

激光等离子体波纹临界面的共振 吸收和二次谐波产生

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提 要

当激光入射到非均匀的各向同性等离子体时,垂直于主密度梯度方向的密度分布会发生变化,使临界面呈现波纹.本文讨论了法向入射情况下波纹临界面的共振吸收和二次谐波产生,导出了吸收系数和转换系数的解析表达式.利用这种机制可以合理地解释线性理论与实验结果的偏离.

一、引 言

关于激光等离子体的共振吸收和二次谐波产生,是近年来人们广泛研究的重要课题之一.但在所有这些工作中^[1-7],一般所考虑的等离子体密度只在一个方向上具有线性变化.按照这种分布,对于法向入射来说,共振吸收为零^[1],二次谐波的发射也为零^[3].显然,这与实验结果是不符合的^[10,11].事实上,在激光产生等离子体的过程中会出现自生磁场,也会在垂直于主密度梯度方向上使密度分布呈现周期性的小波纹.对于自生磁场引起的共振吸收和二次谐波发射,文献[8]已作过分析,指出由于磁场洛伦兹力的作用,即使在法向入射下,也会产生纵场,从而引起共振吸收和发射二次谐波.对于小波纹的出现,文献[9]作了初步分析,指出由于这种波纹的存在,即使在法向入射下,也会有平行于密度梯度方向的电场存在,从而在临界面附近引起共振吸收.然而在文献[9]中,在计算引起共振吸收的微扰场时,采用了一种假设的驱动场分布,这种分布只考虑了一个固定的空间范围内的驱动场贡献,而并未反映等离子体标度长度对驱动场分布的影响,因而得到的解对 L 的依赖关系是不正确的,而且对扰动场的振幅也未给出解析表达式.至于二次谐波的产生,到目前为止,还没有对临界密度面波纹的作用进行过分析.在本文中,驱动场采用了严格的爱里函数解,并利用微扰论方法进行计算,得到了依赖于入射光强度 E_i 和波数 k_0 、等离子体标度长度 L 、临界面波纹波数 k' 的共振吸收系数和二次谐波转换系数的解析表达式,根据所得到的结果,可以合理地解释线性理论与实验结果的偏离.

二、基本方程

假设具有场分量 $\mathbf{k}H_i e^{i(\omega t - k_0 x)}$ 和 $\mathbf{j}E_i e^{i(\omega t - k_0 x)}$ 的平面电磁波射入弱不均匀的各向同性

等离子体,则基波和二次谐波分别满足如下方程组:

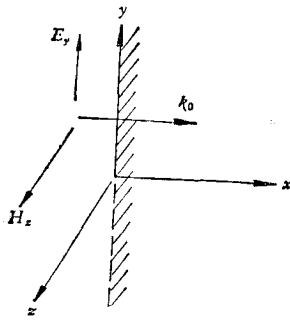


图 1

$$c\nabla \times \mathbf{H}_1 = i\omega \mathbf{D}_1, \quad (1a)$$

$$c\nabla \times \mathbf{E}_1 = -i\omega \mathbf{H}_1, \quad (1b)$$

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1, \quad \varepsilon_1 = 1 - \frac{4\pi e^2 n}{m\omega^2}. \quad (1c)$$

$$c\nabla \times \mathbf{H}_2 = 2i\omega \mathbf{D}_2 + 4\pi \mathbf{j}_2, \quad (2a)$$

$$c\nabla \times \mathbf{E}_2 = -2i\omega \mathbf{H}_2, \quad (2b)$$

$$\mathbf{j}_2 = \frac{ie}{4\pi m\omega} \left(\mathbf{E}_1 \nabla \cdot \mathbf{E}_1 + \frac{\omega_{pe}^2}{4\omega^2} \nabla \mathbf{E}_1^2 \right), \quad (2c)$$

$$\mathbf{D}_2 = \varepsilon_2 \mathbf{E}_2, \quad \varepsilon_2 = 1 - \frac{\pi e^2 n}{m\omega^2}. \quad (2d)$$

这里 n 是等离子体的电子密度,它是 x 和 y 的函数. 我们假定 n 沿主密度梯度方向 x 作线性变化,而在 y 方向上,具有周期性的波纹扰动,即

$$\begin{aligned} n(x, y) &= n_0(x) + \delta n(x, y) = n_e \left[\frac{x}{L} (1 + \mu \cos k'y) \right] \\ &= \frac{m\omega^2}{4\pi e^2} \left[\frac{x}{L} (1 + \mu \cos k'y) \right], \end{aligned}$$

相应的介电常数为

$$\varepsilon_1(x, y) = \varepsilon_0(x) + \delta\varepsilon(x, y) = 1 - \frac{x}{L} (1 + \mu \cos k'y).$$

这里 L 为等离子体标度长度, μ 表示扰动大小, k' 为周期性小波纹的波数. 如果扰动 μ 为零,则 $\varepsilon_1(x, y) = \varepsilon_0(x) = 1 - x/L$; 当 $\mu \neq 0$ 时,介电常数的微扰项 $\delta\varepsilon(x, y)$ 随着远离界面而减小,在边界 $x = 0$ 处为零.

三、基波场及共振吸收

假如 y 方向没有扰动时的基波场用 $\mathbf{E}_0^{(1)}$, $\mathbf{H}_0^{(1)}$ 表示,由 y 方向的波纹产生的微扰场用 $\delta\mathbf{E}^{(1)}$, $\delta\mathbf{H}^{(1)}$ 表示,则基波场 $\mathbf{E}_1 = \mathbf{E}_0^{(1)} + \delta\mathbf{E}^{(1)}$, $\mathbf{H}_1 = \mathbf{H}_0^{(1)} + \delta\mathbf{H}^{(1)}$. 将它们代入方程 (1a) 和 (1b), 得到

$$\frac{d^2 E_{0y}}{dx^2} + k_0^2 \varepsilon_0 E_{0y} = 0, \quad (3)$$

$$\nabla \cdot [\varepsilon_1 \mathbf{E}_0^{(1)}] = -\nabla \cdot [\varepsilon_1 \delta\mathbf{E}^{(1)}]. \quad (4)$$

方程 (3) 有严格的爱里函数解^[4]:

$$E_{0y} = a \zeta^{1/2} \left[J_{1/3} \left(\frac{2}{3} \zeta^{3/2} \right) + J_{-1/3} \left(\frac{2}{3} \zeta^{3/2} \right) \right], \quad \zeta > 0,$$

$$E_{0y} = a (-\zeta)^{1/2} \left[-I_{1/3} \left(\frac{2}{3} (-\zeta)^{3/2} \right) + I_{-1/3} \left(\frac{2}{3} (-\zeta)^{3/2} \right) \right], \quad \zeta < 0,$$

其中

$$a = \alpha E_i e^{-i(\frac{2}{3}\rho_0 - \frac{\pi}{4})}, \quad \alpha = \frac{2\sqrt{\pi}}{3} \rho_0^{1/6},$$

$$\rho_0 = k_0 L, \quad \zeta = \rho_0^{2/3} \left(1 - \frac{x}{L}\right) = -\rho_0^{2/3} \xi.$$

为了求解方程 (4), 引入函数 $\phi(\xi) = \mu \tilde{\phi}(\xi) \sin k'y$, 使

$$\delta \mathbf{E}^{(1)} = \nabla \phi(\xi),$$

代入 (4) 式, 保留到 μ 的一级小量, 得到

$$\frac{d^2 \tilde{\phi}(\xi)}{d\xi^2} + \frac{1}{\xi} \frac{d\tilde{\phi}(\xi)}{d\xi} - \rho'^2 \tilde{\phi}(\xi) = \frac{L e'(\xi+1)}{\xi} E_{0y}, \quad (5)$$

其中 $\rho' = k'L$. 与方程 (5) 相应的齐次方程的两个线性无关的解是零阶汉克尔函数 $H_0^{(1)}(i\rho'\xi)$ 和 $H_0^{(2)}(i\rho'\xi)$, 利用常数变易法求解方程 (5), 得到

$$\tilde{\phi}(\xi) = c_1(\xi) H_0^{(1)}(i\rho'\xi) + c_2(\xi) H_0^{(2)}(i\rho'\xi), \quad (6)$$

其中

$$c_1(\xi) = \int_{-1}^{\xi} \frac{\pi L \rho'(\xi+1) E_{0y} H_0^{(2)}(i\rho'\xi) d\xi}{4i} + c_0(-1), \quad (7)$$

$$c_2(\xi) = \int_{\xi}^{\infty} \frac{\pi L \rho'(\xi+1) E_{0y} H_0^{(1)}(i\rho'\xi) d\xi}{4i}, \quad (8)$$

$c_0(-1)$ 由 $\rho' \rightarrow 0$ 时 $\tilde{\phi}(\xi) \rightarrow 0$ 来确定. 在临界面 $\xi \sim 0$ 附近有

$$\begin{aligned} \tilde{\phi}(0) &= (c_1(0) - c_2(0)) i \frac{2}{\pi} \ln \frac{\rho'\xi}{2} \\ &= A(\rho_0, \rho') E_i L e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \ln \frac{\rho'\xi}{2}, \end{aligned} \quad (9)$$

因而

$$\phi(0) = \mu \tilde{\phi}(0) \sin k'y = \mu A E_i L e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \sin k'y \ln \frac{\rho'\xi}{2}, \quad (10)$$

$$\delta E_x = \nabla_x \phi = \frac{\mu A E_i e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \sin k'y}{\xi}, \quad (11)$$

$$\delta E_y = \nabla_y \phi = \mu A \rho' E_i e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \cos k'y \ln \frac{\rho'\xi}{2}. \quad (12)$$

在临界面附近的基波场

$$\mathbf{E}_1 = \mathbf{E}_0 + \delta \mathbf{E}^{(1)},$$

$$E_{1x} = \delta E_x = \frac{\mu A E_i e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \sin k'y}{\xi}, \quad (13)$$

$$\begin{aligned} E_{1y} &= E_{0y} + \delta E_y = 3\alpha A_i(0) E_i e^{-i(\frac{2}{3}\rho_0 - \frac{\pi}{4})} \\ &\quad + \mu A \rho' E_i e^{-i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)} \cos k'y \ln \frac{\rho'\xi}{2}, \end{aligned} \quad (14)$$

其中 A_i 为爱里函数, $A \equiv A(\rho_0, \rho')$, 而

$$A(\rho_0, \rho') = \frac{i2(c_1(0) - c_2(0)) e^{i(\frac{2}{3}\rho_0 - \frac{3}{4}\pi)}}{\pi E_i L} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{\alpha \rho_0^{-2/3} (\rho')^{2l+1}}{\pi (l!)^2}$$

$$\begin{aligned}
& \cdot \left\{ \frac{\left(\frac{4}{3} + 2k\right)\left(\frac{1}{3} - \frac{2l}{3}\right)_k J_{2k+4/3}\left(\frac{2}{3}\rho_0\right)}{\left(1 + \frac{2l}{3}\right)_{k+1}} \right. \\
& \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s - 1} - \sum_{s=0}^k \frac{1}{2l + 3s + 3} \right) \\
& - \frac{\left(\frac{4}{3} + 2k\right)\left(\frac{2}{3} - \frac{2l}{3}\right)_k J_{2k+4/3}\left(\frac{2}{3}\rho_0\right)}{\left(\frac{2}{3} + \frac{2l}{3}\right)_{k+1}} \\
& \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s + 1} - \sum_{s=0}^k \frac{1}{2l + 3s + 6} \right) \\
& + \frac{\left(\frac{2}{3} + 2k\right)\left(\frac{1}{3} - \frac{2l}{3}\right)_k J_{2k+2/3}\left(\frac{2}{3}\rho_0\right)}{\left(\frac{1}{3} + \frac{2l}{3}\right)_{k+1}} \\
& \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s - 1} - \sum_{s=0}^k \frac{1}{2l + 3s + 4} \right) \\
& - \frac{\left(\frac{2}{3} + 2k\right)\left(-\frac{2l}{3}\right)_k J_{2k+2/3}\left(\frac{2}{3}\rho_0\right)}{\left(\frac{2}{3} + \frac{2l}{3}\right)_{k+1}} \\
& \cdot \left. \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s} - \sum_{s=0}^k \frac{1}{2l + 3s + 5} \right) \right\}. \tag{15}
\end{aligned}$$

利用贝塞耳函数 $J_{2k+4/3}\left(\frac{2}{3}\rho_0\right)$ 和 $J_{2k+2/3}\left(\frac{2}{3}\rho_0\right)$ 在 $\rho_0 \gg 1$ 时的渐近展开式

$$J_{2k+4/3}\left(\frac{2}{3}\rho_0\right) \sim (-1)^k \sqrt{\frac{3}{\pi\rho_0}} \cos\left(\frac{2}{3}\rho_0 - \frac{11}{12}\pi\right),$$

$$J_{2k+2/3}\left(\frac{2}{3}\rho_0\right) \sim (-1)^k \sqrt{\frac{3}{\pi\rho_0}} \cos\left(\frac{2}{3}\rho_0 - \frac{7}{12}\pi\right),$$

(15)式可化为

$$\begin{aligned}
A(\rho_0, \rho') &= \frac{2\rho_0^{-2/3}}{\sqrt{3}\pi} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^k (\rho')^{2l+1}}{(l!)^2} \\
& \cdot \left\{ \cos\left(\frac{2}{3}\rho_0 - \frac{11}{12}\pi\right) \left[\frac{\left(\frac{4}{3} + 2k\right)\left(\frac{1}{3} - \frac{2l}{3}\right)_k}{\left(1 + \frac{2l}{3}\right)_{k+1}} \right. \right. \\
& \cdot \left. \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s - 1} - \sum_{s=0}^k \frac{1}{2l + 3s + 3} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{\left(\frac{4}{3} + 2k\right)\left(\frac{2}{3} - \frac{2l}{3}\right)_k}{\left(\frac{2}{3} + \frac{2l}{3}\right)_{k+1}} \\
& \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s + 1} - \sum_{s=0}^k \frac{1}{2l - 3s + 6} \right) \Bigg] \\
& + \cos\left(\frac{2}{3}\rho_0 - \frac{7}{12}\pi\right) \left[\frac{\left(\frac{2}{3} + 2k\right)\left(\frac{1}{3} - \frac{2l}{3}\right)_k}{\left(\frac{1}{3} + \frac{2l}{3}\right)_{k+1}} \right. \\
& \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s - 1} - \sum_{s=0}^k \frac{1}{2l + 3s + 4} \right) \\
& - \frac{\left(\frac{2}{3} + 2k\right)\left(-\frac{2l}{3}\right)_k}{\left(\frac{2}{3} + \frac{2l}{3}\right)_{k+1}} \\
& \left. \cdot \left(\ln \frac{\rho'}{2} + \sum_{s=0}^{k-1} \frac{1}{2l - 3s} - \sum_{s=0}^k \frac{1}{2l + 3s + 5} \right) \right] \Bigg\}. \quad (16)
\end{aligned}$$

考虑到碰撞阻尼, ξ 用 $\xi + i\nu/\omega$ 来代替, 于是可以算出垂直入射光束的单位截面平均吸收能流

$$S_{\text{吸}} = \frac{c}{4} \pi \rho_0 \mu^2 A^2 E_i^2, \quad (17)$$

而平均吸收系数为

$$\frac{\pi}{2} \mu^2 F(\rho_0, \rho'), \quad F(\rho_0, \rho') \equiv \rho_0 A^2. \quad (18)$$

四、二次谐波发射强度的计算

由于临界面的波纹产生了纵场, 因而在波纹临界面附近必然会产生二次谐波。

把 \mathbf{E}_1 [(13)和(14)式]代入(2c)式, 得到

$$\begin{aligned}
(\nabla \times \mathbf{j}_2)_z &= \frac{ie}{4\pi m \omega L^2} \left[\frac{6\alpha \mu A A_i(0) E_i^2 e^{-i(\frac{4}{3}\rho_0 - \pi)} \sin k'y}{(\xi + i\nu/\omega)^3} \right. \\
& + \frac{\mu^2 A^2 \rho' E_i^2 e^{-i(\frac{4}{3}\rho_0 - \frac{3\pi}{2})} \sin 2k'y \ln(\xi + i\nu/\omega)}{(\xi + i\nu/\omega)^3} \\
& \left. + \frac{\mu^2 A^2 \rho' E_i^2 e^{-i(\frac{4}{3}\rho_0 - \frac{3\pi}{2})} \left(\ln \frac{\rho'}{2} + \frac{1}{2}\right) \sin 2k'y}{\left(\xi + \frac{i\nu}{\omega}\right)^3} \right]. \quad (19)
\end{aligned}$$

上式代入 (2a) 式, 并与 (2b) 式联立, 得到二次谐波场所满足的方程

$$\nabla^2 \mathbf{H}_2 + \frac{i2\omega}{c} \nabla \varepsilon_2 \times \mathbf{E}_2 + \frac{4\omega^2}{c^2} \varepsilon_2 \mathbf{H}_2 = -\frac{4\pi}{c} \nabla \times \mathbf{j}_2, \quad (20)$$

忽略 $\nabla \varepsilon_2$ 的项, (20) 式变成

$$\nabla^2 \mathbf{H}_2 + \frac{4\omega^2}{c^2} \varepsilon_2 \mathbf{H}_2 = -\frac{4\pi}{c} \nabla \times \mathbf{j}_2. \quad (21)$$

对于 z 分量, 方程 (21) 取如下形式:

$$\nabla^2 H_{2z} + k_z^2 H_{2z} = -\frac{4\pi}{c} (\nabla \times \mathbf{j}_2)_z, \quad k_z^2 = \frac{4\omega^2}{c^2} \varepsilon_2. \quad (22)$$

考虑 $H_{2z} = H'_{2z}(\xi) \sin k'y + H''_{2z}(\xi) \sin 2k'y$ 形式的特解, 由方程 (22) 得到函数 $H'_{2z}(\xi)$ 和 $H''_{2z}(\xi)$ 满足的方程

$$\frac{d^2 H'_{2z}}{d\xi^2} + \rho_1'^2 H'_{2z} = \frac{-i6\pi\alpha\mu A A_i(0) E_i^2 e^{-i(\frac{4}{3}\rho_0 - \pi)}}{m\omega c (\xi + i\nu/\omega)^3}, \quad (23)$$

$$\begin{aligned} \frac{d^2 H''_{2z}}{d\xi^2} + \rho_1''^2 H''_{2z} = & -\frac{ie\mu^2 A^2 \rho' E_i^2 e^{-i(\frac{4}{3}\rho_0 - \frac{3}{2}\pi)}}{m\omega c (\xi + i\nu/\omega)^3} \ln(\xi + i\nu/\omega) \\ & - \frac{ie\mu^2 A^2 \rho' E_i^2 e^{-i(\frac{4}{3}\rho_0 - \frac{3}{2}\pi)}}{m\omega c (\xi + i\nu/\omega)^3} \ln\left(\frac{\rho'}{2} + \frac{1}{2}\right), \end{aligned} \quad (24)$$

其中

$$\rho_1'^2 = k_z^2 L^2 - k'^2 L^2 = \frac{4\omega^2}{c^2} \varepsilon_2 L - k'^2 L^2,$$

$$\rho_1''^2 = k_z^2 L^2 - 4k'^2 L^2 = \frac{4\omega^2}{c^2} \varepsilon_2 L - 4k'^2 L^2.$$

求解方程 (23), 得到

$$H'_{2z}(\xi) = \frac{3\pi\alpha\mu A A_i(0) \rho_1' E_i^2}{2m\omega c} \exp\left[-\rho_1' \frac{\nu}{\omega} + i\left(\rho_1' \xi - \frac{4}{3} \rho_0 + \frac{\pi}{2}\right)\right], \quad (25)$$

求解方程 (24), 得到

$$\begin{aligned} H''_{2z}(\xi) = & \frac{\pi c \mu^2 \rho_1' \rho_1'' \left(\ln \frac{\rho_1'}{2} + c_0 - 1 + i \frac{\pi}{2}\right) A^2 E_i^2}{4m\omega c} \\ & \cdot \exp\left[-\rho_1'' \frac{\nu}{\omega} + i\left(\rho_1'' \xi - \frac{4}{3} \rho_0 + \pi\right)\right], \end{aligned} \quad (26)$$

故

$$\begin{aligned} H_{2z}(x, y) = & H'_{2z}\left(\frac{x}{L} - 1\right) \sin k'y + H''_{2z}\left(\frac{x}{L} - 1\right) \sin 2k'y \\ = & \frac{i3\pi\alpha\mu A A_i(0) \rho_1' E_i^2 \sin k'y}{2m\omega c} \\ & \cdot \exp\left[-\rho_1' \frac{\nu}{\omega} + i\left(\rho_1' \left(\frac{x}{L} - 1\right) - \frac{4}{3} \rho_0\right)\right] \end{aligned}$$

$$\begin{aligned}
& - \frac{\pi e \mu^2 \rho' \rho_2'' \left(\ln \frac{\rho'}{2} + c_0 - 1 + i \frac{\pi}{2} \right) A^2 E_i^2 \sin 2k'y}{4m\omega c} \\
& \cdot \exp \left[-\rho_2'' \frac{\nu}{\omega} + i \left(\rho_2'' \left(\frac{x}{L} - 1 \right) - \frac{4}{3} \rho_0 \right) \right], \quad (27) \\
\text{Re } H_{2x}(x, y, t) = & - \frac{3\pi e \alpha \mu A A_i(0) \rho_2' E_i^2 \sin k'y \sin \left[2\omega t + \rho_2' \left(\frac{x}{L} - 1 \right) - \frac{4}{3} \rho_0 \right]}{2m\omega c} e^{-\rho_2' \nu / \omega} \\
& - \left\{ \left[\pi e \mu^2 A^2 \left(\ln \frac{\rho'}{2} + c_0 - 1 \right) \rho_2' \rho_2'' E_i^2 \sin 2k'y \right. \right. \\
& \cdot \cos \left[2\omega t + \rho_2'' \left(\frac{x}{L} - 1 \right) - \frac{4}{3} \rho_0 \right] \left. \right] / [4m\omega c] \right\} e^{-\rho_2'' \nu / \omega} \\
& + \frac{\pi^2 e \mu^2 A^2 \rho_2' \rho_2'' E_i^2 \sin 2k'y \sin \left[2\omega t + \rho_2'' \left(\frac{x}{L} - 1 \right) - \frac{4}{3} \rho_0 \right]}{8m\omega c} e^{-\rho_2'' \nu / \omega}, \quad (28)
\end{aligned}$$

其中 c_0 是欧勒数. 由于源项 $\nabla \times \mathbf{j}_2$ 的 x, y 分量为零, 所以 H_{2x}, H_{2y} 为零. 由 (2a) 式得到真空中的电场 $E_{2y} = -H_{2x}$, 于是, 经过对 t 和 ν 求平均, 得到二次谐波的平均能流

$$\begin{aligned}
\bar{S}_{2x} &= \frac{c \overline{(\text{Re } \mathbf{E}_2 \times \text{Re } \mathbf{H}_2)_x}}{4\pi} \\
&= - \left(\frac{c}{4\pi} \right) \frac{\pi^2 e^2 \mu^2 A^2 E_i^4}{64 m^2 \omega^2 c^2} \left\{ 16\pi \rho_0^{1/3} \rho_2'^2 A_i^2(0) e^{-2\rho_2' \nu / \omega} \right. \\
&\quad \left. + \mu^2 A^2 \rho_2' \rho_2''^2 \left[\left(\ln \frac{\rho'}{2} + c_0 - 1 \right)^2 + \frac{\pi^2}{4} \right] e^{-2\rho_2'' \nu / \omega} \right\}, \quad (29)
\end{aligned}$$

而入射基波场的平均能流为

$$\bar{S}_\lambda = \frac{c}{4\pi} (\text{Re } \mathbf{j} E_i \times \text{Re } \mathbf{k} H_i)_x = \frac{c}{8\pi} E_i^2.$$

因而, 基波转换成二次谐波的转换系数为

$$\begin{aligned}
Q &= \left| \frac{\bar{S}_{2x}}{\bar{S}_\lambda} \right| = \frac{\pi^2 e^2 \mu^2 A^2 E_i^4}{32 m^2 \omega^2 c^2} \left\{ 16\pi \rho_0^{1/3} \rho_2'^2 A_i^2(0) e^{-2\rho_2' \nu / \omega} \right. \\
&\quad \left. + \mu^2 A^2 \rho_2' \rho_2''^2 \left[\left(\ln \frac{\rho'}{2} + c_0 - 1 \right)^2 + \frac{\pi^2}{4} \right] e^{-2\rho_2'' \nu / \omega} \right\}. \quad (30)
\end{aligned}$$

五、结 论

我们研究了垂直于主密度梯度的临界面波纹所产生的共振吸收和二次谐波. 通过求解基波和二次谐波所满足的偏微分方程组, 得到了与通常线性转换理论不同的结论: 当临界密度面呈现波纹 (即 $\mu \neq 0$) 时, 即使对于法向入射, 也将有共振吸收和二次谐波产生. 这是因为垂直于主密度梯度的波纹, 使电场沿主密度梯度方向有一分量. 这种机

制对于实验上观察到法向入射时也有一定吸收和二次谐波发射提供了一种合理的解释^[10,11]。

(18)和(30)式表明,吸收系数和二次谐波转换系数随扰动 μ 的增大而增大,二次谐波转换系数与入射光强度成正比,这正是我们所期待的。要指出的是,共振吸收系数(18)式中的 $F(\rho_0, \rho')$ 与文献[9]中的系数 $L^{-2/3}F(k)$ 不同,这是由于我们选取了不同的驱动场,该驱动场是方程(3)的严格的爱里函数解,具有驻波结构,因而算得的吸收系数和二次谐波转换系数对等离子体标度长度、激光波数与临界面波纹波数的依赖关系比较精确,而且,由(29)式可以看出,随着等离子体标度长度变短,或称密度分布变陡,二次谐波发射强度减弱,这与实验上所观察到的结果相一致^[12]。因此,如果临界面波纹引起的共振吸收和二次谐波发射的机制在实验上得到进一步证实,就可以根据二次谐波的测量精细地了解临界面的结构和行为。

参 考 文 献

- [1] V. L. Ginzburg, "Propagation of Electromagnetic Waves in Plasmas", Oxford, Pergamon, (1970).
- [2] R. B. White, *et al.*, *Plasma Phys.*, **16**(1974), 565.
- [3] Н. С. Ерохин, *et al.*, *ЖЭТФ*, **56** (1969) 179.
- [4] А. Д. Пиляя, *ЖТФ*, **36** (1966), 818.
- [5] Н. С. Ерохин, *et al.*, *ЖТФ*, **40** (1970), 1144.
- [6] А. В. Виноградов, *et al.*, *ЖЭТФ*, **63** (1972), 940.
- [7] N. S. Erokhin, *et al.*, *Nucl. Fusion*, **14**(1974), 333.
- [8] W. L. Krueger, *et al.*, *Phys. Fluids*, **20**(1977), 1688.
- [9] R. A. Cairns, *Plasma Phys.*, **20**(1978), 991.
- [10] J. E. Balmer, *et al.*, *Phys. Rev. Lett.*, **39**(1977), 1084.
- [11] A. Caruso, *et al.*, *Phys. Lett.*, **33A**(1970), 29.
- [12] J. Bobin, In *Plasma Physics: Nonlinear Theory and Experiments*, ed. by H. Wilhelmsson, New York, Plenum, (1977).

RESONANT ABSORPTION AND SECOND-HARMONIC GENERATION AT A RIPPLED CRITICAL SURFACE IN THE LASER PLASMA

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ABSTRACT

When laser beam is incident on an inhomogeneous isotropic plasma, the variations in density profile perpendicular to main density gradient may develop, producing a rippling of critical surface. We have discussed resonant absorption and second-harmonic generation at the rippled critical surface in the case of normal incidence. Analytic expressions for absorption and transformation coefficients are derived. It is possible to explain reasonably the deviation of linear theory from experimental results with this mechanism.