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CONTENTS

Research Notes

存在恒定外电、磁场时的等离子体动力学方程

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提 要

在有恒定外电、磁场的情况下，采用微观相空间密度方法导出了均匀等离子体的动力学方程.

一、引 言

有一系列工作[1—8]以多种方法讨论处于外电、磁场中的等离子体动力学方程. 由于每种方法都有自己的独特近似，所得结果不尽相同. Силин[5] 用按平均相互作用能量与平均动能之比作展开的微扰理论，讨论了同时存在外电、磁场时的等离子体动力学方程. 我们知道，这种方法在碰撞积分中没有考虑到等离子体的极化效应.

Климонтович[7] 用等离子体各成份 (a) 的六维相空间 ($\boldsymbol{q}, \boldsymbol{p}$) 微观密度

$$N_a(\boldsymbol{q}, \boldsymbol{p}, t) = \sum_{1\leqslant i\leqslant N_a} \delta(\boldsymbol{q} - \boldsymbol{q}_{ai}(t))\delta(\boldsymbol{p} - \boldsymbol{p}_{ai}(t)) \tag{1.1}$$

和微观电、磁场强度 $\boldsymbol{E}^m(\boldsymbol{q}, t)$, $\boldsymbol{H}^m(\boldsymbol{q}, t)$ 描述等离子体的微观状态. 从 $N_a(\boldsymbol{q}, \boldsymbol{p}, t)$ 的刘维方程和 $\boldsymbol{E}^m(\boldsymbol{q}, t)$, $\boldsymbol{H}^m(\boldsymbol{q}, t)$ 的洛仑兹方程出发，讨论了等离子体非平衡过程统计理论的一系列问题，其中包括导出处于外磁场[7]或外电场中[8]的等离子体的动力学方程. Климонтович 方法自动地考虑到等离子体的极化效应. 本文用这个方法导出同时存在均匀恒定外电、磁场时的等离子体动力学方程.

对库仑等离子体，系综平均 $N_a(\boldsymbol{q}, \boldsymbol{p}, t)$ 的刘维方程给出[7]

$$\frac{\partial f_a}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f_a}{\partial \boldsymbol{q}} + e_a\left(\boldsymbol{E} + \frac{1}{c}[\boldsymbol{v}\boldsymbol{B}]\right)\cdot\frac{\partial f_a}{\partial \boldsymbol{p}} = S_a(\boldsymbol{q}, \boldsymbol{p}, t), \tag{1.2}$$

式中，$f_a(\boldsymbol{q}, \boldsymbol{p}, t) \equiv \langle N_a(\boldsymbol{q}, \boldsymbol{p}, t)\rangle / n_a$ 是单粒子分布函数 (n_a 表示 a 成分粒子的平均密度，$\langle\ \rangle$ 表示系综平均)，$S_a(\boldsymbol{q}, \boldsymbol{p}, t)$ 是库仑碰撞项.

$$S_a(\boldsymbol{q}, \boldsymbol{p}, t) = -\frac{e_a}{n_a}\frac{\partial}{\partial \boldsymbol{p}}\cdot\langle \delta N_a(\boldsymbol{q}, \boldsymbol{p}, t)\delta \boldsymbol{E}(\boldsymbol{q}, t)\rangle. \tag{1.3}$$

它决定于扰动量

$$\delta N_a(\boldsymbol{q}, \boldsymbol{p}, t) \equiv N_a(\boldsymbol{q}, \boldsymbol{p}, t) - \langle N_a(\boldsymbol{q}, \boldsymbol{p}, t)\rangle$$

和

$$\delta \boldsymbol{E}(\boldsymbol{q}, t) \equiv \boldsymbol{E}^m(\boldsymbol{q}, t) - \langle \boldsymbol{E}^m(\boldsymbol{q}, t)\rangle = \boldsymbol{E}^m(\boldsymbol{q}, t) - \boldsymbol{E}(\boldsymbol{q}, t)$$

的二级关联函数. 在均匀空间条件下,同时刻二级关联函数的傅氏展开式为

$$\langle \delta N_a(\boldsymbol{q}, \boldsymbol{p}, t)\delta \boldsymbol{E}(\boldsymbol{q}', t)\rangle = \frac{1}{(2\pi)^3}\int \langle \delta N_a(\boldsymbol{p}, t)\delta \boldsymbol{E}(t)\rangle_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{q}')} d\boldsymbol{k}. \tag{1.4}$$

这时,碰撞项可用空间谱函数 $\langle \delta N_a(\boldsymbol{p}, t)\delta \boldsymbol{E}(t)\rangle_{\boldsymbol{k}}$ 表示如下:

$$S_a(\boldsymbol{p}, t) = -\frac{1}{(2\pi)^3 n_a}\frac{\partial}{\partial \boldsymbol{p}}\cdot\int \mathrm{Re}\langle \delta N_a(\boldsymbol{p}, t)\delta \boldsymbol{E}(t)\rangle_{\boldsymbol{k}} d\boldsymbol{k}. \tag{1.5}$$

当德拜球内的粒子数非常大于 1 的时候(稀薄等离子体),对碰撞项作出主要贡献的那些空间谱函数的最大弛豫时间 $\tau_{\max}$(弛豫时间依赖于 k)有可能非常小于单粒子分布函数的弛豫时间 τ_p. 这时,如果单粒子分布函数是时间的慢变函数,则在时间范围

$$\tau_{\max} \ll t - t_0 \sim \frac{1}{\Delta} \ll \tau_p \tag{1.6}$$

内(t_0 表示初始时刻),可以通过单粒子分布函数表示空间谱函数,从而得到关于单粒子分布函数的封闭方程,即等离子体动力学方程.

带电粒子在均匀恒定外电、磁场中的运动是三种运动的合成: 电场平行分量 $(\boldsymbol{Eb})\boldsymbol{b}$ 作用下的匀加速直线运动,拉莫尔旋转和沿 $[\boldsymbol{Eb}]$ 方向的匀速直线运动($\boldsymbol{b} = \boldsymbol{B}/B$ 是外磁场方向单位矢量). 因此,在强场条件下分布函数 $f_a(\boldsymbol{p}, t)$ 是时间的快变函数; 不可能只用这样的分布函数来表示空间谱函数.

在迴旋频率 Ω_a 非常大于粒子碰撞频率的情况下,粒子在两次连续碰撞之间可以完成多次拉莫尔旋转. 对拉莫尔旋转进行了平均的分布函数可以足够精确地描述特征时间非常大于迴旋周期的动力学过程. 这时,方便的是讨论粒子在瞬时运动坐标系(其速度为 $e_a(\boldsymbol{Eb})\boldsymbol{b}t/m_a + c[\boldsymbol{Eb}]/B = e_a E_{\parallel}\boldsymbol{b}t/m_a + e_a[\boldsymbol{Eb}]/m_a\Omega_a$)中的轴对称分布函数

$$f_a(\boldsymbol{p}(t), t) = f_a(p_{\parallel}(t), p_{\perp}, t) = f_a(p_{\parallel} - e_a E_{\parallel} t, |\boldsymbol{p}_{\perp} - e_a[\boldsymbol{Eb}]/\Omega_a|, t). \tag{1.7}$$

根据(1.2)式,它满足方程(限于讨论空间均匀的等离子体)

$$\frac{\partial}{\partial t} f_a(\boldsymbol{p}(t), t) = S_a(p_{\parallel}, p_{\perp}, t), \tag{1.8}$$

式中 $S_a(p_{\parallel}, p_{\perp}, t)$ 是按拉莫尔旋转平均的碰撞项,

$$S_a(p_{\parallel}, p_{\perp}, t) = -\frac{e_a}{(2\pi)^3 n_a}\int_0^{2\pi}\frac{d\varphi}{2\pi}\frac{\partial}{\partial \boldsymbol{p}}\cdot\int \mathrm{Re}\langle \delta N_a(\boldsymbol{p}, t)\delta \boldsymbol{E}(t)\rangle_{\boldsymbol{k}} d\boldsymbol{k}, \tag{1.9}$$

φ 是矢量 $\boldsymbol{p}_{\perp} = \boldsymbol{p}_{\perp} - e_a[\boldsymbol{Eb}]/\Omega_a$ 的相位角. 分布函数(1.7)式是时间的慢变函数;碰撞项(1.9)式可以通过这样的分布函数来表示.

二、空间谱函数

在二级关联近似下,库仑等离子体微观密度扰动量 $\delta N_a(\boldsymbol{q}, \boldsymbol{p}, t)$ 满足方程

$$\frac{\partial \delta N_a}{\partial t} + \boldsymbol{v}\cdot\frac{\partial \delta N_a}{\partial \boldsymbol{q}} + \frac{e_a}{m_a c}[\boldsymbol{pB}]\cdot\frac{\partial \delta N_a}{\partial \boldsymbol{p}} = -n_a e_a \delta \boldsymbol{E}\cdot\frac{\partial f_a}{\partial \boldsymbol{p}}. \tag{2.1}$$

这个方程的特征方程为

$$m_a\frac{d\boldsymbol{q}}{dt} = \boldsymbol{p}, \quad \frac{d\boldsymbol{p}}{dt} = \frac{e_a}{m_a c}[\boldsymbol{pB}]. \tag{2.2}$$

可以把它的解表示如下：

$$\boldsymbol{p}_a(0, t, \boldsymbol{p}) = (\boldsymbol{bp})\boldsymbol{b} + [\boldsymbol{bp}]\sin\Omega_a t - [\boldsymbol{b}[\boldsymbol{bp}]]\cos\Omega_a t, \tag{2.3}$$

$$\boldsymbol{q}_a(0, t, \boldsymbol{p}, \boldsymbol{q}) = \boldsymbol{q} - \frac{(\boldsymbol{bp})\boldsymbol{b}}{m_a}t - \frac{[\boldsymbol{bp}]}{m_a\Omega_a}(1 - \cos\Omega_a t) + \frac{[\boldsymbol{b}[\boldsymbol{bp}]]}{m_a\Omega_a}\sin\Omega_a t$$
$$+ \frac{e_a}{m_a\Omega_a}[\boldsymbol{bE}]t - \frac{e_a(\boldsymbol{bE})\boldsymbol{b}}{2m_a}t^2, \tag{2.4}$$

式中 $\boldsymbol{p}_a(0, t, \boldsymbol{p})$ 是 t 时刻(在运动坐标系)的动量为 $\boldsymbol{p}$ 的粒子在初始时刻（设为零时刻）的动量，$\boldsymbol{q}_a(0, t, \boldsymbol{p}, \boldsymbol{q})$ 是 t 时刻的动量为 $\boldsymbol{p}$、（在实验室坐标系的）空间坐标为 $\boldsymbol{q}$ 的粒子在初始时刻的空间坐标. 通过(2.3)和(2.4)式可以把方程(2.1)的解表示成

$$\delta N_a(\boldsymbol{q}, \boldsymbol{p}, t) = \delta N_a(\boldsymbol{q}_a(0, t, \boldsymbol{p}, \boldsymbol{q}), \boldsymbol{p}_a(0, t, \boldsymbol{p}), 0)$$
$$- n_a e_a \int_0^t dt' \delta\boldsymbol{E}(\boldsymbol{q}_a(0, t-t', \boldsymbol{p}, \boldsymbol{q}), t') \cdot \left(\frac{\partial f_a(\boldsymbol{p}, t')}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0, t-t', \boldsymbol{p})}. \tag{2.5}$$

在条件(1.6)式下，分布函数 $f_a(\boldsymbol{p}, t)$ 是时间的慢变函数. 忽略分布函数对时间的依赖关系，对(2.5)式进行傅氏变换，例如

$$\delta N_a(\boldsymbol{q}, \boldsymbol{p}, t) = \frac{1}{(2\pi)^4}\int \delta N_a(\omega + i\Delta, \boldsymbol{k}, \boldsymbol{p}) e^{-i(\omega+i\Delta)t + i\boldsymbol{k}\cdot\boldsymbol{q}} d\omega d\boldsymbol{k}, \tag{2.6}$$

$$\delta N_a(\omega + i\Delta, \boldsymbol{k}, \boldsymbol{p}) = \int_0^\infty dt \int d\boldsymbol{q}\, \delta N_a(\boldsymbol{q}, \boldsymbol{p}, t) e^{i(\omega+i\Delta)t - i\boldsymbol{k}\cdot\boldsymbol{q}}. \tag{2.7}$$

（Δ^{-1} 是(1.6)式中定义的时间参数，在最后的计算结果中将作极限过渡 $\Delta \to 0$）就得到

$$\delta N_a(\omega, \boldsymbol{k}, \boldsymbol{p}) + n_a e_a \int_0^\infty d\tau\, e^{-\Delta\tau + i(\omega\tau + \boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))} \delta\boldsymbol{E}(\omega, \boldsymbol{k}) \cdot \left(\frac{\partial f_a(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}$$
$$= (\delta N_a(\boldsymbol{q}_a(0, t, \boldsymbol{p}, \boldsymbol{q}), \boldsymbol{p}_a(0, t, \boldsymbol{p}), 0))_{\omega, \boldsymbol{k}}. \tag{2.8}$$

电场扰动量方程

$$\nabla \times \delta\boldsymbol{E} = 0, \quad \nabla \cdot \delta\boldsymbol{E} = 4\pi \sum_a e_a \int \delta N_a d\boldsymbol{p}. \tag{2.9}$$

给出

$$\delta\boldsymbol{E}(\omega, \boldsymbol{k}) = -i \sum_a 4\pi e_a \frac{\boldsymbol{k}}{k^2}\int \delta N_a(\omega, \boldsymbol{k}, \boldsymbol{p}) d\boldsymbol{p}. \tag{2.10}$$

将(2.8)式代入后得到

$$\delta\boldsymbol{E}(\omega, \boldsymbol{k}) = -i \sum_a \frac{4\pi e_a \boldsymbol{k}}{k^2 \varepsilon(\omega, \boldsymbol{k})}\int d\boldsymbol{p}(\delta N_a(\boldsymbol{q}_a(0, t, \boldsymbol{p}, \boldsymbol{q}), \boldsymbol{p}_a(0, t, \boldsymbol{p}), 0))_{\omega, \boldsymbol{k}}, \tag{2.11}$$

式中 $\varepsilon(\omega, \boldsymbol{k})$ 是等离子体介电常数，

$$\varepsilon(\omega, \boldsymbol{k}) = 1 - i \sum_a \frac{4\pi n_a e_a^2}{k^2}\int_0^\infty d\tau \int d\boldsymbol{p}\, e^{-\Delta\tau + i(\omega\tau + \boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}\left(\boldsymbol{k}\frac{\partial f_a(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}. \tag{2.12}$$

在条件(1.6)式之下，空间谱函数达到稳态. 这时，均匀等离子体扰动量傅氏分量乘积的系综平均与扰动量乘积的系综平均的傅氏分量之间有以下简单关系：

$$\lim_{\Delta\to 0} 2\Delta\langle \delta L(\omega, \boldsymbol{k})\delta M(\omega, \boldsymbol{k})\rangle = \langle \delta L(\boldsymbol{q}, t)\delta M(\boldsymbol{q}', t')\rangle_{\omega, \boldsymbol{k}}. \tag{2.13}$$

把(2.8)式和(2.11)式左右两边分别相乘，利用性质(2.13)式得到

$$\langle \delta N_a(\boldsymbol{p})\delta \boldsymbol{E}\rangle_{\omega,\boldsymbol{k}} = -n_a e_a \int_0^\infty d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}\langle \delta\boldsymbol{E}\delta\boldsymbol{E}\rangle_{\omega,\boldsymbol{k}}\cdot\left(\frac{\partial f_a(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}$$
$$+ i\sum_b \frac{4\pi e_b \boldsymbol{k}}{k^2\varepsilon^*(\omega,\boldsymbol{k})}\int d\boldsymbol{p}' D_{ab}(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'). \quad (2.14)$$

式中，初始密度扰动量二级关联函数的傅氏分量

$$\begin{aligned} D_{ab}(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}') &\equiv \langle \delta N_a(\boldsymbol{q}_a(0,t,\boldsymbol{p},\boldsymbol{q}),\boldsymbol{p}_a(0,t,\boldsymbol{p}),0) \\ &\quad\times \delta N_b(\boldsymbol{q}_b(0,t',\boldsymbol{p}',\boldsymbol{q}'),\boldsymbol{p}_b(0,t',\boldsymbol{p}'),0)\rangle_{\omega,\boldsymbol{k}} \\ &= \langle \delta N_a(\boldsymbol{q}_a(0,t,\boldsymbol{p},\boldsymbol{q}),\boldsymbol{p}_a(0,t,\boldsymbol{p}),0) \\ &\quad\times \delta N_b(\boldsymbol{q}_b(0,t-\tau,\boldsymbol{p}',\boldsymbol{q}'),\boldsymbol{p}_b(0,t-\tau,\boldsymbol{p}'),0)\rangle_{\omega,\boldsymbol{k}} \\ &= n_a\delta_{ab}(\delta(\boldsymbol{q}_a(0,t,\boldsymbol{p},\boldsymbol{q})-\boldsymbol{q}_a(0,t-\tau,\boldsymbol{p}',\boldsymbol{q}')) \\ &\quad\times \delta(\boldsymbol{p}_a(0,t,\boldsymbol{p})-\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}'))f_a(\boldsymbol{p}(\tau),t-\tau))_{\omega,\boldsymbol{k}} \\ &\approx n_a\delta_{ab}(\delta(\boldsymbol{q}_a(0,t,\boldsymbol{p},\boldsymbol{q})-\boldsymbol{q}_a(0,t-\tau,\boldsymbol{p}',\boldsymbol{q}')) \\ &\quad\times \delta(\boldsymbol{p}_a(0,t,\boldsymbol{p})-\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}'))f_a(\boldsymbol{p}(\tau)))_{\omega,\boldsymbol{k}}. \end{aligned} \quad (2.15)$$

因为 $\boldsymbol{q}_a(0,t,\boldsymbol{p},\boldsymbol{q}) = \boldsymbol{q}+\boldsymbol{q}_a(0,t,\boldsymbol{p},0) = \boldsymbol{q}-\boldsymbol{q}_a(0,-t,\boldsymbol{p}_a(0,t,\boldsymbol{p}),0)$，所以

$$\begin{aligned} D_{ab}(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}') &= n_a\delta_{ab}(\delta(\boldsymbol{q}-\boldsymbol{q}'+\boldsymbol{q}_a(0,t,\boldsymbol{p},0) \\ &\quad+\boldsymbol{q}_a(0,-t+\tau,\boldsymbol{p}_a(0,t,\boldsymbol{p}),0)) \\ &\quad\times \delta(\boldsymbol{p}_a(0,t,\boldsymbol{p})-\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}'))f_a(\boldsymbol{p}(\tau)))_{\omega,\boldsymbol{k}}. \end{aligned} \quad (2.16)$$

再利用性质 $\boldsymbol{q}_a(0,t_1,\boldsymbol{p}_a(0,t_2,\boldsymbol{p}),0) = \boldsymbol{q}_a(0,t_1+t_2,\boldsymbol{p},0)-\boldsymbol{q}_a(0,t_2,\boldsymbol{p},0)$ 得到

$$\begin{aligned} D_{ab}(\omega,\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}') &= n_a\delta_{ab}(\delta(\boldsymbol{q}-\boldsymbol{q}'+\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))\delta(\boldsymbol{p}_a(0,t,\boldsymbol{p}) \\ &\quad-\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}'))f_a(\boldsymbol{p}(\tau)))_{\omega,\boldsymbol{k}} \\ &= n_a\delta_{ab}2\mathrm{Re}\int_0^\infty d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}\delta(\boldsymbol{p}_a(0,t,\boldsymbol{p}) \\ &\quad-\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}'))f_a(\boldsymbol{p}(\tau)). \end{aligned} \quad (2.17)$$

现在将(2.17)式代入(2.14)式，完成 $\boldsymbol{p}'$ 积分 ($\boldsymbol{p}'\to\boldsymbol{p}_a(0,t-\tau,\boldsymbol{p}')$ 变换的雅可毕行列式等于 1)，该式变为

$$\langle \delta N_a(\boldsymbol{p})\delta \boldsymbol{E}\rangle_{\omega,\boldsymbol{k}} = -n_a e_a \int_0^\infty d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}\langle \delta\boldsymbol{E}\delta\boldsymbol{E}\rangle_{\omega,\boldsymbol{k}}\cdot\left(\frac{\partial f_a(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}$$
$$+ i\frac{8\pi n_a e_a \boldsymbol{k}}{k^2\varepsilon^*(\omega,\boldsymbol{k})}\mathrm{Re}\int_0^\infty d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}f_a(\boldsymbol{p}(\tau)). \quad (2.18)$$

(2.11)和(2.17)式给出式中的 $\langle \delta\boldsymbol{E}\delta\boldsymbol{E}\rangle_{\omega,\boldsymbol{k}}$ 为

$$\langle \delta\boldsymbol{E}\delta\boldsymbol{E}\rangle_{\omega,\boldsymbol{k}} = \frac{\boldsymbol{k}\boldsymbol{k}}{k^2}\langle|\delta\boldsymbol{E}|^2\rangle_{\omega,\boldsymbol{k}}. \quad (2.19)$$

$$\langle|\delta\boldsymbol{E}|^2\rangle_{\omega,\boldsymbol{k}} = \sum_a \frac{2(4\pi)^2 n_a e_a^2}{k^2|\varepsilon(\omega,\boldsymbol{k})|^2}\mathrm{Re}\int d\boldsymbol{p}\int_0^\infty d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}f_a(\boldsymbol{p}(\tau)). \quad (2.20)$$

借助于(2.12)，(2.18)—(2.20)式，就能用单粒子分布函数来表示空间谱函数：

$$\langle \delta N_a(\boldsymbol{p},t)\delta \boldsymbol{E}(t)\rangle_{\boldsymbol{k}} = \int_{-\infty}^\infty \frac{d\omega}{2\pi}\langle \delta N_a(\boldsymbol{p})\delta\boldsymbol{E}\rangle_{\omega,\boldsymbol{k}}. \quad (2.21)$$

三、碰撞项

根据(1.9)，(2.18)和(2.21)式，碰撞项具有如下形式：

$$S_a(p_{\parallel}, p_{\perp}, t) = \frac{e_a^2}{(2\pi)^4}\int_0^{2\pi}\frac{d\varphi}{2\pi}\int\frac{d\omega d\boldsymbol{k}}{k^2}\left(\boldsymbol{k}\frac{\partial}{\partial \boldsymbol{p}}\right)\mathrm{Re}\int_0^{\infty}d\tau\, e^{-\Delta\tau+i(\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0))}$$
$$\times\left\{\langle|\delta \boldsymbol{E}|^2\rangle_{\omega,\boldsymbol{k}}\left(\boldsymbol{k}\frac{\partial f_a(\boldsymbol{p},t)}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}\right.$$
$$\left.+\,8\pi\frac{\mathrm{Im}\,\varepsilon(\omega,\boldsymbol{k})}{|\varepsilon(\omega,\boldsymbol{k})|^2}f_a(\boldsymbol{p}(\tau),t)\right\}. \tag{3.1}$$

代入(2.12)和(2.20)式，取极限 $\Delta\to 0$ 后得到

$$S_a(p_{\parallel}, p_{\perp}, t) = \sum_b\frac{2n_b e_a^2 e_b^2}{\pi^2}\int_0^{2\pi}\frac{d\varphi}{2\pi}\int\frac{d\omega d\boldsymbol{k}d\boldsymbol{p}'}{k^4|\varepsilon(\omega,\boldsymbol{k})|^2}$$
$$\times\left(\boldsymbol{k}\frac{\partial}{\partial \boldsymbol{p}}\right)\int_0^{\infty}d\tau\int_0^{\infty}d\tau'\cos[\omega\tau+\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0)]$$
$$\times\cos[\omega\tau'+\boldsymbol{k}\cdot\boldsymbol{q}_b(0,\tau',\boldsymbol{p}',0)]$$
$$\times\left\{\left(\boldsymbol{k}\frac{\partial}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})}-\left(\boldsymbol{k}\frac{\partial}{\partial \boldsymbol{p}'}\right)_{\boldsymbol{p}'\to\boldsymbol{p}_b(0,\tau',\boldsymbol{p}')}\right\}f_a(\boldsymbol{p},t)f_b(\boldsymbol{p}',t). \tag{3.2}$$

引进柱坐标系 $(k, \psi, k_{\parallel})$ 和 $(p_{\perp}, \varphi, p_{\parallel})$. 设矢量 $\boldsymbol{E}$ 处于 $\psi=\varphi=0$ 的平面内并与矢量 $\boldsymbol{b}$ 呈夹角 ϕ，则根据(2.3)和(2.4)式有

$$\boldsymbol{k}\cdot\boldsymbol{q}_a(0,\tau,\boldsymbol{p},0) = -\frac{k_{\parallel}p_{\parallel}}{m_a}\tau+\frac{e_a k_{\parallel}E\cos\phi}{2m_a}\tau^2$$
$$-\frac{k_{\perp}p_{\perp}}{m_a\Omega_a}[\sin(\varphi-\psi+\Omega_a\tau)-\sin(\varphi-\psi)]$$
$$+\frac{e_a k_{\perp}E\sin\phi}{m_a\Omega_a^2}[(1-\cos\Omega_a\tau)\cos\psi$$
$$+(\Omega_a\tau-\sin\Omega_a\tau)\sin\psi]. \tag{3.3}$$

$$\left(\boldsymbol{k}\frac{\partial}{\partial \boldsymbol{p}}\right)_{\boldsymbol{p}\to\boldsymbol{p}_a(0,\tau,\boldsymbol{p})} = k_j\frac{\partial p_i}{\partial p_{aj}(0,\tau,\boldsymbol{p})}\frac{\partial}{\partial p_i}$$
$$= k_{\parallel}\frac{\partial}{\partial p_{\parallel}}+k_{\perp}\cos(\varphi-\psi+\Omega_a\tau)\frac{\partial}{\partial p_{\perp}}$$
$$-k_{\perp}\sin(\varphi-\psi+\Omega_a\tau)\frac{\partial}{p_{\perp}\partial\varphi}. \tag{3.4}$$

并且 $[\boldsymbol{p}_a(0,0,\boldsymbol{p})=\boldsymbol{p}]$

$$\boldsymbol{k}\cdot\frac{\partial}{\partial \boldsymbol{p}} = k_{\parallel}\frac{\partial}{\partial p_{\parallel}}+k_{\perp}\cos(\varphi-\psi)\frac{\partial}{\partial p_{\perp}}-k_{\perp}\sin(\varphi-\psi)\frac{\partial}{p_{\perp}\partial\varphi}. \tag{3.5}$$

将(3.3)—(3.5)式代入(3.2)式，然后把含有 φ 的正弦的指数函数按贝塞耳函数展开

$$e^{i\alpha\sin\vartheta} = \sum_{n=-\infty}^{\infty}J_n(\alpha)e^{in\vartheta}. \tag{3.6}$$

完成φ角积分，再借助于贝塞耳函数递推公式

$$J_{n-1}(\alpha)+J_{n+1}(\alpha)=\frac{2n}{\alpha}J_n(\alpha), \tag{3.7}$$

整理所得表达式，最终得到

$$\begin{aligned}S_a(p_\parallel, p_\perp, t) = 2\pi\sum_b n_b \sum_{n,n'=-\infty}^{\infty}\int k_\perp dk_\perp dk_\parallel d\psi\cdot p'_\perp dp'_\perp dp'_\parallel \\ \times\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}\int_0^\infty\frac{d\tau}{\pi}\int_0^\infty\frac{d\tau'}{\pi}G_{ab}^{n,n'}(\boldsymbol{k}; p_\parallel, p_\perp, \tau; p'_\parallel, p'_\perp, \tau') \\ \times\left\{\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}-\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}'}\right]_{bn'}\right\} \\ \times f_a(p_\parallel-e_aE_\parallel\tau, p_\perp, t)f_b(p'_\parallel-e_bE_\parallel\tau', p'_\perp, t),\end{aligned} \tag{3.8}$$

式中

$$\begin{aligned}G_{ab}^{n,n'}(\boldsymbol{k}; p_\parallel, p_\perp, \tau; p'_\parallel, p'_\perp, \tau') = 2e_a^2e_b^2\int_{-\infty}^{\infty}\frac{d\omega}{k^4|\varepsilon(\omega,\boldsymbol{k})|^2}J_n^2\left(\frac{k_\perp p_\perp}{m_a\Omega_a}\right)J_{n'}^2\left(\frac{k_\perp p'_\perp}{m_b\Omega_b}\right) \\ \times\cos\left\{\left(\omega-\frac{k_\parallel p_\parallel}{m_a}-n\Omega_a\right)\tau+\frac{e_ak_\parallel E\cos\phi}{2m_a}\tau^2\right. \\ \left.+\frac{e_ak_\perp E\sin\phi}{m_a\Omega_a^2}[(1-\cos\Omega_a\tau)\cos\psi+(\Omega_a\tau-\sin\Omega_a\tau)\sin\psi]\right\} \\ \times\cos\left\{\left(\omega-\frac{k_\parallel p'_\parallel}{m_b}-n'\Omega_b\right)\tau'+\frac{e_bk_\parallel E\cos\phi}{2m_b}\tau'^2\right. \\ \left.+\frac{e_bk_\perp E\sin\phi}{m_b\Omega_b^2}[(1-\cos\Omega_b\tau')\cos\psi+(\Omega_b\tau'-\sin\Omega_b\tau')\sin\psi]\right\}.\end{aligned} \tag{3.9}$$

而

$$\begin{aligned}\varepsilon(\omega,\boldsymbol{k}) = 1-i\sum_a\frac{4\pi n_ae_a^2}{k^2}\sum_{n=-\infty}^{\infty}\int 2\pi p_\perp dp_\perp dp_\parallel J_n^2\left(\frac{k_\perp p_\perp}{m_a\Omega_a}\right) \\ \times\int_0^\infty d\tau\exp\left[-\Delta\tau+i\left\{\left(\omega-\frac{k_\parallel p_\parallel}{m_a}-n\Omega_a\right)\tau+\frac{e_ak_\parallel E\cos\phi}{2m_a}\tau^2\right.\right. \\ \left.\left.+\frac{e_ak_\perp E\sin\phi}{m_a\Omega_a^2}[(1-\cos\Omega_a\tau)\cos\psi+(\Omega_a\tau-\sin\Omega_a\tau)\sin\psi]\right\}\right] \\ \times\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}f_a(p_\parallel-e_aE_\parallel\tau, p_\perp, t),\end{aligned} \tag{3.10}$$

$$\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}=k_\parallel\frac{\partial}{\partial p_\parallel}+\frac{nm_a\Omega_a}{p_\perp}\frac{\partial}{\partial p_\perp},\quad \left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}'}\right]_{bn'}=k_\parallel\frac{\partial}{\partial p'_\parallel}+\frac{n'm_b\Omega_b}{p'_\perp}\frac{\partial}{\partial p'_\perp}. \tag{3.11}$$

当电场平行或反平行于磁场的时候，(3.9)式简化为

$$\begin{aligned}G_{ab}^{n,n'}(k_\parallel, k_\perp; p_\parallel, p_\perp, \tau; p'_\parallel, p'_\perp, \tau') = 2e_a^2e_b^2\int_{-\infty}^{\infty}\frac{d\omega}{k^4|\varepsilon(\omega,\boldsymbol{k})|^2}J_n^2\left(\frac{k_\perp p_\perp}{m_a\Omega_a}\right)J_{n'}^2\left(\frac{k_\perp p'_\perp}{m_b\Omega_b}\right) \\ \times\cos\left\{\left(\omega-\frac{k_\parallel p_\parallel}{m_a}-n\Omega_a\right)\tau+\frac{e_ak_\parallel E_\parallel}{2m_a}\tau^2\right\} \\ \times\cos\left\{\left(\omega-\frac{k_\parallel p'_\parallel}{m_b}-n'\Omega_b\right)\tau'+\frac{e_bk_\parallel E_\parallel}{2m_b}\tau'^2\right\}.\end{aligned} \tag{3.12}$$

在没有外电场的情况下，

$$f_a(p_{\parallel}-e_aE_{\parallel}\tau,\ p_{\perp},\ t)=f_a(p_{\parallel},\ p_{\perp},\ t),\ f_b(p'_{\parallel}-e_bE_{\parallel}\tau',\ p'_{\perp},\ t)=f_b(p'_{\parallel},\ p'_{\perp},\ t),$$

可以完成(3.8)式中的 τ、τ' 积分．这时有

$$\int_0^{\infty}\frac{d\tau}{\pi}\int_0^{\infty}\frac{d\tau'}{\pi}G_{ab}^{n,n'}(k_{\parallel},\ k_{\perp};\ p_{\parallel},\ p_{\perp},\ \tau;\ p'_{\parallel},\ p'_{\perp},\ \tau')$$

$$=2e_a^2e_b^2J_n^2\left(\frac{k_{\perp}p_{\perp}}{m_a\Omega_a}\right)J_{n'}^2\left(\frac{k_{\perp}p'_{\perp}}{m_b\Omega_b}\right)\frac{\delta\left(\dfrac{k_{\parallel}p_{\parallel}}{m_a}+n\Omega_a-\dfrac{k_{\parallel}p'_{\parallel}}{m_b}-n'\Omega_b\right)}{k^4\left|\varepsilon\left(\dfrac{k_{\parallel}p_{\parallel}}{m_a}+n\Omega_a;\ k_{\parallel},\ k_{\perp}\right)\right|^2}$$

$$\equiv Q_{ab}^{n,n'}(k_{\parallel},\ k_{\perp};\ p_{\parallel},\ p_{\perp};\ p'_{\parallel},\ p'_{\perp}).\tag{3.13}$$

把这个式子代入(3.8)式就得到文献[7]的结果：

$$S_a(p_{\parallel},\ p_{\perp},\ t)=(2\pi)^2\sum_b n_b\sum_{n,n'=-\infty}^{\infty}\int k_{\perp}dk_{\perp}dk_{\parallel}\cdot p'_{\perp}dp'_{\perp}dp'_{\parallel}$$

$$\times\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}Q_{ab}^{n,n'}(k_{\parallel},\ k_{\perp};\ p_{\parallel},\ p_{\perp};\ p'_{\parallel},\ p'_{\perp})$$

$$\times\left\{\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right]_{an}-\left[\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}'}\right]_{bn'}\right\}f_a(p_{\parallel},\ p_{\perp},\ t)f_b(p'_{\parallel},\ p'_{\perp},\ t).\tag{3.14}$$

在没有外磁场的情况下，(2.3)和(2.4)式简化成

$$\boldsymbol{p}_a(0,\ t,\ \boldsymbol{p})=\boldsymbol{p}(t)=\boldsymbol{p}-e_a\boldsymbol{E}t,\tag{3.15}$$

$$\boldsymbol{q}_a(0,\ t,\ \boldsymbol{p},\ 0)=-\frac{\boldsymbol{p}(t)}{m_a}t-\frac{e_a\boldsymbol{E}}{2m_a}t^2=-\frac{\boldsymbol{p}}{m_a}t+\frac{e_a\boldsymbol{E}}{2m_a}t^2.\tag{3.16}$$

将这两个式子代入(3.2)式，最终得到

$$S_a(\boldsymbol{p},\ t)=\sum_b n_b\int d\boldsymbol{k}d\boldsymbol{p}'\left(\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right)\int_0^{\infty}\frac{d\tau}{\pi}\int_0^{\infty}\frac{d\tau'}{\pi}S_{ab}(\boldsymbol{k};\ \boldsymbol{p},\ \tau;\ \boldsymbol{p}',\ \tau')$$

$$\times\left\{\left(\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}}\right)-\left(\boldsymbol{k}\frac{\partial}{\partial\boldsymbol{p}'}\right)\right\}f_a(\boldsymbol{p}-e_a\boldsymbol{E}\tau,\ t)f_b(\boldsymbol{p}'-e_b\boldsymbol{E}\tau',\ t),\tag{3.17}$$

式中

$$S_{ab}(\boldsymbol{k};\ \boldsymbol{p},\ \tau;\ \boldsymbol{p}',\tau')=2e_a^2e_b^2\int_{-\infty}^{\infty}\frac{d\omega}{k^2|\varepsilon(\omega,\ \boldsymbol{k})|^2}$$

$$\times\cos\left\{\left(\omega-\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{m_a}\right)\tau+\frac{e_a\boldsymbol{k}\cdot\boldsymbol{E}}{2m_a}\tau^2\right\}\cos\left\{\left(\omega-\frac{\boldsymbol{k}\cdot\boldsymbol{p}'}{m_b}\right)\tau'+\frac{e_b\boldsymbol{k}\cdot\boldsymbol{E}}{2m_b}\tau'^2\right\}.\tag{3.18}$$

这个结果与文献[8]一致．

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PLASMA KINETIC EQUATION IN THE PRESENCE OF BOTH CONSTANT ELECTRIC AND MAGNETIC FIELDS

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ABSTRACT

A kinetic equation for uniform plasma in the presence of both constant electric and magnetic fields was obtained by the method of micro-density in the phase space.