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正氬子 ($^3S_1$) 三光子衰变的顶角及电子自能辐射修正*

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一

近年来,由于实验精度提高,发觉正氬子(即电子偶素,也有叫"正电子代核原子"的)($^3S_1$)三光子衰变率的初级微扰理论值对于实验值有肯定的偏离,以致提出了计算这个衰变率的次一级微扰值即辐射修正的问题[1].

这个衰变率的初级微扰值实际上算的是静止自由电子偶的三光子湮灭反应率,只是取不同的归一化体积以反映束缚态的影响.本文也是计算静止自由电子偶三光子湮灭反应率的辐射修正.按照量子电动力学,这个反应过程的辐射修正($\propto e^5$)应当包括图 1 所列各 Feynman 图,其它有关的 e^5 级图可略去(不是含具有奇数顶点的闭电子线就是对外

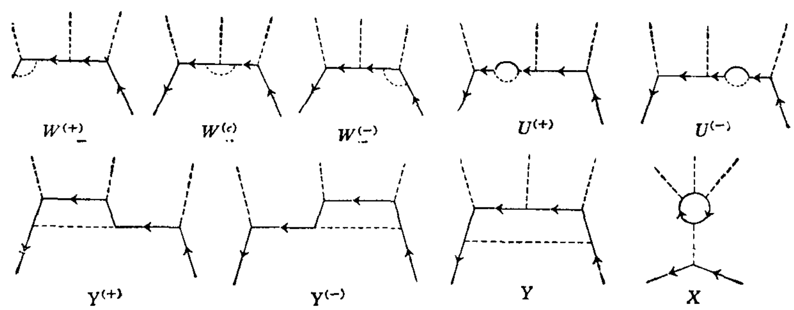

图 1

部线的自能修正).本文计算了由二级电子自能及顶角引起的辐射修正,即图 1 中头五个图的贡献.

按照量子场论,自由电子偶三光子湮灭微分反应率[2]是

$$dW_{3\gamma} = \frac{e^6}{8\omega_1\omega_2\omega_3} \cdot \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^5 \left(\frac{E_+E_-}{m^2}\right)\Omega}$$

$$\cdot \frac{1}{4}\sum_{(r)}\sum_{(s)} \left|\bar{v}_r(-\boldsymbol{p}_+)\sum_P Q(123)u_s(\boldsymbol{p}_-)\right|^2 \delta(p_+ + p_- - k_1 - k_2 - k_3), \quad (1)$$

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其中 $p_\pm=(\boldsymbol{p}_\pm,\ iE_\pm)$ 是正、负电子四维动量，$k_j=(\boldsymbol{k}_j,\ i\omega_j)$ $(j=1,2,3)$ 是三个光子的四维动量；$\bar{v}_r$, u_s 是对应于正、负电子的 Dirac 平面波幅；第一个求和号 $\left(\times\frac{1}{4}\right)$ 是对始态正、负电子极化状态取平均，第二个求和号是对末态三个光子的极化状态求和，第三个求和号是对三个光子之所有可能的排列求和；而 Ω 是归一化体积，e, m 是电子的电荷与质量；至于 $Q(123)$，则是散射矩阵中的四维矩阵部份，可按微扰论的 Feynman 规则写出.

我们计算 $dW_{3\gamma}$ 的办法是把 Dirac 矩阵(采用通常表象)化为 2×2 的块，再利用 Pauli 矩阵的性质直接算出 $\bar{v}_rQ(123)u_s$，而不用通常的四维求迹办法．看来，我们所用的办法对粒子数较多的某些情形反而会简便些，但所付的代价是需具体分析三个光子的极化矢量、波矢量间的几何关系．把 $Q(123)$ 写成分块的形式：

$$Q(123)=\begin{pmatrix}K(123) & L(123)\\ M(123) & N(123)\end{pmatrix},\tag{2}$$

其中每个块 K, L, M, N 都是作用在二维旋量空间(记其基底旋量为 ξ_r, $r=1,2$)上的二维矩阵，就会有

$$\begin{aligned}\bar{v}_r(-\boldsymbol{p}_+)Q(123)u_s(\boldsymbol{p}_-)&=\xi_r^+\left\{M(123)+\frac{\boldsymbol{p}_+\cdot\boldsymbol{\sigma}K(123)}{|E_+|+m}+\frac{N(123)\boldsymbol{p}_-\cdot\boldsymbol{\sigma}}{|E_-|+m}\right.\\&\left.+\frac{\boldsymbol{p}_+\cdot\boldsymbol{\sigma}L(123)\boldsymbol{p}_-\cdot\boldsymbol{\sigma}}{(|E_+|+m)(|E_-|+m)}\right\}\xi_s=\sqrt{\frac{(|E_+|+m)(|E_-|+m)}{4m^2}}\\&\cdot\xi_r^+\{A(123)I+\boldsymbol{B}(123)\cdot\boldsymbol{\sigma}\}\xi_s,\end{aligned}\tag{3}$$

最后一步是把所得二维矩阵按幺矩阵及 Pauli 矩阵 $\boldsymbol{\sigma}$ 展开，以下有时称展开系数 A 为有关二维矩阵的标量部份，称系数 $\boldsymbol{B}$ 为其矢量部份，另外还对任意三维矢量 $\boldsymbol{a}$ 引入记号 $\check{a}\equiv\boldsymbol{a}\cdot\boldsymbol{\sigma}$. (3) 式中的 A, $\boldsymbol{B}$ 可通过运用 Pauli 矩阵性质 $(\check{a}\check{b}=(\boldsymbol{a}\cdot\boldsymbol{b})+i(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{\sigma})$ 而由(3)式之第二步求得．在本文的 $\boldsymbol{p}_+=\boldsymbol{p}_-=0$ 情形，只需计算M块，故大为简化.

简记 $A\equiv\sum\limits_P A(123)$, $\boldsymbol{B}\equiv\sum\limits_P\boldsymbol{B}(123)$，不对电子极化状态求和，由 (3) 式则有

$$\begin{aligned}\left|\bar{v}_r(0)\sum_P Q(123)u_s(0)\right|^2&=|A|^2\delta_{rs}\\&+2\delta_{rs}\sum_{j=1}^{3}\mathrm{Re}(AB_j^*)(\sigma_j)_{rs}+\sum_{j,k=1}^{3}B_j^*B_k(\sigma_j)_{rs}(\sigma_k)_{sr};\end{aligned}\tag{4}$$

当对电子极化状态取平均时，就有

$$\begin{aligned}\frac{1}{4}\sum_{(r)}\left|v_r(0)\sum_P Q(123)u_s(0)\right|^2&=\frac{1}{4}\mathrm{Sp}((A+\check{B})(A^*+\check{B}^*))\\&=\frac{1}{2}(|A|^2+\boldsymbol{B}\cdot\boldsymbol{B}^*).\end{aligned}\tag{5}$$

如果 $Q(123)$ 分为数量级不同 (e^3 及 e^5) 的两部份 $Q(123)=Q^{(3)}(123)+Q^{(5)}(123)$，对应地也有 $A(123)=A^{(3)}(123)+A^{(5)}(123)$, $\boldsymbol{B}(123)=\boldsymbol{B}^{(3)}(123)+\boldsymbol{B}^{(5)}(123)$，从而准确至 e^8 量级有

$$\frac{1}{4}\sum_{(r)}\left|\bar{v}_r(0)\sum_P Q(123)u_s(0)\right|^2=\left\{\frac{1}{2}(|A^{(3)}|^2+\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*})\right\}$$

$$+\{\mathrm{Re}(A^{(3)*}\cdot A^{(5)})+\mathrm{Re}(\boldsymbol{B}^{(3)*}\cdot\boldsymbol{B}^{(5)})\},\tag{6}$$

其中第一部份正是初级微扰值，第二部份就是辐射修正的贡献.

把上述办法用到本问题的初级微扰，可求得

$$\bar{v}_r(0)Q^{(3)}(123)u_s(0)=\frac{i}{4m^2}\xi_r^+(\check{e}_1\check{e}_2\check{e}_3+\check{e}_1'\check{e}_2'\check{e}_3')\xi_s,$$

其中 $\boldsymbol{e}_i$ 是第 i 个光子的一个极化基底矢量，$\boldsymbol{e}_i'=\boldsymbol{n}_i\times\boldsymbol{e}_i$ 则是另一个极化基底矢量，$\boldsymbol{n}_i=\boldsymbol{k}_i/\omega_i$ 是波矢量，由此可以看出：

$$A^{(3)}=\sum_P A^{(3)}(123)=0,$$

$$\boldsymbol{B}^{(3)}=\sum_P\boldsymbol{B}^{(3)}(123)=\frac{i}{2m^2}\{\boldsymbol{e}_1(\boldsymbol{e}_2\cdot\boldsymbol{e}_3-\boldsymbol{e}_2'\cdot\boldsymbol{e}_3')$$

$$+\boldsymbol{e}_1'(\boldsymbol{e}_2\cdot\boldsymbol{e}_3'+\boldsymbol{e}_2'\cdot\boldsymbol{e}_3)+\text{循环项}\}\equiv\frac{i}{2m^2}\boldsymbol{B}_0^{(3)}.\tag{7}$$

将 (7) 式代入 (4) 和 (5) 式，可以得到允许考虑电子、光子处于特定极化状态的衰变率. 进一步再对光子极化状态求和(注意，如取 $\boldsymbol{e}_i'$ 为一个独立极化基底矢量，则应取 $\boldsymbol{n}_i\times\boldsymbol{e}_i'=-\boldsymbol{e}_i$ 为另一个独立极化基底矢量！)，并利用 ($\boldsymbol{e}_i$, $\boldsymbol{e}_i'$, $\boldsymbol{n}_i$) 组成空间基底的性质，就能得到

$$\frac{1}{4}\sum_{(r)}\sum_{(v)}\left|\bar{v}_r(0)\sum_P Q^{(3)}(123)u_s(0)\right|^2=\frac{1}{2}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*}$$

$$=\frac{1}{m^4}\{(1-n_1\cdot n_2)^2+\text{循环项}\},$$

这与文献 [2] 中 § 33 引述的结果完全一致.

应用 (6), (7) 及 (1) 式，衰变率辐射修正与初级微扰值 $W_{3\gamma}^{(3)}$ 之比即可表示为

$$\frac{\delta W^{(5)}}{W_{3\gamma}^{(3)}}=\frac{\int_0^m d\omega_1\int_{m-\omega_1}^m d\omega_3\sum_P\mathrm{Re}[\boldsymbol{B}^{(5)}(123)\cdot\boldsymbol{B}^{(3)*}]}{\int_0^m d\omega_1\int_{m-\omega_1}^m d\omega_3\left(\frac{1}{2}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*}\right)}$$

$$=\left(\frac{3!}{16}m^4\right)\frac{\int_0^2 d\tau_1\int_{2-\tau_1}^2 d\tau_3\mathrm{Re}[\boldsymbol{B}^{(3)*}\cdot\boldsymbol{B}^{(5)}(123)]}{(\pi^2-9)},\tag{8}$$

这里利用了 $\int_0^m d\omega_1\int_{m-\omega_1}^m d\omega_3\left(\frac{1}{2}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)*}\right)=\frac{4}{m^4}(\pi^2-9)$ (参见文献 [2] § 33)，$\tau_i=\frac{2\omega_i}{m}$.

二

现在我们进入主题. 按 Feynman 规则，图 1 中头五个图对 $Q^{(5)}(123)$ 的贡献是

$$U^{(+)}(123)=\hat{e}_1\frac{i\hat{f}_1'-m}{f_1'^2+m^2}\Sigma_R^{(2)}(f_1')\frac{i\hat{f}_1'-m}{f_1'^2+m^2}\hat{e}_2\frac{i\hat{f}_3-m}{f_3^2+m^2}\hat{e}_3,$$

$$U^{(-)}(123)=\hat{e}_1\frac{i\hat{f}'_1-m}{f'^2_1+m^2}\hat{e}_2\frac{i\hat{f}_3-m}{f_3^2+m^2}\Sigma_R^{(2)}(f_3)\frac{i\hat{f}_3-m}{f_3^2+m^2}\hat{e}_3,$$

$$W^{(+)}(123)=e_{1\mu}\Lambda_{R\mu}^{(2)}(f'_1,p_+,k_1)\frac{i\hat{f}'_1-m}{f'^2_1+m^2}\hat{e}_2\frac{i\hat{f}_3-m}{f_3^2+m^2}\hat{e}_3,$$

$$W^{(c)}(123)=\hat{e}_1\frac{i\hat{f}'_1-m}{f'^2_1+m^2}e_{2\mu}\Lambda_{R\mu}^{(2)}(f_3,f'_1,k_2)\frac{i\hat{f}_3-m}{f_3^2+m^2}\hat{e}_3,$$

$$W^{(-)}(123)=\hat{e}_1\frac{i\hat{f}'_1-m}{f'^2_1+m^2}\hat{e}_2\frac{i\hat{f}_3-m}{f_3^2+m^2}e_{3\mu}\Lambda_{R\mu}^{(2)}(p_-,f_3,k_3),\tag{9}$$

其中 $f'_i=k_i-p_+$, $f_i=p_--k_i$. 再记 $Q_R^{(5)}(123)=U^{(+)}(123)+U^{(-)}(123)+W^{(+)}(123)+W^{(c)}(123)+W^{(-)}(123)$.

关于重整化二级电子自能 $\Sigma_R^{(2)}(p)$ 及重整化二级顶角 $\Lambda_{R\mu}^{(2)}(p,p',k)$(约定 $p=p'+k$),我们按文献[3]中§24,§32及文献[4]中提到的步骤分别计算过,得到相同结果:

$$\Sigma_R^{(2)}(p)=\left(\frac{\alpha}{2\pi}\right)m\left\{\left[-1-\ln\frac{\lambda^2}{m^2}+\frac{2\rho\ln\rho}{1-\rho}\right]+\frac{i\hat{p}}{m}\left[-\frac{3}{2}-\ln\frac{\lambda^2}{m^2}+\frac{1}{2(1-\rho)}\right.\right.$$
$$\left.\left.+\frac{\rho(2-\rho)}{2(1-\rho)^2}\ln\rho\right]\right\}=(i\hat{p}+m)^2\left(\frac{1}{m}\right)\left(\frac{\alpha}{2\pi}\right)\left\{\left[\frac{-2}{\rho}-\frac{1}{\rho}\ln\frac{\lambda^2}{m^2}+\frac{2-\rho}{\rho(1-\rho)}\ln\rho\right]\right.$$
$$\left.+\frac{i\hat{p}}{m}\left[\frac{2}{\rho}+\frac{1}{\rho}\ln\frac{\lambda^2}{m^2}+\frac{1}{2(1-\rho)}-\frac{(4-4\rho-\rho^2)}{2\rho(1-\rho)^2}\ln\rho\right]\right\}$$
$$\equiv(i\hat{p}+m)^2\left(\frac{\alpha}{2\pi}\right)\left\{\frac{F(p)}{m}+\frac{G(p)}{m}\frac{i\hat{p}}{m}\right\},\tag{10}$$

$$\Lambda_{R\mu}^{(2)}(p,p',k)=\left(\frac{\alpha}{2\pi}\right)\left\{\left(-3-\ln\frac{\lambda^2}{m^2}-J[\ln R(u,v)]+J\left[\frac{1}{R(u,v)}\right]\right)\gamma_\mu\right.$$
$$-J\left[\frac{(1-u)^2}{R(u,v)}\right]\frac{\hat{p}\gamma_\mu\hat{p}}{m^2}+J\left[\frac{uv(1-uv)}{R(u,v)}\right]\frac{\hat{k}\gamma_\mu\hat{k}}{m^2}+J\left[\frac{(1-u)(1-uv)}{R(u,v)}\right]\frac{\hat{p}\gamma_\mu\hat{k}}{m^2}$$
$$\left.-J\left[\frac{uv(1-u)}{R(u,v)}\right]\frac{\hat{k}\gamma_\mu\hat{p}}{m^2}+4iJ\left[\frac{(1-u)}{R(u,v)}\right]\frac{p_\mu}{m}-2iJ\left[\frac{(1-2uv)}{R(u,v)}\right]\frac{k_\mu}{m}\right\}$$
$$\equiv\left(\frac{\alpha}{2\pi}\right)\left\{A(p,p')\gamma_\mu-C(p,p')\frac{\hat{p}\gamma_\mu\hat{p}}{m^2}+[D(p,p')-E(p,p')]\frac{\hat{p}\gamma_\mu\hat{k}}{m^2}\right.$$
$$\left.-E(p,p')\frac{\hat{k}\gamma_\mu\hat{p}}{m^2}+B(p,p')\frac{\hat{k}\gamma_\mu\hat{k}}{m^2}+4D(p,p')i\left(\frac{p_\mu}{m}\right)-2H(p,p')\frac{ik_\mu}{m}\right\},\tag{11}$$

其中 $\rho=(p^2+m^2)/m^2$, $\rho'=(p'^2+m^2)/m^2$, $\sigma=k^2/m^2$, $\eta=\lambda^2/m^2$(λ 为红外发散引入的光子"质量"), $\rho'-\rho-\sigma=-2pk/m^2$, 而 $R(u,v)\equiv\eta+u[(\rho-\eta)+(\rho'-\rho)v]+u^2[(1-\rho)+(\rho-\rho'+\sigma)v-\sigma v^2]$, $J[f(u,v)]\equiv\int_0^1du\int_0^1dvu[f(u,v)]$. 可以证明(11)式中系数满足恒等式:

$$A+\sigma B+(\rho-1)C=-\frac{3}{2}-\ln\frac{\lambda^2}{m^2}+\frac{1}{2(1-\rho')}+\frac{\rho'(2-\rho')}{2(1-\rho')^2}\ln\rho',$$

$$A+(\rho'-\sigma-1)C+\sigma(B+D-2E)=-\frac{3}{2}-\ln\frac{\lambda^2}{m^2}$$
$$+\frac{1}{2(1-\rho)}+\frac{\rho(2-\rho)}{2(1-\rho)^2}\ln\rho,$$

$$\sigma H + (\rho' - \rho - \sigma)D = \frac{\rho \ln \rho}{1-\rho} - \frac{\rho' \ln \rho'}{1-\rho'}.$$

从而可知，表式(10)和(11)满足广义的 Ward 恒等式[5] $ik_\mu \Lambda_{R\mu}(p, p', k) = \Sigma_R(p) - \Sigma_R(p')$. 当顶角的外光子是自由光子时，(11) 式中 k_μ 及 $\hat{k}\gamma_\mu\hat{k}$ 项无贡献，其余的系数都可以积出来(脚标 0 表示限于自由光子)：

$$J[\ln R_0(uv)] = -\frac{3}{2} - \frac{1}{\rho-\rho'}[F(\rho'-1) - F(\rho-1)]$$
$$- \frac{\rho'(2-\rho')}{2(\rho-\rho')(1-\rho')}\ln \rho' + \frac{\rho(2-\rho)\ln \rho}{2(\rho-\rho')(1-\rho)},$$
$$J\left[\frac{1}{R_0(u, v)}\right] = \frac{F(\rho-1) - F(\rho'-1)}{\rho-\rho'},$$
$$J\left[\frac{1-u}{R_0(u, v)}\right] = \frac{\rho' \ln \rho'}{(\rho-\rho')(1-\rho')} - \frac{\rho \ln \rho}{(\rho-\rho')(1-\rho)},$$
$$J\left[\frac{(1-u)^2}{R_0(u, v)}\right] = \frac{-1}{2(1-\rho)(1-\rho')} + \frac{\rho'(2-\rho')\ln \rho'}{2(\rho-\rho')(1-\rho')^2} - \frac{\rho(2-\rho)\ln \rho}{2(\rho-\rho')(1-\rho)^2},$$
$$J\left[\frac{uv(1-u)}{R_0(u, v)}\right] = \frac{-(2-\rho')}{2(\rho-\rho')(1-\rho')} - \frac{[F(\rho'-1) - F(\rho-1)]}{(\rho-\rho')^2}$$
$$- \frac{\rho'\left[2(2-\rho) - (2-\rho')\dfrac{1-\rho}{1-\rho'}\right]}{2(\rho-\rho')^2(1-\rho')^2}\ln \rho' + \frac{\rho(2-\rho)\ln \rho}{2(\rho-\rho')^2(1-\rho)},$$
$$J\left[\frac{(1-u)(1-uv)}{R_0(u, v)}\right] = \frac{2-\rho'}{2(\rho-\rho')(1-\rho')} + \frac{F(\rho'-1) - F(\rho-1)}{(\rho-\rho')^2}$$
$$+ \frac{\rho'(2-\rho')(1-2\rho'+\rho)}{2(\rho-\rho')^2(1-\rho')^2}\ln \rho' - \frac{\rho(2-2\rho'+\rho)\ln \rho}{2(\rho-\rho')^2(1-\rho)}, \tag{12}$$

其中 $F(y) = \int_0^y \frac{\ln(1+x)}{x} dx$. 由 (12) 式容易看出，当光子线及一条电子线为自由的时候，(11) 式将过渡到文献 [2] 的 (47—56) 式；另外也容易验证，对于系数 (12) 式，广义 Ward 恒等式亦成立. 把 (11), (12) 式具体用于 $W^{(+)}$, $W^{(-)}$, $W^{(c)}$, 则有

$$\Lambda_{R\mu}^{(2)}(f_1', p_+, k_1) = \frac{\alpha}{2\pi}\left\{\left[-2 - \ln\frac{\lambda^2}{m^2} + \frac{1}{\tau_1}[F(\tau_1 - 1) - F(-1)]\right.\right.$$
$$\left.\left. + \frac{(\tau_1 - 2)}{2(\tau_1 - 1)}\ln \tau_1\right]\gamma_\mu - i\left(\frac{\ln \tau_1}{\tau_1 - 1}\right)\frac{\hat{k}_1}{m}\gamma_\mu\right\}$$
$$\equiv \left(\frac{\alpha}{2\pi}\right)\left\{A(\tau_1)\gamma_\mu + iB(\tau_1)\frac{\hat{k}_1\gamma_\mu}{m}\right\}, \tag{13.1}$$

$$\Lambda_{R\mu}^{(2)}(p_-, f_3, k_3) = \frac{\alpha}{2\pi}\left\{\left[-2 - \ln\frac{\lambda^2}{m^2} + \frac{1}{\tau_3}[F(\tau_3 - 1) - F(-1)]\right.\right.$$
$$\left.\left. + \frac{\tau_3 - 2}{2(\tau_3 - 1)}\ln \tau_3\right]\gamma_\mu + i\left(\frac{\ln \tau_3}{\tau_3 - 1}\right)\frac{\gamma_\mu\hat{k}_3}{m}\right\}$$
$$\equiv \left(\frac{\alpha}{2\pi}\right)\left\{A(\tau_3)\gamma_\mu - iB(\tau_3)\frac{\gamma_\mu\hat{k}_3}{m}\right\}, \tag{13.2}$$

$$\Lambda_{R\mu}^{(2)}(f_3, f_1', k_2) = \frac{\alpha}{2\pi}\Big\{A(\tau_1, \tau_3)\gamma_\mu - C(\tau_1, \tau_3)\frac{\hat{f}_3\gamma_\mu\hat{f}_3}{m^2}$$
$$+ [D(\tau_1, \tau_3) - E(\tau_1, \tau_3)]\frac{\hat{f}_3\gamma_\mu\hat{k}_2}{m^2}$$
$$- E(\tau_1, \tau_3)\frac{\hat{k}_2\gamma_\mu\hat{f}_3}{m^2} + 4D(\tau_1, \tau_3)\frac{if_{3\mu}}{m}\Big\}, \tag{13.3}$$

其中

$$A(\tau_1, \tau_3) = -\Big\{\ln\frac{\lambda^2}{m^2} + \frac{3}{2} + \frac{\tau_1(2-\tau_1)\ln\tau_1}{2(\tau_1-\tau_3)(1-\tau_1)} + \frac{\tau_3(2-\tau_3)\ln\tau_3}{2(\tau_3-\tau_1)(1-\tau_3)}\Big\},$$

$$C(\tau_1, \tau_3) = \frac{-1}{2(1-\tau_1)(1-\tau_3)} + \frac{\tau_1(2-\tau_1)\ln\tau_1}{2(\tau_3-\tau_1)(1-\tau_1)^2} + \frac{\tau_3(2-\tau_3)\ln\tau_3}{2(\tau_1-\tau_3)(1-\tau_3)^2},$$

$$D(\tau_1, \tau_3) = \frac{\tau_1\ln\tau_1}{(\tau_3-\tau_1)(1-\tau_1)} + \frac{\tau_3\ln\tau_3}{(\tau_1-\tau_3)(1-\tau_3)},$$

$$E(\tau_1, \tau_3) = \frac{-(2-\tau_1)}{2(\tau_3-\tau_1)(1-\tau_1)} - \frac{[F(\tau_1-1) - F(\tau_3-1)]}{(\tau_3-\tau_1)^2}$$
$$- \frac{\tau_1\Big[2(2-\tau_3) - (2-\tau_1)\dfrac{1-\tau_3}{1-\tau_1}\Big]\ln\tau_1}{2(1-\tau_1)(\tau_3-\tau_1)^2} + \frac{\tau_3(2-\tau_3)\ln\tau_3}{2(1-\tau_3)(\tau_1-\tau_3)^2}.$$

将 (13.1—13.3) 式代入 $Q_R^{(5)}(123)$ [(9) 式],经过分块矩阵运算可以得到

$$\bar{v}_r(0)Q_R^{(5)}(123)u_s(0) = \left(\frac{\alpha}{2\pi}\right)\left(\frac{i}{4m^2}\right)\xi_r^+\Big\{A(123)\check{e}_1\check{e}_2\check{e}_3$$
$$+ A(1'23')\check{e}_1'\check{e}_2\check{e}_3' + A(12'3' - 1'2'3)(\check{e}_1\check{e}_2'\check{e}_3' - \check{e}_1'\check{e}_2'\check{e}_3)$$
$$+ A(31)\Big[\frac{\boldsymbol{k}_1\cdot\boldsymbol{e}_2}{m}(\check{e}_1\check{n}_1\check{e}_3) + \frac{\boldsymbol{k}_3\cdot\boldsymbol{e}_2}{m}(\check{e}_1\check{n}_3\check{e}_3)\Big]\Big\}\xi_s, \tag{14}$$

其中

$$A(123) \equiv A(\tau_1) + A(\tau_3) + A(\tau_1,\tau_3) + \tau_3G(f_3) + \tau_1G(f_1') - 2[F(f_3) + F(f_1')$$
$$+ G(f_3) + G(f_1')] + \left(1 - \frac{\tau_2}{2}\right)C(\tau_1, \tau_3) - \frac{\tau_2}{2}D(\tau_1, \tau_3)$$
$$= -\ln\frac{\lambda^2}{m^2} - \frac{5}{2} - \frac{1}{4}\left(\frac{1}{1-\tau_1} + \frac{1}{1-\tau_3}\right) + \frac{1}{\tau_1}[F(\tau_1-1) - F(-1)]$$
$$+ \frac{1}{\tau_3}[F(\tau_3-1) - F(-1)] + \left(\frac{3}{4} + \frac{1}{2(1-\tau_1)} - \frac{1}{4(1-\tau_1)^2}\right)\ln\tau_1$$
$$+ \left(\frac{3}{4} + \frac{1}{2(1-\tau_3)} - \frac{1}{4(1-\tau_3)^2}\right)\ln\tau_3 + \frac{\tau_2}{\tau_1-\tau_3}\left(\frac{\tau_1\ln\tau_1}{1-\tau_1} - \frac{\tau_3\ln\tau_3}{1-\tau_3}\right),$$

$$A(1'23') \equiv A(\tau_1) + A(\tau_3) + A(\tau_1,\tau_3) + \tau_3G(f_3) + \tau_1G(f_1') + 2[B(\tau_1) + B(\tau_3)]$$
$$+ \left(1 - \frac{\tau_2}{2}\right)C(\tau_1, \tau_3) - \frac{\tau_2}{2}D(\tau_1, \tau_3) = -\ln\frac{\lambda^2}{m^2} - \frac{5}{2} + \frac{3}{4}\Big(\frac{1}{1-\tau_1}$$
$$+ \frac{1}{1-\tau_3}\Big) + \frac{1}{\tau_1}[F(\tau_1-1) - F(-1)] + \frac{1}{\tau_3}[F(\tau_3-1) - F(-1)]$$
$$+ \left[\frac{3}{4} - \frac{1}{2(1-\tau_1)} + \frac{3}{4(1-\tau_1)^2}\right]\ln\tau_1 + \Big[\frac{3}{4} - \frac{1}{2(1-\tau_3)}$$

$$+\frac{3}{4(1-\tau_3)^2}\Big]\ln\tau_3+\frac{\tau_2}{2(\tau_1-\tau_3)}\left[\frac{\tau_1\ln\tau_1}{1-\tau_1}-\frac{\tau_3\ln\tau_3}{1-\tau_3}\right],$$

$$A(12'3'-1'2'3)\equiv\frac{\tau_2}{2}[D(\tau_1,\tau_3)-C(\tau_1,\tau_3)-2E(\tau_1,\tau_3)]=\frac{3}{2(1-\tau_1)(1-\tau_3)}$$

$$+\frac{3}{4}\left(\frac{1}{1-\tau_1}+\frac{1}{1-\tau_3}\right)+\frac{\tau_2}{2(\tau_3-\tau_1)}+\frac{\tau_2[F(\tau_1-1)-F(\tau_3-1)]}{(\tau_1-\tau_3)^2}$$

$$+\frac{\tau_2\tau_1\ln\tau_1}{2(\tau_3-\tau_1)(1-\tau_1)}\left[\frac{1}{2}+\frac{1}{1-\tau_1}+\frac{2-\tau_1}{\tau_3-\tau_1}\right]$$

$$-\frac{\tau_2\tau_3\ln\tau_3}{2(1-\tau_3)(1-\tau_3)}\left[\frac{1}{2}+\frac{1}{1-\tau_3}+\frac{2-\tau_3}{\tau_1-\tau_3}\right],$$

$$A(31)\equiv 4D(\tau_1\tau_3)=\frac{4\tau_1\ln\tau_1}{(\tau_3-\tau_1)(1-\tau_1)}+\frac{4\tau_3\ln\tau_3}{(\tau_1-\tau_3)(1-\tau_3)}. \qquad (14')$$

显然，取（14）式右边花弧括中二维矩阵的矢量部份就得到 $\boldsymbol{B}_K^{(5)}(123)$.

不难证明对光子极化状态求和后有下列等式：

$$[\alpha;i,j]\equiv\sum_{(\nu)}(\boldsymbol{e}_i\cdot\boldsymbol{e}_j)(\boldsymbol{e}_i\cdot\boldsymbol{e}_j-\boldsymbol{e}_i'\cdot\boldsymbol{e}_j')=2(1-\boldsymbol{n}_i\cdot\boldsymbol{n}_j)^2=\frac{32(2-\tau_k)}{\tau_i^2\tau_j^2},$$

$$[\beta;i,j]\equiv\sum_{(\nu)}(\boldsymbol{e}_i\cdot\boldsymbol{e}_k)(\boldsymbol{e}_k\cdot\boldsymbol{e}_j)(\boldsymbol{e}_i\cdot\boldsymbol{e}_j-\boldsymbol{e}_i'\cdot\boldsymbol{e}_j')=\frac{[\tau_k^2+(\tau_i-\tau_j)^2]}{4\tau_k^2}[\alpha;i,j],$$

$$[\gamma;i,j]\equiv\sum_{(\nu)}(\boldsymbol{e}_i\cdot\boldsymbol{e}_k)(\boldsymbol{e}_k'\cdot\boldsymbol{e}_j)(\boldsymbol{e}_i\boldsymbol{e}_j'+\boldsymbol{e}_i'\cdot\boldsymbol{e}_j)=\frac{(\tau_j-\tau_i)}{2\tau_k}[\alpha;i,j],$$

其中 $i,j,k=1,2,3$ 且 (i,j,k) 组成循环关系；从而有

$$\sum_{(\nu)}\{\check{e}_1\check{e}_2\check{e}_3\}\cdot\boldsymbol{B}_0^{(3)*}=\{[\alpha;1,2]+[\alpha;2,3]-[\alpha;3,1]\}+2\{[\beta;3,1]$$

$$+[\gamma;2,3]-[\gamma;1,2]\}=\frac{64(2-\tau_2)}{\tau_1^2\tau_2^2\tau_3^2}\{(2-\tau_3)^2\tau_3+(2-\tau_1)^2\tau_1$$

$$-(2-\tau_1)(2-\tau_2)(2-\tau_3)\}\equiv 64f(\tau_1,\tau_3),$$

$$\sum_{(\nu)}\{\check{e}_1'\check{e}_2\check{e}_3'\}\cdot\boldsymbol{B}_0^{(3)*}=\{[\alpha;1,2]+[\alpha;2,3]+[\alpha;3,1]\}+2\{[\gamma;2,3]$$

$$-[\gamma;1,2]-[\beta;3,1]\}=\frac{64(2-\tau_2)}{\tau_1^2\tau_2^2\tau_3^2}\{(2-\tau_3)^2\tau_3+(2-\tau_1)^2\tau_1$$

$$+(2-\tau_1)(2-\tau_2)(2-\tau_3)\}\equiv 64g(\tau_1,\tau_3),$$

$$\sum_{(\nu)}\{\check{e}_1\check{e}_2'\check{e}_3'-\check{e}_1'\check{e}_2'\check{e}_3\}\cdot\boldsymbol{B}_0^{(3)*}=2\{[\alpha;1,2]-[\alpha;2,3]\}+4[\gamma;3,1]$$

$$=\frac{128(2-\tau_2)(\tau_1-\tau_3)}{\tau_1^2\tau_2^2\tau_3^2}[\tau_3\tau_1-(2-\tau_2)^2]\equiv 64h(\tau_1,\tau_3),$$

$$\sum_{(\nu)}\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{e}_2}{m}\{\check{e}_1\check{n}_1\check{e}_3\}+\frac{\boldsymbol{k}_3\cdot\boldsymbol{e}_2}{m}\{\check{e}_1\check{n}_3\check{e}_3\}\right)=\frac{1}{4\tau_1}[\tau_1^2-(\tau_2-\tau_3)^2]\{[\alpha;2,3]$$

$$-([\beta;1,2]+[\gamma;1,2])\}-\frac{2(2-\tau_1)}{\tau_1}\{([\beta;3,1]+[\gamma;3,1])$$

$$+([\beta;1,2]-[\gamma;1,2])\}-\frac{2(2-\tau_1)(2-\tau_2)(2-\tau_3)}{\tau_1\tau_2^2}[\alpha;31]$$

$$+\frac{1}{4\tau_3}[\tau_3^2-(\tau_1-\tau_2)^2]\{[\alpha;1,2]-([\beta;2,3]-[\gamma;2,3])\}$$
$$-\frac{2(2-\tau_3)}{\tau_3}\{([\beta;2,3]+[\gamma;2,3])+([\beta;3,1]-[\gamma;3,1])\}$$
$$-\frac{2(2-\tau_1)(2-\tau_2)(2-\tau_3)}{\tau_3\tau_2^2}[\alpha;3,1]=\frac{-128(2-\tau_2)^2(2-\tau_1)(2-\tau_3)}{\tau_1^2\tau_2^2\tau_3^2}$$
$$\equiv -64r(\tau_1,\tau_3), \tag{15}$$

其中 $\{abc\}$ 是 abc 的矢量部份,并利用了 $\tau_1+\tau_2+\tau_3=4, \tau_1\boldsymbol{n}_1+\tau_2\boldsymbol{n}_2+\tau_3\boldsymbol{n}_3=0$.

由(14),(14′),(15)式,便得到我们需要的

$$\int_0^2 d\tau_1\int_{2-\tau_1}^2 d\tau_3\,\mathrm{Re}[\boldsymbol{B}_R^{(5)}(123)\cdot\boldsymbol{B}^{(3)*}]=\left(\frac{\alpha}{\pi}\right)\left(\frac{4}{m^4}\right)\int_0^2 d\tau_1\int_{2-\tau_1}^2 d\tau_3\{f(\tau_1,\tau_3)A(123)$$
$$+g(\tau_1,\tau_3)A(1'23')+h(\tau_1,\tau_3)A(12'3'-1'2'3)-r(\tau_1,\tau_3)A(31)\}$$
$$\equiv\left(\frac{\alpha}{\pi}\right)\left(\frac{4}{m^4}\right)J. \tag{16}$$

J 的实质部分经用辛卜森公式在电子计算机上算出,结果为 0.408 ± 0.001,精度是用改变步长等经验方法估出的. J 中红外发散项 $-\ln\left(\frac{\lambda^2}{m^2}\right)$ 的系数是 $f(\tau_1,\tau_3)+g(\tau_1,\tau_3)$ 对 τ_1, τ_3 的积分,手算出其准确值是 $2(\pi^2-9)/3$,用计算机也算过,在千分之二内符合. 把这些计算结果代入(8)式即得所要结果:

$$\frac{\delta W_R}{W_{3\gamma}^{(3)}}=\left(\frac{\alpha}{\pi}\right)\frac{3J}{2(\pi^2-9)}=\left(\frac{\alpha}{\pi}\right)\left(-\ln\frac{\lambda^2}{m^2}+0.704\pm0.001\right). \tag{17}$$

它只消除了 $W_{3\gamma}^{(3)}$ 对实验值的偏离约四分之一[1].

参 考 文 献

[1] B. E. Lautrup, A. Peterman, Recent Developments in the Comparison between Theory and Experiments in QED, *Physics Reports*, **3C**, No. 4 (1972).

[2] А. И. 阿希叶泽尔, В. Б. 别列斯捷茨基, 量子电动力学, 第二版, 中译本, 科学出版社出版.

[3] Н. Н. Боголюбов, Д. В. Ширков, Введение в теорию квантованных полей (1957), ГИТТЛ, Москва.

[4] J. M. Jauch, F. Rohrlich, The Theory of Photons and Electrons (1955), Addison-Wesley Publishing Company, Inc.

[5] J. D. Bjorken, S. D. Drell, Relativistic Quantumfields (1965), § 19.6, McGraw-Hill Book Company.

VERTEX ANGLE AND ELECTRON SELF-ENERGY RADIATION CORRECTIONS TO THE THREE-PHOTON DECAY RATE OF ORTHOPOSITRONIUM ($^3S_1$)

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