\mathbf{E} = \mathbf{E}_e + \mathbf{E}_m = \nabla \times \nabla \times \Pi - \mu \nabla \times \frac{\partial \Pi'}{\partial t}, \tag{1}$$

$$\mathbf{H} = \mathbf{H}_e + \mathbf{H}_m = \nabla \times \left(\varepsilon \frac{\partial \Pi}{\partial t} + \sigma \Pi \right) + \nabla \times \nabla \times \Pi', \tag{2}$$

其中 π 和 π' 滿足下列方程:

$$\nabla \times \nabla \times \Pi - \nabla\nabla\cdot\Pi + \mu\varepsilon \frac{\partial^2 \Pi}{\partial t^2} + \mu\sigma \frac{\partial \Pi}{\partial t} = 0, \tag{3}$$

$$\nabla \times \nabla \times \Pi' - \nabla\nabla\cdot\Pi' + \mu\varepsilon \frac{\partial^2 \Pi'}{\partial t^2} + \mu\sigma \frac{\partial \Pi'}{\partial t} = 0. \tag{4}$$

利用椭圓柱形坐标系 (ξ, η, z), ξ 为常数的曲面代表一系列同焦点的椭圓柱面, η 为常数的曲面代表一系列与上述椭圓柱面正交的双曲綫柱面, z 为常数的平面代表一系列与椭圓柱軸垂直的截面. 設 $2f$ 为所研究的无限长理想导电的椭圓柱体 $(\xi = \xi_0)$ 的焦距, $2a$ 为其长軸, $2b$ 为其短軸. 令該椭圓柱体的軸与 z 軸重合. 入射的均匀平面波如图 1 所示. 这时,我們有

$$\xi \geqslant 1, \quad -1 \leqslant \eta \leqslant 1, \tag{5}$$

$$h_\xi = f\left(\frac{\xi^2 - \eta^2}{\xi^2 - 1}\right)^{\frac{1}{2}}, \quad h_\eta = f\left(\frac{\xi^2 - \eta^2}{1 - \eta^2}\right)^{\frac{1}{2}}, \quad h_z = 1, \tag{6}$$

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其中 h_ξ, h_η, h_z 是度规系数. 因此,无限小的长度元可以表示为

$$\mathbf{ds} = \mathbf{i}_\xi h_\xi d\xi + \mathbf{i}_\eta h_\eta d\eta + \mathbf{i}_z dz. \tag{7}$$

从方程(1)和(2)得电型和磁型的場分量如下 ($\Pi = \mathbf{i}_z \pi_z$, $\Pi' = \mathbf{i}_z \pi'_z$):

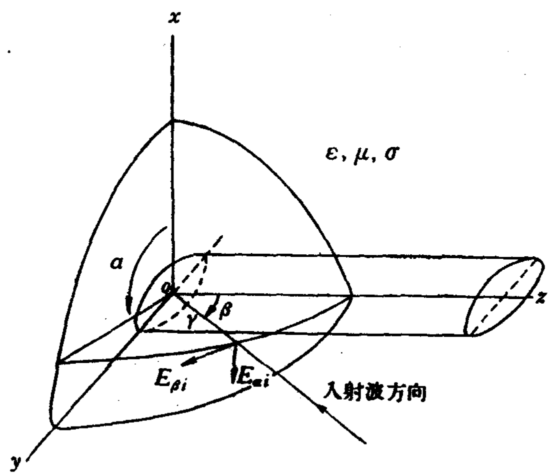

图 1

$$\left.\begin{aligned}
&E_{e\xi} = \frac{1}{h_\xi}\frac{\partial^2 \pi_z}{\partial z\,\partial\xi}, && H_{e\xi} = \left(\varepsilon\frac{\partial}{\partial t} + \sigma\right)\frac{1}{h_\eta}\frac{\partial \pi_z}{\partial\eta},\\
&E_{e\eta} = \frac{1}{h_\eta}\frac{\partial^2 \pi_z}{\partial z\,\partial\eta}, && H_{e\eta} = -\left(\varepsilon\frac{\partial}{\partial t} + \sigma\right)\frac{1}{h_\xi}\frac{\partial \pi_z}{\partial\xi},\\
&E_{ez} = -\frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial\xi}\left(\frac{h_\eta}{h_\xi}\frac{\partial\pi_z}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\frac{h_\xi}{h_\eta}\frac{\partial\pi_z}{\partial\eta}\right)\right], && H_{ez} = 0;
\end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned}
&E_{m\xi} = -\frac{\mu}{h_\eta}\frac{\partial^2\pi'_z}{\partial t\,\partial\eta}, && H_{m\xi} = \frac{1}{h_\xi}\frac{\partial^2\pi'_z}{\partial z\,\partial\xi},\\
&E_{m\eta} = \frac{\mu}{h_\xi}\frac{\partial^2\pi'_z}{\partial t\,\partial\xi}, && H_{m\eta} = \frac{1}{h_\eta}\frac{\partial^2\pi'_z}{\partial z\,\partial\eta},\\
&E_{mz} = 0, && H_{mz} = -\frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial\xi}\left(\frac{h_\eta}{h_\xi}\frac{\partial\pi'_z}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\frac{h_\xi}{h_\eta}\frac{\partial\pi'_z}{\partial\eta}\right)\right].
\end{aligned}\right\} \tag{9}$$

其中 π_z 和 π'_z 都必須满足下列的标量波动方程($\Pi = \mathbf{i}_z \pi_z$, $\Pi' = \mathbf{i}_z \pi'_z$ 代入方程(3)和(4)得):

$$\nabla^2\psi - \mu\varepsilon\frac{\partial^2\psi}{\partial t^2} - \mu\sigma\frac{\partial\psi}{\partial t} = 0. \tag{10}$$

表示为椭圓柱形坐标,它的形式为:

$$\frac{1}{h_\xi h_\eta}\frac{\partial}{\partial\xi}\left(\frac{h_\eta}{h_\xi}\frac{\partial\psi}{\partial\xi}\right) + \frac{1}{h_\xi h_\eta}\frac{\partial}{\partial\eta}\left(\frac{h_\xi}{h_\eta}\frac{\partial\psi}{\partial\eta}\right) + \frac{\partial^2\psi}{\partial z^2} - \mu\varepsilon\frac{\partial^2\psi}{\partial t^2} - \mu\sigma\frac{\partial\psi}{\partial t} = 0. \tag{11}$$

在正弦情况下,(11)的解可以写成

$$\psi = F(\xi,\eta)e^{ihz - i\omega t}. \tag{12}$$

将(12)式代入(11)式得

$$\frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \xi}\left(\frac{h_\eta}{h_\xi}\frac{\partial F}{\partial \xi}\right)+\frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \eta}\left(\frac{h_\xi}{h_\eta}\frac{\partial F}{\partial \eta}\right)+(k^2-h^2)F=0, \tag{13}$$

其中

$$k^2=\omega^2\mu\varepsilon+i\,\omega\mu\sigma. \tag{14}$$

这里所用的时間因子是 $e^{-i\omega t}$. 将(12)式代入(8)和(9)式得

电型場 (TM 型):　　　　磁型場 (TE 型):

$$\left.\begin{aligned}
E_{e\xi}&=\frac{ih}{h_\xi}\frac{\partial\psi}{\partial\xi}, & E_{m\xi}&=i\,\mu\omega\frac{1}{h_\eta}\frac{\partial\psi'}{\partial\eta},\\
E_{e\eta}&=\frac{ih}{h_\eta}\frac{\partial\psi}{\partial\eta}, & E_{m\eta}&=-i\,\mu\omega\frac{1}{h_\xi}\frac{\partial\psi'}{\partial\xi},\\
E_{ez}&=(k^2-h^2)\psi, & E_{mz}&=0,\\
H_{e\xi}&=-\frac{ik^2}{\mu\omega}\frac{1}{h_\eta}\frac{\partial\psi}{\partial\eta}, & H_{m\xi}&=\frac{ih}{h_\xi}\frac{\partial\psi'}{\partial\xi},\\
H_{e\eta}&=\frac{ik^2}{\mu\omega}\frac{1}{h_\xi}\frac{\partial\psi}{\partial\xi}, & H_{m\eta}&=\frac{ih}{h_\eta}\frac{\partial\psi'}{\partial\eta},\\
H_{ez}&=0; & H_{mz}&=(k^2-h^2)\psi'.
\end{aligned}\right\} \tag{15}$$

(15)式中的 $\psi=\pi_z$, $\psi'=\pi'_z$ 都是方程(10)的解.

现在的問題就是要求解(13)式. 将(6)式代入(13)式得:

$$\sqrt{\xi^2-1}\frac{\partial}{\partial\xi}\left(\sqrt{\xi^2-1}\frac{\partial F}{\partial\xi}\right)+\sqrt{1-\eta^2}\frac{\partial}{\partial\eta}\left(\sqrt{1-\eta^2}\frac{\partial F}{\partial\eta}\right)+ \\ +f^2(k^2-h^2)(\xi^2-\eta^2)F=0. \tag{16}$$

利用分离变数法,令

$$F(\xi,\eta)=F_1(\xi)\,F_2(\eta), \tag{17}$$

并代入方程(16)中得常微分方程:

$$(\xi^2-1)\frac{d^2F_1}{d\xi^2}+\xi\frac{dF_1}{d\xi}+(f^2\lambda^2\xi^2-b)F_1=0, \tag{18}$$

$$(1-\eta^2)\frac{d^2F_2}{d\eta^2}-\eta\frac{dF_2}{d\eta}+(b-f^2\lambda^2\eta^2)F_2=0, \tag{19}$$

其中 $\lambda^2=k^2-h^2$.

作簡单的变换,令

$$\xi=\cosh u,\quad \eta=\cos v,\quad z=z, \tag{20}$$

則 u 与 v 和直角坐标的关系是:

$$x=f\cosh u\cos v,\quad y=f\sinh u\sin v,\quad z=z. \tag{21}$$

将(20)式代入(18)和(19)式得

$$\frac{d^2F_1(u)}{du^2}+(f^2\lambda^2\cosh^2u-b)\,F_1(u)=0, \tag{22}$$

$$\frac{dF_2(v)}{dv^2}+(b-f^2\lambda^2\cos^2v)\,F_2(v)=0. \tag{23}$$

这两个方程有同样的形式，称为“馬許微分方程”(Mathieu differential equations), $F_1(u)$

称为馬許幅径函数，$F_2(v)$ 称为馬許角函数，式中 b 为分离常数，后者具有一系列的特征値 $b_1, b_2, \cdots, b_m, \cdots$. 对应于每个特征値，方程(23)具有奇数周期解和偶数周期解．用 b_m^0 代表一系列分离常数，其对应的解为[3,4]：

$$So_m^1(f\lambda,\ \cos v) = \sum_{n=1,2}^{\infty}{}' F_n^m \sin nv,\quad (m = 1, 2, 3, \cdots), \tag{24}$$

称为第一类奇馬許角函数，式中一撇表示当 m 为奇数时 n 取奇数而当 m 为偶数时 n 取偶数．用 b_m^e 代表另一系列分离常数，其对应的解为[3,4]

$$Se_m^1(f\lambda,\ \cos v) = \sum_{n=0,1}^{\infty}{}' D_n^m \cos nv, \tag{25}$$

称为第一类偶馬許角函数．(24)式的归正法（Normalization）是在 $v = 0$ 处[3]，則

$$\left[\frac{d}{dv} So_m^1(f\lambda,\ \cos v)\right]_{v=0} = 1, \tag{26}$$

从而得

$$\sum_{n=1,2}^{\infty}{}' n F_n^m = 1. \tag{27}$$

(25)式的归正法是在 $v = 0$ 处，則

$$Se_m^1(f\lambda, 1) = 1, \tag{28}$$

从而得

$$\sum_{n=0,1}^{\infty}{}' D_n^m = 1. \tag{29}$$

对于角函数，为了在 $v = 0$ 处函数为有限値，我們选取第一类馬許角函数 $So_m^1(f\lambda,\ \cos v)$ 和 $Se_m^1(f\lambda,\ \cos v)$；对于幅径函数，为了在无穷远处为正规，我們选取第四类馬許幅径函数[4]：

$$Ro_m^4(f\lambda, \cosh u) = \frac{\sqrt{\cosh^2 u - 1}}{\cosh u}\sqrt{\frac{\pi}{2}}\sum_{n=1,2}^{\infty}{}' i^{m-n} n F_n^m H_n^{(2)}(f\lambda, \cosh u), \tag{30}$$

$$Re_m^4(f\lambda, \cosh u) = \sqrt{\frac{\pi}{2}}\sum_{n=0,1}^{\infty}{}' i^{m-n} D_n^m H_n^{(2)}(f\lambda, \cosh u), \tag{31}$$

式中 $H_n^{(2)}(f\lambda \cosh u)$ 为汉克耳函数．

因此，(13)式的普遍解可以写成：

$$\begin{aligned} F(\xi, \eta) &= F_1(\xi) F_2(\eta) = F_1(u) F_2(v) = \\ &= \sum_{m=0}^{\infty} \{ A_m^e(\lambda) Se_m^1(f\lambda,\ \cos v) Re_m^4(f\lambda, \cosh u) + \\ &\quad + A_m^0(\lambda) So_m^1(f\lambda,\ \cos v) Ro_m^4(f\lambda, \cosh u)\}. \end{aligned} \tag{32}$$

从而得 ψ 和 ψ' 如下：

$$\begin{aligned} \psi = \sum_{m=0}^{\infty} \{ &A_m^e(\lambda) Se_m^1(f\lambda,\ \cos v) Re_m^4(f\lambda, \cosh u) + \\ &+ A_m^0(\lambda) So_m^1(f\lambda,\ \cos v) Ro_m^4(f\lambda, \cosh u)\}\, e^{ihz-i\omega t}, \end{aligned} \tag{33}$$

$$\psi' = \sum_{m=0}^{\infty} \{ B_m^e(\lambda)\, Se_m^1(f\lambda,\ \cos v)\, Re_m^4(f\lambda, \cosh u) + $$
$$+ B_m^0(\lambda)\, So_m^1(f\lambda,\ \cos v)\, Ro_m^4(f\lambda, \cosh u) \}\, e^{ihz - i\omega t}. \qquad (34)$$

(32)到(34)式中的系数 $A_m^e(\lambda)$, $A_m^0(\lambda)$, $B_m^e(\lambda)$ 和 $B_m^0(\lambda)$ 是待定的系数．它們可以利用柱体表面的边界条件来确定．

二、問題的解(求散射場)

設入射平面波的传播方向用 (α, β) 表示如图 1，将电場強度分解为两个互相垂直的分量 E_α 和 E_β．E_α 激起磁型散射波，E_β 激起电型散射波．$E_{\alpha i}$ 和 $E_{\beta i}$ 的矢量和代表任意偏振的入射平面波的电場矢量．$E_{\beta i}$ 可以写成：

$$E_{\beta i} = E_0\, e^{-ikr - i\omega t}, \qquad (35)$$

其中 r 是到坐标原点的距离如图 1 所示：

$$r = f \sin\beta\, (\cosh u\, \cos v \cos\alpha + \sinh u\, \sin v \sin\alpha) + z\cos\beta. \qquad (36)$$

$E_0\, e^{-ikr}$ 可以展开为馬許函数如下[3]：

$$E_0\, e^{-ikr} = \sqrt{8\pi}\, E_0 \sum_{m=0}^{\infty} (-i)^m \Big\{ \frac{1}{N_m^e} Re_m^1(f\lambda, \cosh u)\, Se_m^1(f\lambda,\ \cos v)\, Se_m^1(f\lambda,\ \cos\alpha) + $$
$$+ \frac{1}{N_m^0} Ro_m^1(f\lambda, \cosh u)\, So_m^1(f\lambda,\ \cos v)\, So_m^1(f\lambda,\ \cos\alpha) \Big\}\, e^{-ikz\cos\beta}, \qquad (37)$$

其中

$$N_m^e = \pi \Big\{ 2\,(D_n^m)^2 + \sum_{m=2}^{\infty}{}' (D_n^m)^2 \Big\}, \quad n = \text{偶数}, \qquad (38)$$

$$N_m^e = \pi \sum_{m=1}^{\infty}{}' (D_n^m)^2, \qquad n = \text{奇数}, \qquad (39)$$

$$N_m^0 = \pi \sum_{m=1,2}^{\infty}{}' (F_n^m)^2. \qquad (40)$$

这时 $E_{\beta i}$ 沿 z 軸的分量 $E_{\beta iz}$ 可以写成(暫时弃去时間因子 $e^{-i\omega t}$)：

$$E_{\beta iz} = -E_{\beta i} \sin\beta = -\sqrt{8\pi}\, E_0\, \sin\beta \sum_{m=0}^{\infty} (-i)^m \Big\{ \frac{1}{N_m^e} Re_m^1(f\lambda, \cosh u)\, Se_m^1(f\lambda, \cos v) \times $$
$$\times Se_m^1(f\lambda,\ \cos\alpha) + \frac{1}{N_m^0} Ro_m^1(f\lambda, \cosh u) So_m^1(f\lambda, \cos v) So_m^1(f\lambda, \cos\alpha) \Big\}\, e^{-ikz\cos\beta}. \qquad (41)$$

为了方便起見，电型散射場中的 E_{ez} 可以表示为(弃去时間因子 $e^{-i\omega t}$)：

$$E_{ez} = \lambda^2 \psi = \lambda^2 \sum_{m=0}^{\infty} \{ A_m^e(\lambda)\, Se_m^1(f\lambda,\ \cos v)\, R_{em}^4(f\lambda, \cosh u) + $$
$$+ A_m^0(\lambda)\, So_m^1(f\lambda,\ \cos v) \times Ro_m^4(f\lambda, \cosh u) \}\, e^{ihz}$$
$$= -\sqrt{8\pi}\, E_0 \sin\beta \sum_{m=0}^{\infty} (-i)^m \Big\{ \frac{c_n}{N_m^e} Re_m^4(f\lambda, \cosh u)\, Se_m^1(f\lambda, \cos v)\, Se_m^1(f\lambda, \cos\alpha) + $$
$$+ \frac{d_n}{N_m^0} Ro_m^4(f\lambda, \cosh u)\, So_m^1(f\lambda,\ \cos v)\, So_m^1(f\lambda,\ \cos\alpha) \Big\}\, e^{-ikz\cos\beta}, \qquad (42)$$

其中已将常数 A_m^e 和 A_m^0 分别用另两个常数 c_n 和 d_n 来表示:

$$\left.\begin{aligned} A_m^e &= \frac{-\sqrt{8\pi}\,E_0 \sin\beta\, Se_m^1(f\lambda, \cos\alpha)}{\lambda^2 N_m^e}(-i)^m c_n, \\ A_m^0 &= \frac{-\sqrt{8\pi}\,E_0 \sin\beta\, So_m^1(f\lambda, \cos\alpha)}{\lambda^2 N_m^0}(-i)^m d_n; \end{aligned}\right\} \tag{43}$$

同时已令 $h = -k\cos\beta$. 以上这样作的目的就是要将入射场分量 $E_{\beta iz}$ 和散射场分量 E_{ez} 表示为同样的形式,以便于利用柱面上的边界条件来求常数 c_n 和 d_n.

根据理想导体柱面上的边界条件 ($\xi = \xi_0$ 或 $u = u_0$):

$$(E_{ez} + E_{\beta iz})_{u=u_0} = 0, \tag{44}$$

$$(E_{e\eta} + E_{\beta i\eta})_{u=u_0} = 0, \tag{45}$$

并将(41)和(42)式代入即得常数 c_n 和 d_n 如下:

$$c_n = -\frac{Re_m^1(f\lambda, \cosh u_0)}{Re_m^4(f\lambda, \cosh u_0)}, \tag{46}$$

$$d_n = -\frac{Ro_m^1(f\lambda, \cosh u_0)}{Ro_m^4(f\lambda, \cosh u_0)}, \tag{47}$$

因而得电型散射场如下:

$$\begin{aligned} E_{ez} = \sqrt{8\pi}\,E_0 \sin\beta \sum_{m=0}^{\infty}(-i)^m \cdot \\ \cdot \left\{ \frac{Re_m^1(f\lambda, \cosh u_0)}{N_m^e Ro_m^1(f\lambda, \cosh u_0)} Re_m^4(f\lambda, \cosh u)\, Se_m^1(f\lambda, \cos v)\, Se_m^1(f\lambda, \cos\alpha) + \right. \\ \left. + \frac{Ro_m^1(f\lambda, \cosh u_0)}{N_m^0 Ro_m^4(f\lambda, \cosh u_0)} Ro_m^4(f\lambda, \cosh u) So_m^1(f\lambda, \cos v)\, So_m^1(f\lambda, \cos\alpha)\right\} e^{-ihz\cos\beta}, \end{aligned} \tag{48}$$

$$\begin{aligned} \psi &= E_{ez}/\lambda^2, & H_{ez} &= 0, \\ E_{e\eta} &= \frac{ih}{h_\eta}\frac{\partial\psi}{\partial\eta}, & H_{e\eta} &= \frac{ik^2}{\omega\mu}\frac{1}{h_\xi}\frac{\partial\psi}{\partial\xi}, \\ E_{e\xi} &= \frac{ih}{h_\xi}\frac{\partial\psi}{\partial\xi}, & H_{e\xi} &= -\frac{ik^2}{\omega\mu}\frac{1}{h_\eta}\frac{\partial\psi}{\partial\eta}. \end{aligned} \tag{49}$$

激起磁型的平面波可以写成:

$$E_{\alpha i} = E_1\, e^{-ikr-i\omega t}, \tag{50}$$

对应的标量函数可以写成:

$$\psi_i' = \frac{-E_1 e^{-ikr}}{z\,k^2 \sin\beta}. \tag{51}$$

因此, ψ' 可以表示为:

$$\begin{aligned} \psi' = \psi_i' + \psi_s = \frac{-\sqrt{8\pi}\,E_1}{z\,k^2 \sin\beta}\sum_{m=0}^{\infty}(-i)^m \left\{ \frac{1}{N_m^e}[Re_m^1(f\lambda, \cosh u) + \right. \\ + c_n' Re_m^4(f\lambda, \cosh u)] \times Se_m^1(f\lambda, \cos v)\, Se_m^1(f\lambda, \cos\alpha) + \\ + \frac{1}{N_m^0}[Ro_m^1(f\lambda, \cosh u) + d_n' Ro_m^4(f\lambda, \cosh u)] \times \end{aligned}$$

$$\times\ So^1_m(f\lambda,\ \cos\ v)\, So^1_m(f\lambda,\ \cos\ \alpha)\Big\}\ e^{-ikz\cos\beta} \tag{52}$$

其中 ψ_s 为对应于散射磁型場的标量函数，z 是媒質的波阻抗.

根据理想导体柱面上的边界条件（$\xi=\xi_0$ 或 $u=u_0$）：

$$(E_{m\eta})_{u=u_0}=\left[-i\,\omega\mu\frac{1}{h_\xi}\frac{\partial\psi'}{\partial\xi}\right]_{u=u_0}=0, \tag{53}$$

$$(E_{mz})_{u=u_0}=0, \tag{54}$$

得常数 c'_n 和 d'_n 如下：

$$c'_n=-\frac{\left[\dfrac{d}{du}Re^1_m(f\lambda,\cosh u)\right]_{u=u_0}}{\left[\dfrac{d}{du}Re^4_m(f\lambda,\cosh u)\right]_{u=u_0}}, \tag{55}$$

$$d'_n=-\frac{\left[\dfrac{d}{du}Ro^1_m(f\lambda,\cosh u)\right]_{u=u_0}}{\left[\dfrac{d}{du}Ro^4_m(f\lambda,\cosh u)\right]_{u=u_0}}. \tag{56}$$

因此 ψ' 和磁型的散射場可以写成：

$$\begin{aligned}\psi'=&\frac{-\sqrt{8\pi}\,E_1}{E\,k^2\sin\beta}\sum_{m=0}^{\infty}(-i)^m\Bigg\{\frac{1}{N^e_m}\Bigg[Re^1_m(f\lambda,\cosh u)-\\&-\frac{\left[\dfrac{d}{du}Re^1_m(f\lambda,\cosh u)\right]_{u=u_0}}{\left[\dfrac{d}{du}Re^4_m(f\lambda,\cosh u)\right]_{u=u_0}}Re^4_m(f\lambda,\cosh u)\Bigg]\times\\&\times\ Se^1_m(f\lambda,\ \cos\ v)\,Se^1_m(f\lambda,\ \cos\ \alpha)+\frac{1}{N^0_m}\Bigg[Ro^1_m(f\lambda,\cosh u)+\\&+\frac{\left[\dfrac{d}{du}Ro^1_m(f\lambda,\cosh u)\right]_{u=u_0}}{\left[\dfrac{d}{du}Ro^4_m(f\lambda,\cosh u)\right]_{u=u_0}}Ro^4_m(f\lambda,\cosh u)\Bigg]\times\\&\times\ So^1_m(f\lambda,\ \cos\ v)\,So^1_m(f\lambda,\ \cos\ \alpha)\Bigg\}\ e^{-ikz\cot\beta},\end{aligned} \tag{57}$$

$$\left.\begin{aligned}E_{m\xi}&=\ \ i\,\omega\mu\frac{1}{h_\eta}\frac{\partial\psi'}{\partial\eta}, & H_{m\xi}&=\frac{ih}{h_\xi}\frac{\partial\psi'}{\partial\xi},\\E_{m\eta}&=-i\,\omega\mu\frac{1}{h_\xi}\frac{\partial\psi'}{\partial\xi}, & H_{m\eta}&=\frac{ih}{h_\eta}\frac{\partial\psi'}{\partial\eta},\\E_{mz}&=0, & H_{mz}&=\lambda^2\psi'.\end{aligned}\right\} \tag{58}$$

总的散射場是电型場和磁型場的迭加，即(49)式和(58)式的迭加.

参 考 文 献

[1] S. A. Schelkunoff, Electromagnetic Waves (1943).
[2] A. Nisbet, *Proc. Roy. Soc.*, A, **231** (1955), 250.
[3] J. A. Stratton, Electromagnetic Theory (1941), 32, 375—387.
[4] J. A. Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, F. J. Corbato, Spheroidal Wave Functions, p. 45—51.
[5] B. Seiger, *Ann. der Phys.* **27**, p 626, Nov. 1908.
[6] P. M. Morse and P. J. Rubenstein, *Phys. Rev.*, **54** (1938), 895.

THE SCATTERING OF AN ARBITRARY UNIFORM PLANE ELECTROMAGNETIC WAVE BY AN INFINITE METAL ROD OF ELLIPTICAL CROSS SECTION IN AN INFINITE HOMOGENEOUS ISOTROPIC MEDIUM WITH FINITE CONDUCTIVITY

Jen Lang

Abstract

The scattering of an arbitrarily polarized uniform plane electromagnetic wave by an infinite metal rod of elliptical cross section with any eccentricity in a homogeneous isotropic medium of infinite extent with finite conductivity is analized rigorously as a boundary value problem using elliptical cylinder coordinates, the scattered field arround the rod is obtained in terms of calculable Mathieu functions.