第 32 卷 第 12 期 物 理 学 报 Vol. 32, No. 12
1983 年 12 月 ACTA PHYSICA SINICA Dec., 1983

阴极透镜象差的变分理论

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1982 年 10 月 7 日收到

提 要

本文运用变分原理研究了电磁复合聚焦阴极透镜的象差理论，导出了在任意象平面中象差系数的普遍公式．它们可以用 Glaser 导出的象差系数的线性组合来表示．这就表明了宽束与细束电子光学系统的象差都可以用统一的变分方法处理，并建立了两者之间的联系．

本文采用矢量形式描写阴极透镜的近轴轨迹，并采用矩阵形式描写象差．本文所得的象差系数较之文献上已有结果形式更为简洁且适用于计算机计算．本文考虑了阴极面上磁场和横向电场不为零的情况，因而所得结果较为普遍．

一、引 言

阴极透镜宽束电子光学系统中，阴极处在电场、磁场中，发射初速很小的电子束．从阴极发出的斜率很大(甚至无穷大)的电子也参与成象．因此，通常都要引用准规范化电位，利用近轴方程的解、借助轨迹法来计算阴极透镜的象差[1-3]．文献[1]首先全面地建立了电磁复合聚焦阴极透镜象差的严格理论．文献[2]基本上沿用文献[1]的方法再次计算了阴极透镜的象差．文献[3]则进一步发展了文献[1]的结果，导出了在任意象平面中的象差系数．然而在传统的电子光学文献中，宽束电子光学系统与细束电子光学系统中计算象差的方法和象差系数的公式在形式上似乎很不相同．

为了解决这个问题，本文运用变分原理研究了电磁复合聚焦阴极透镜的象差理论，导出了在任意象平面中象差系数的普遍公式．它们可以用 Glaser[4] 的几何象差系数和色差系数的线性组合来表征．从而表明了宽束和细束电子光学系统的象差都可以用统一的变分方法来处理，并建立了两者之间的联系．

本文采用了矢量形式描写阴极透镜的近轴轨迹，采用了矩阵形式描写象差[5,6]，并运用了 Glaser 的象差系数[4](即 9 个三级几何象差系数和 4 个一级色差系数)来描写任意象平面中全部的三级横向象差．

由此可见，本文将 Glaser 的细束电子光学象差理论推广到宽束电子光学中，并考虑了阴极上磁场和横向电场不为零的情况．因而本文较之文献 [1—3] 结果更为普遍、形式更为简单．此外，近年来还有一些文献[7—9]也给出了阴极透镜的象差公式．但是其公式过于复杂，或表达不够准确，或结果不够普遍，在此不再评述．

二、阴极透镜的变分函数和光程函数

根据阴极透镜宽束电子光学系统的上述特点，通常引用准规范化轴上电位(设旋转对称轴为 z 轴)为

$$V(z) = \varepsilon_z + \phi(z), \tag{1}$$

式中 ε_z 为从阴极发出电子的轴向初能量所对应的电位，$\phi(z)$ 为从阴极面 $z = z_0$ 算起的轴上电位．于是近轴轨迹方程为[1]

$$V\boldsymbol{u}'' + \frac{1}{2}V'\boldsymbol{u}' + \frac{1}{4}(V'' + \mathscr{H}^2)\boldsymbol{u} = 0, \tag{2}$$

其中

$$\mathscr{H}(z) = \sqrt{\eta/2}\,H(z) \quad \eta = e/m_0. \tag{3}$$

e，m_0 和 η 分别为电子电荷绝对值、静止质量和荷质比．撇号一律是对 z 求导数．在国际单位制中，$H(z)$ 是旋转对称轴上的磁感应强度．

设 $v(z)$ 和 $w(z)$ 是近轴轨迹方程一对线性无关的特解，它们在阴极面 $z = z_0$ 上满足以下初始条件：

$$v(z_0) = 0,\ v'(z_0)\sqrt{\varepsilon_z} = 1;\ w(z_0) = 1,\ w'(z_0)\sqrt{\varepsilon_z} = 0. \tag{4}$$

在旋转直角坐标系 xy 平面中近轴轨迹 $\boldsymbol{u}$ 及其斜率 $\boldsymbol{u}'$ 可以表示如下：

$$\boldsymbol{u} = \boldsymbol{u}_0 w(z) + (2\eta)^{-1/2}\dot{\boldsymbol{u}}_0 v(z),\ \boldsymbol{u}' = \boldsymbol{u}_0 w'(z) + (2\eta)^{-1/2}\dot{\boldsymbol{u}}_0 v'(z). \tag{5}$$

注意到旋转直角坐标系中轨迹 $\boldsymbol{u}$ 相对于固定直角坐标系中轨迹 $\boldsymbol{r}$ 有一旋转角 $\chi(z)$，即

$$\boldsymbol{r} = \boldsymbol{u}e^{i\chi(z)}, \qquad \boldsymbol{u} = \boldsymbol{r}e^{-i\chi(z)}, \tag{6}$$

其中

$$\chi(z) = \frac{1}{2}\int_{z_0}^{z}\frac{\mathscr{H}}{V^{1/2}}dz. \tag{7}$$

利用(6)，(7)式，轨迹的初始位置和速度 $\boldsymbol{u}_0$，$\dot{\boldsymbol{u}}_0$ 可以写成

$$\begin{aligned}(x_0, y_0) &= \boldsymbol{u}_0 = \boldsymbol{u}(z_0) = \boldsymbol{r}_0,\\ (\dot{x}_0, \dot{y}_0) &= \dot{\boldsymbol{u}}_0 = \dot{\boldsymbol{u}}(z_0) = \dot{\boldsymbol{r}}_0 - \dot{\chi}_0\smile{\boldsymbol{r}}_0,\end{aligned} \tag{8}$$

其中

$$\dot{\chi}_0 = \sqrt{\eta/2}\,\mathscr{H}(z_0) = \frac{\eta}{2}H(z_0), \quad \underset{\smile}{\boldsymbol{r}_0} = \boldsymbol{k}\times\boldsymbol{r}_0. \tag{9}$$

上述诸式中圆点号一律表示对时间求导数，$\boldsymbol{k}$ 是旋转对称 z 轴方向的单位矢量．$\underset{\smile}{\boldsymbol{r}_0}$ 是 $\boldsymbol{r}_0$ 的正交矢量(今后正交矢量一律如此定义)．

根据 Glaser 的象差理论[4,6]计算几何象差所用的四级变分函数为

$$\begin{aligned}-F_4 = (2\eta)^{1/2}\Big\{&\frac{1}{4}L(\boldsymbol{u}\cdot\boldsymbol{u})^2 + \frac{1}{2}M(\boldsymbol{u}\cdot\boldsymbol{u})(\boldsymbol{u}'\cdot\boldsymbol{u}')\\ &+ \frac{1}{4}N(\boldsymbol{u}'\cdot\boldsymbol{u}')^2 + PV^{1/2}(\boldsymbol{u}\cdot\boldsymbol{u})(\boldsymbol{u}\times\boldsymbol{u}')\boldsymbol{k} + QV^{1/2}(\boldsymbol{u}'\cdot\boldsymbol{u}')(\boldsymbol{u}\times\boldsymbol{u}')\boldsymbol{k}\end{aligned}$$

$$+KV[(\boldsymbol{u}\times\boldsymbol{u}')\boldsymbol{k}]^2\Big\}. \tag{10}$$

上式中 $\boldsymbol{u}$, $\boldsymbol{u}'$ 由(5)式给出；K, L, M, N, P, Q 为包括轴上电位和磁感应强度的场分布函数：

$$K=\frac{1}{8}\frac{\mathscr{H}^2}{V^{3/2}},\quad L=\frac{1}{32V^{1/2}}\left(\frac{1}{V}(V''+\mathscr{H}^2)^2-V''''-4\mathscr{H}\mathscr{H}''\right),$$
$$M=\frac{1}{8V^{1/2}}(V''+\mathscr{H}^2),$$
$$N=\frac{1}{2}V^{1/2},\quad P=\frac{1}{16V^{1/2}}\left[\frac{\mathscr{H}}{V}(V''+\mathscr{H}^2)-\mathscr{H}''\right],$$
$$Q=\frac{1}{4V^{1/2}}\mathscr{H}. \tag{11}$$

于是从阴极平面(即物平面) z_0 到任意象平面 z_i 的四级光程函数为

$$-\int_{z_0}^{z_i}F_4dz=\frac{1}{4}A(\boldsymbol{u}_0\cdot\boldsymbol{u}_0)^2+\frac{1}{4}B(\dot{\boldsymbol{u}}_0\cdot\dot{\boldsymbol{u}}_0)^2+C(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)^2$$
$$+\frac{1}{2}D(\boldsymbol{u}_0\cdot\boldsymbol{u}_0)(\dot{\boldsymbol{u}}_0\cdot\dot{\boldsymbol{u}}_0)+E(\boldsymbol{u}_0\cdot\boldsymbol{u}_0)(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)+F(\dot{\boldsymbol{u}}_0\cdot\dot{\boldsymbol{u}}_0)(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)$$
$$+2c(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k}+e(\boldsymbol{u}_0\cdot\boldsymbol{u}_0)(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k}+f(\dot{\boldsymbol{u}}_0\cdot\dot{\boldsymbol{u}}_0)(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k}, \tag{12}$$

式中 $(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)$, $(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k}$ 定义如下:

$$(\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0)=(x_0\dot{x}_0+y_0\dot{y}_0),\quad(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k}=(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\cdot\boldsymbol{k}=(x_0\dot{y}_0-y_0\dot{x}_0). \tag{13}$$

(12)式中 A, B, C, D, E, F 以及 c, e, f 是类似于 Glaser 导出的 9 个三级几何象差系数[4,6]. 对于阴极透镜，它们的定义如下:

$$A=(2\eta)^{1/2}\int_{z_0}^{z_i}[Lw^4+2Mw^2w'^2+Nw'^4]dz,$$
$$B=(2\eta)^{-3/2}\int_{z_0}^{z_i}[Lv^4+2Mv^2v'^2+Nv'^4]dz,$$
$$C=(2\eta)^{-1/2}\int_{z_0}^{z_i}[Lv^2w^2+2Mvv'ww'+Nv'^2w'^2-K]dz,$$
$$D=(2\eta)^{-1/2}\int_{z_0}^{z_i}[Lv^2w^2+M(v^2w'^2+w^2v'^2)+Nv'^2w'^2+2K]dz,$$
$$E=\int_{z_0}^{z_i}[Lvw^3+Mww'(vw)'+Nv'w'^3]dz,$$
$$F=(2\eta)^{-1}\int_{z_0}^{z_i}[Lv^3w+Mvv'(vw)'+Nw'v'^3]dz,$$
$$c=(2\eta)^{-1/2}\int_{z_0}^{z_i}[Pvw+Qv'w']dz \quad (\text{此处 } c \text{ 为 Glaser 定义的一半}),$$
$$e=\int_{z_0}^{z_i}[Pw^2+Qw'^2]dz,$$
$$f=(2\eta)^{-1}\int_{z_0}^{z_i}[Pv^2+Qv'^2]dz. \tag{14}$$

将四级光程函数对 $\boldsymbol{u}_0$, $\dot{\boldsymbol{u}}_0$ 求梯度可以算出任意象平面 z_i 中的三级几何象差[4,6]

$$\Delta \boldsymbol{u}_3(z_i) = (2\eta)^{-1/2} v(z_i) \frac{\partial}{\partial \boldsymbol{u}_0} \left[\int_{z_0}^{z_i} F_4 dz \right] - w(z_i) \frac{\partial}{\partial \dot{\boldsymbol{u}}_0} \left[\int_{z_0}^{z_i} F_4 dz \right] + (2\eta)^{-1/2} v(z_i) \left\{ \frac{\partial}{\partial \boldsymbol{u}'} F_4 \right\}_{z_0}. \tag{15}$$

将(10)和(12)式代入上式可以得到

$$\Delta \boldsymbol{u}_3(z_i) = w(z_i) \begin{bmatrix} \dot{\boldsymbol{u}}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2\boldsymbol{u}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2(\boldsymbol{u}_0 \times \dot{\boldsymbol{u}}_0)\boldsymbol{k} \\ \boldsymbol{u}_0 \cdot \boldsymbol{u}_0 \end{bmatrix}^* \begin{bmatrix} B & F & f \\ F & C & c \\ f & c & 0 \\ D & E & e \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}}_0 \\ \boldsymbol{u}_0 \\ \underset{\smile}{\boldsymbol{u}_0} \end{bmatrix} - (2\eta)^{-1/2} v(z_i) \begin{bmatrix} \dot{\boldsymbol{u}}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2\boldsymbol{u}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2(\boldsymbol{u}_0 \times \dot{\boldsymbol{u}}_0)\boldsymbol{k} \\ \boldsymbol{u}_0 \cdot \boldsymbol{u}_0 \end{bmatrix}^* \begin{bmatrix} F & -f & D \\ C & -c & E \\ c & 0 & e \\ E & -e & A \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}}_0 \\ \underset{\smile}{\dot{\boldsymbol{u}}_0} \\ \boldsymbol{u}_0 \end{bmatrix} + (2\eta)^{-1/2} v(z_i) \left\{ \frac{\mathscr{H}_0''}{16} (\boldsymbol{u}_0 \cdot \boldsymbol{u}_0) \underset{\smile}{\boldsymbol{u}_0} \right\}. \tag{16}$$

这里＊表示转置矩阵．(15)和(16)式中最后带有{ }的项可参阅文献[10]．上式是三级几何象差的公式，包括两个象差系数矩阵，第一个称为 w 矩阵，第二个称为 v 矩阵．

下面再讨论计算色差所用的二级变分函数[6]：

$$F_{2c} = (2\eta)^{-1/2} \left[\frac{1}{16 V^{3/2}} (V'' + \mathscr{H}^2) \boldsymbol{u}^2 + \frac{1}{4 V^{1/2}} \boldsymbol{u}'^2 + \frac{\mathscr{H}}{4V} (\boldsymbol{u} \times \boldsymbol{u}')\boldsymbol{k} \right]. \tag{17}$$

于是 z_0，z_i 之间的二级光程函数为

$$\int_{z_0}^{z_i} F_{2c} dz = \frac{1}{2} C_d (\dot{\boldsymbol{u}}_0 \cdot \dot{\boldsymbol{u}}_0) + C_m (\boldsymbol{u}_0 \cdot \dot{\boldsymbol{u}}_0) + \frac{1}{2} C_a (\boldsymbol{u}_0 \cdot \boldsymbol{u}_0) + C_r (\boldsymbol{u}_0 \times \dot{\boldsymbol{u}}_0)\boldsymbol{k}. \tag{18}$$

将二级光程函数对 $\boldsymbol{u}_0$，$\dot{\boldsymbol{u}}_0$ 求梯度，得出任意平面 z_i 中的一级色差[6]

$$\begin{aligned} \Delta \boldsymbol{u}_c(z_i) &= (2\eta\Delta\varepsilon) \left[(2\eta)^{-1/2} v(z_i) \frac{\partial}{\partial \boldsymbol{u}_0} \int_{z_0}^{z_i} F_{2c} dz - w(z_i) \frac{\partial}{\partial \dot{\boldsymbol{u}}_0} \int_{z_0}^{z_i} F_{2c} dz \right] \\ &= (2\eta\Delta\varepsilon) \{ (2\eta)^{-1/2} v(z_i) [C_m \dot{\boldsymbol{u}}_0 + C_a \boldsymbol{u}_0 - C_r \underset{\smile}{\dot{\boldsymbol{u}}_0}] \\ &\quad - w(z_i) [C_d \dot{\boldsymbol{u}}_0 + C_m \boldsymbol{u}_0 + C_r \underset{\smile}{\boldsymbol{u}_0}] \}, \end{aligned} \tag{19}$$

式中 $\Delta\varepsilon$ 为初能量的变化量，C_d，C_m，C_a 和 C_r 为一级色差系数[4,6]，

$$\begin{aligned} C_d &= \frac{1}{2} (2\eta)^{-3/2} \int_{z_0}^{z_i} \left[\frac{v'^2}{\sqrt{V}} + \frac{V'' + \mathscr{H}^2}{4V^{3/2}} v^2 \right] dz, \\ C_m &= \frac{1}{2} (2\eta)^{-1} \int_{z_0}^{z_i} \left[\frac{v'w'}{\sqrt{V}} + \frac{V'' + \mathscr{H}^2}{4V^{3/2}} vw \right] dz, \\ C_a &= \frac{1}{2} (2\eta)^{-1/2} \int_{z_0}^{z_i} \left[\frac{w'^2}{\sqrt{V}} + \frac{V'' + \mathscr{H}^2}{4V^{3/2}} w^2 \right] dz, \\ C_r &= \frac{1}{4} (2\eta)^{-1} \int_{z_0}^{z_i} \frac{\mathscr{H}}{V^{3/2}} dz. \end{aligned} \tag{20}$$

至此，(14)式给出了 9 个三级几何象差系数，(20) 式给出了 4 个一级色差系数，它们与 Glaser[4] 对于细束电子光学系统导出的象差系数是一致的，可以用来描写复合阴极透

镜宽束电子光学系统的三级象差.

现在按照文献[2,3],引进以下函数:

$$h = \mathscr{H}/2,\quad T = \frac{1}{4}V'' + h^2,\quad W = \frac{1}{16}V'''' + hh'',\quad K_{vv} = Tv^2 + Vv'^2,$$
$$K_{vw} = Tvw + Vv'w',\quad K_{ww} = Tw^2 + Vw'^2, \tag{21}$$

则(14),(20)式中各个象差系数可以表示如下:

$$A = -\frac{1}{2}(2\eta)^{1/2}\int_{z_0}^{z_i}\left[Ww^4 - \frac{K_{ww}^2}{V}\right]\frac{dz}{\sqrt{V}},$$
$$B = -\frac{1}{2}(2\eta)^{-3/2}\int_{z_0}^{z_i}\left[Wv^4 - \frac{K_{vv}^2}{V}\right]\frac{dz}{\sqrt{V}},$$
$$C = -\frac{1}{2}(2\eta)^{-1/2}\int_{z_0}^{z_i}\left[Wv^2w^2 - \frac{K_{vw}^2}{V} + \frac{h^2}{V}\right]\frac{dz}{\sqrt{V}},$$
$$D = -\frac{1}{2}(2\eta)^{-1/2}\int_{z_0}^{z_i}\left[Wv^2w^2 - \frac{K_{vv}K_{ww}}{V} - \frac{2h^2}{V}\right]\frac{dz}{\sqrt{V}},$$
$$E = -\frac{1}{2}\int_{z_0}^{z_i}\left[Wvw^3 - \frac{K_{vw}K_{ww}}{V}\right]\frac{dz}{\sqrt{V}},$$
$$F = -\frac{1}{2}(2\eta)^{-1}\int_{z_0}^{z_i}\left[Wv^3w - \frac{K_{vv}K_{vw}}{V}\right]\frac{dz}{\sqrt{V}},$$
$$c = -\frac{1}{2}(2\eta)^{-1/2}\int_{z_0}^{z_i}\left[\frac{h''vw}{4} - \frac{hK_{vw}}{V}\right]\frac{dz}{\sqrt{V}},$$
$$e = -\frac{1}{2}\int_{z_0}^{z_i}\left[\frac{h''w^2}{4} - \frac{hK_{ww}}{V}\right]\frac{dz}{\sqrt{V}},$$
$$f = -\frac{1}{2}(2\eta)^{-1}\int_{z_0}^{z_i}\left[\frac{h''v^2}{4} - \frac{hK_{vv}}{V}\right]\frac{dz}{\sqrt{V}},$$
$$C_d = \frac{1}{2}(2\eta)^{-3/2}\int_{z_0}^{z_i}\frac{K_{vv}}{V^{3/2}}dz,\quad C_m = \frac{1}{2}(2\eta)^{-1}\int_{z_0}^{z_i}\frac{K_{vw}}{V^{3/2}}dz,$$
$$C_a = \frac{1}{2}(2\eta)^{-1/2}\int_{z_0}^{z_i}\frac{K_{ww}}{V^{3/2}}dz,\quad C_r = \frac{1}{2}(2\eta)^{-1}\int_{z_0}^{z_i}\frac{h}{V^{3/2}}dz. \tag{22}$$

以上 13 个系数与 Glaser 形式的系数(见(14)和(20)式)完全等价,但是形式较为简单,便于计算.

三、阴极透镜三级象差的计算

为了利用上述三级几何象差和一级色差来计算阴极透镜宽束电子光学系统的象差,我们需要进行以下三个步骤的处理.

第一步 上面用旋转坐标系中表观的轨迹参量 $\boldsymbol{u}_0$, $\dot{\boldsymbol{u}}_0$ 来表征象差. 现在要换用固定坐标系中实在的轨迹参量 $\boldsymbol{r}_0$, $\dot{\boldsymbol{r}}_0$ 来描述. 为此须将 $\boldsymbol{u}_0$, $\dot{\boldsymbol{u}}_0$ 按照(8)式作线性变换而得 $\boldsymbol{r}_0$, $\dot{\boldsymbol{r}}_0$. 当轨迹参量受到线性变换后导致的象差可按文献[5]处理.

第二步 细束电子光学系统通常的规范化轴上电位 $\varepsilon + \phi(z)$ 减去垂直于轴的横向初能量 $\varepsilon_\perp$ 就等于宽束电子光学系统的准规范化轴上电位 $\varepsilon_z + \phi(z)$ (见(1)式). 由此

推论，宽束电子光学系统的三级横向象差可以用通常细束电子光学系统的三级几何象差减去 $\varepsilon_\perp$ 的色差来求得．

第三步　为普遍起见，我们不仅考虑阴极面上磁场 $\mathscr{H}_0 \neq 0$，而且考虑到其上横向电场也不为零，即 $V''(z_0) = V_0'' \neq 0$ 的情况．这时 $z = z_0$ 平面阴极不是等位平面．从这种阴极平面上起始离轴位置 $\boldsymbol{r}_0$ 处发出的电子其初能量的变化量为 $V_0''\boldsymbol{r}_0^2/4$．这将导致附加的色差，并要从三级几何象差中减去．

1. w 矩阵所代表的象差

在(16)式中 w 矩阵所代表的象差为

$$\Delta \boldsymbol{u}_w(z_i) = w(z_i)\begin{bmatrix} \dot{\boldsymbol{u}}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2\boldsymbol{u}_0 \cdot \dot{\boldsymbol{u}}_0 \\ 2(\boldsymbol{u}_0 \times \dot{\boldsymbol{u}}_0)\boldsymbol{k} \\ \boldsymbol{u}_0 \cdot \boldsymbol{u}_0 \end{bmatrix}^* \begin{bmatrix} B & F & f \\ F & C & c \\ f & c & 0 \\ D & E & e \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{u}}_0 \\ \boldsymbol{u}_0 \\ \underset{\smile}{\boldsymbol{u}_0} \end{bmatrix}. \tag{23}$$

第一步处理即将 $\boldsymbol{u}_0$，$\dot{\boldsymbol{u}}_0$ 换成 $\boldsymbol{r}_0$，$\dot{\boldsymbol{r}}_0$，并按文献[5]计算得出

$$\Delta \boldsymbol{u}_w(z_i) = w(z_i)\begin{bmatrix} \dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2\boldsymbol{r}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2(\boldsymbol{r}_0 \times \dot{\boldsymbol{r}}_0)\boldsymbol{k} \\ \boldsymbol{r}_0 \cdot \boldsymbol{r}_0 \end{bmatrix}^*$$

$$\cdot\begin{bmatrix} B & 0 & F & f - \dot{\chi}_0 B \\ F & 0 & C + 2\dot{\chi}_0 f - \dot{\chi}_0^2 B & c - \dot{\chi}_0 F \\ f - \dot{\chi}_0 B & 0 & c - \dot{\chi}_0 F & 0 \\ D - 6\dot{\chi}_0 f + 3\dot{\chi}_0^2 B & 0 & E - 2\dot{\chi}_0 c + \dot{\chi}_0^2 F & e - \dot{\chi}_0 D + 3\dot{\chi}_0^2 f - \dot{\chi}_0^3 B \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \underset{\smile}{\dot{\boldsymbol{r}}_0} \\ \boldsymbol{r}_0 \\ \underset{\smile}{\boldsymbol{r}_0} \end{bmatrix}. \tag{24}$$

第二步处理即按照(19)式算出 $\varepsilon_\perp$ 的附加色差为

$$\Delta \boldsymbol{u}_w(z_i)|_{\varepsilon_\perp} = -w(z_i)(\dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0)[C_d \dot{\boldsymbol{r}}_0 + C_m \boldsymbol{r}_0 + (C_r - \dot{\chi}_0 C_d)\underset{\smile}{\boldsymbol{r}_0}]. \tag{25}$$

第三步处理即按照(19)式求得 $V_0''\boldsymbol{r}_0^2/4$ 的附加色差为

$$\Delta \boldsymbol{u}_w(z_i)\Big|_{V_0''} = -w(z_i)(2\eta)\left(\frac{1}{4}V_0''\boldsymbol{r}_0 \cdot \boldsymbol{r}_0\right)[C_d \dot{\boldsymbol{r}}_0 + C_m \boldsymbol{r}_0 + (C_r - \dot{\chi}_0 C_d)\underset{\smile}{\boldsymbol{r}_0}]. \tag{26}$$

将以上三式求和即得到与 $w(z_i)$ 有关的三级象差为

$$\Delta \boldsymbol{u}_{3w}(z_i) = \Delta \boldsymbol{u}_w(z_i) + \Delta \boldsymbol{u}_w(z_i)|_{\varepsilon_\perp} + \Delta \boldsymbol{u}_w(z_i)|_{V_0''}. \tag{27}$$

由于(25)式含有 $\dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0$ 的幂次，则可以并入 w 矩阵的第一行相应的各元素中；同理，由于(26)式含有 $\boldsymbol{r}_0 \cdot \boldsymbol{r}_0$ 的幂次，则可以并入 w 矩阵的第四行相应的元素中．于是(27)式求和的结果可以用广义的 w 矩阵表示如下：

$$\Delta \boldsymbol{u}_{3w}(z_i) = w(z_i)\begin{bmatrix} \dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2\boldsymbol{r}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2(\boldsymbol{r}_0 \times \dot{\boldsymbol{r}}_0)\boldsymbol{k} \\ \boldsymbol{r}_0 \cdot \boldsymbol{r}_0 \end{bmatrix}^*$$

$$\cdot\begin{bmatrix} B-C_d & 0 & F-C_m & f-\dot{\chi}_0B-C_r+\dot{\chi}_0C_d \\ F & 0 & C+2\dot{\chi}_0f-\dot{\chi}_0^2B & c-\dot{\chi}_0F \\ f-\dot{\chi}_0B & 0 & c-\dot{\chi}_0F & 0 \\ \begin{matrix} D-6\dot{\chi}_0f+3\dot{\chi}_0^2B \\ -(2\eta V_0''/4)C_d \end{matrix} & 0 & \begin{matrix} E-2\dot{\chi}_0c+\dot{\chi}_0^2F \\ -(2\eta V_0''/4)C_m \end{matrix} & \begin{matrix} e-\dot{\chi}_0D+3\dot{\chi}_0^2f-\dot{\chi}_0^3B \\ -(2\eta V_0''/4)(C_r-\dot{\chi}_0C_d) \end{matrix} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \underset{\smile}{\dot{\boldsymbol{r}}_0} \\ \boldsymbol{r}_0 \\ \underset{\smile}{\boldsymbol{r}_0} \end{bmatrix}. \tag{28}$$

若逸出电子束具有相同的 ε_z，与此对应的象平面为 z_i，则

$$v(z_i,\varepsilon_z)=0,\qquad w(z_i,\varepsilon_z)=(-1)^n m, \tag{29}$$

式中 m 为横向放大率的绝对值，n 为成象次数．可见，在 z_i 象平面中，只有 w 矩阵的象差．当 $V_0''=0$ 时，(28)式就是文献 [1—3] 所给出的阴极透镜在 $v(z_i,\varepsilon_z)=0$ 对应的象平面中的三级象差．

2. v 矩阵所代表的象差

在(16)式中 v 矩阵所代表的象差为

$$\Delta\boldsymbol{u}_v(z_i)=-(2\eta)^{-1/2}v(z_i)\begin{bmatrix} \dot{\boldsymbol{u}}_0\cdot\dot{\boldsymbol{u}}_0 \\ 2\boldsymbol{u}_0\cdot\dot{\boldsymbol{u}}_0 \\ 2(\boldsymbol{u}_0\times\dot{\boldsymbol{u}}_0)\boldsymbol{k} \\ \boldsymbol{u}_0\cdot\boldsymbol{u}_0 \end{bmatrix}^*\begin{bmatrix} F & -f & D \\ C & -c & E \\ c & 0 & e \\ E & -e & A \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{u}}_0 \\ \underset{\smile}{\dot{\boldsymbol{u}}_0} \\ \boldsymbol{u}_0 \end{bmatrix}. \tag{30}$$

按照文献[5]经过第一步处理即得

$$\Delta\boldsymbol{u}_v(z_i)=-(2\eta)^{-1/2}v(z_i)\begin{bmatrix} \dot{\boldsymbol{r}}_0\cdot\dot{\boldsymbol{r}}_0 \\ 2\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0 \\ 2(\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k} \\ \boldsymbol{r}_0\cdot\boldsymbol{r}_0 \end{bmatrix}^*$$

$$\cdot\begin{bmatrix} F & -f & D-\dot{\chi}_0f & -\dot{\chi}_0F \\ C & -c & E-\dot{\chi}_0c & -\dot{\chi}_0C \\ c-\dot{\chi}_0F & \dot{\chi}_0f & e-\dot{\chi}_0D+\dot{\chi}_0^2f & -\dot{\chi}_0c+\dot{\chi}_0^2F \\ E-2\dot{\chi}_0c+\dot{\chi}_0^2F & -e-\dot{\chi}_0^2f & A-3\dot{\chi}_0e+\dot{\chi}_0^2D-\dot{\chi}_0^3f & -\dot{\chi}_0E+2\dot{\chi}_0^2c-\dot{\chi}_0^2F \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \underset{\smile}{\dot{\boldsymbol{r}}_0} \\ \boldsymbol{r}_0 \\ \underset{\smile}{\boldsymbol{r}_0} \end{bmatrix}. \tag{31}$$

首先将 v 矩阵中的(2,2)，(3,2)，(4,2)及(3,4)诸元素变换如下：

$$\begin{aligned} -c:&\ (\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0)\underset{\smile}{\dot{\boldsymbol{r}}_0}=(\dot{\boldsymbol{r}}_0\cdot\dot{\boldsymbol{r}}_0)\underset{\smile}{\boldsymbol{r}_0}-((\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k})\dot{\boldsymbol{r}}_0, \\ \dot{\chi}_0f:&\ ((\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k})\underset{\smile}{\dot{\boldsymbol{r}}_0}=-(\dot{\boldsymbol{r}}_0\cdot\dot{\boldsymbol{r}}_0)\boldsymbol{r}_0+(\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0)\dot{\boldsymbol{r}}_0, \\ -e-\dot{\chi}_0^2f:&\ (\boldsymbol{r}_0\cdot\boldsymbol{r}_0)\underset{\smile}{\dot{\boldsymbol{r}}_0}=-((\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k})\boldsymbol{r}_0+(\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0)\underset{\smile}{\boldsymbol{r}_0}, \\ -\dot{\chi}_0c+\dot{\chi}_0^2F:&\ ((\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k})\underset{\smile}{\boldsymbol{r}_0}=(\boldsymbol{r}_0\cdot\boldsymbol{r}_0)\dot{\boldsymbol{r}}_0-(\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0)\boldsymbol{r}_0. \end{aligned} \tag{32}$$

再将 $-(2\eta)^{-1/2}$ 因子的负号移入 v 矩阵中，并将(16)式{}括号的项并入到 v 矩阵的(4,4)元素中，最后得到

$$\Delta \boldsymbol{u}_v(z_i) = (2\eta)^{-1/2} v(z_i) \begin{bmatrix} \dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2\boldsymbol{r}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2(\boldsymbol{r}_0 \times \dot{\boldsymbol{r}}_0)\boldsymbol{k} \\ \boldsymbol{r}_0 \cdot \boldsymbol{r}_0 \end{bmatrix}^*$$

$$\cdot \begin{bmatrix} -F & f & -D + 3\dot{\chi}_0 f & 2c + \dot{\chi}_0 F \\ -C - \dot{\chi}_0 f & 0 & -E + \dot{\chi}_0^2 F & (e/2) + \dot{\chi}_0 C + \dot{\chi}_0^2 f/2 \\ -2c + \dot{\chi}_0 F & 0 & -(3/2)(e + \dot{\chi}_0^2 f) + \dot{\chi}_0 D & 0 \\ -E + 4\dot{\chi}_0 c - 3\dot{\chi}_0^2 F & 0 & -A + 3\dot{\chi}_0 e - \dot{\chi}_0^2 D + \dot{\chi}_0^3 F & \dot{\chi}_0(E - 2\dot{\chi}_0 c + \dot{\chi}_0^2 F) + \mathscr{H}_0''/16 \end{bmatrix}$$

$$\cdot \begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \underset{\smile}{\dot{\boldsymbol{r}}_0} \\ \boldsymbol{r}_0 \\ \underset{\smile}{\boldsymbol{r}_0} \end{bmatrix}. \tag{33}$$

经过第二步处理算出 $\varepsilon_\perp$ 的附加色差为

$$\Delta \boldsymbol{u}_v(z_i)|_{\varepsilon_\perp} = (2\eta)^{-1/2} v(z_i)(\dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0)[C_m \dot{\boldsymbol{r}}_0 - C_r \underset{\smile}{\dot{\boldsymbol{r}}_0} + (C_a - C_r \dot{\chi}_0)\boldsymbol{r}_0 - \dot{\chi}_0 C_m \underset{\smile}{\boldsymbol{r}_0}]. \tag{34}$$

经过第三步处理算出 $V_0''\boldsymbol{r}_0^2/4$ 的附加色差为

$$\Delta \boldsymbol{u}_v(z_i)|_{V_0''} = (2\eta)^{-1/2} v(z_i)(2\eta V_0'' \boldsymbol{r}_0 \cdot \boldsymbol{r}_0/4)[C_m \dot{\boldsymbol{r}}_0 - C_r \underset{\smile}{\dot{\boldsymbol{r}}_0} + (C_a - C_r \dot{\chi}_0)\boldsymbol{r}_0 - \dot{\chi}_0 C_m \underset{\smile}{\boldsymbol{r}_0}]. \tag{35}$$

同理可得与 $v(z_i)$ 有关的三级象差为

$$\Delta \boldsymbol{u}_{3v}(z_i) = \Delta \boldsymbol{u}_v(z_i) + \Delta \boldsymbol{u}_v(z_i)|_{\varepsilon_\perp} + \Delta \boldsymbol{u}_v(z_i)|_{V_0''}. \tag{36}$$

经过整理，由上式可以得到广义的 v 矩阵如下:

$$\Delta \boldsymbol{u}_{3v}(z_i) = (2\eta)^{-1/2} v(z_i) \begin{bmatrix} \dot{\boldsymbol{r}}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2\boldsymbol{r}_0 \cdot \dot{\boldsymbol{r}}_0 \\ 2(\boldsymbol{r}_0 \times \dot{\boldsymbol{r}}_0)\boldsymbol{k} \\ \boldsymbol{r}_0 \cdot \boldsymbol{r}_0 \end{bmatrix}^*$$

$$\cdot \begin{bmatrix} -F + C_m & f - C_r & -D + 3\dot{\chi}_0 f + C_a - \dot{\chi}_0 C_r & 2c + \dot{\chi}_0 F - \dot{\chi}_0 C_m \\ -C - \dot{\chi}_0 f & 0 & -E + \dot{\chi}_0^2 F & e/2 + \dot{\chi}_0 C + \dot{\chi}_0^2 f/2 - (\eta/4)V_0'' C_r \\ -2c + \dot{\chi}_0 F & 0 & -(3/2)(e + \dot{\chi}_0^2 f) + \dot{\chi}_0 D + (\eta/4)V_0'' C_r & 0 \\ \begin{gathered} -E + 4\dot{\chi}_0 c - 3\dot{\chi}_0^2 F \\ +(2\eta V_0''/4)C_m \end{gathered} & 0 & \begin{gathered} -A + 3\dot{\chi}_0 e - \dot{\chi}_0^2 D + \dot{\chi}_0^3 f \\ +(2\eta V_0''/4)(C_a - \dot{\chi}_0 C_r) \end{gathered} & \begin{gathered} \dot{\chi}_0(E - 2\dot{\chi}_0 c + \dot{\chi}_0^2 F) \\ -\dot{\chi}_0(2\eta V_0''/4)C_m + \mathscr{H}_0''/16 \end{gathered} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \underset{\smile}{\dot{\boldsymbol{r}}_0} \\ \boldsymbol{r}_0 \\ \underset{\smile}{\boldsymbol{r}_0} \end{bmatrix}. \tag{37}$$

至此，(28)和(37)式给出了复合阴极透镜在任意平面 z_i 中全部的三级象差为

$$\Delta\boldsymbol{u}_3(z_i) = \Delta\boldsymbol{u}_{3w}(z_i) + \Delta\boldsymbol{u}_{3v}(z_i), \tag{38}$$

式中 w 矩阵和 v 矩阵所代表的三级象差系数总共包括 9 个几何象差系数和 4 个色差系数，形式简洁，便于数值计算。可以验证这两个广义的 w, v 矩阵中所有的三级象差系数(见(41)和(42)式)，不论 ε_z, $\mathscr{H}_0$, V_0'' 和 $v(z_i)$ 是否为零，都是不发散的(确切地说，即使有发散项也一定能互相抵消)。

现将三级象差用复数形式表示，以便与文献[3,10]对照。为此引进以下复数表示式：

$$\boldsymbol{r}_0 = r_0 e^{i\theta_0},\ \underset{\smile}{\boldsymbol{r}_0} = i r_0 e^{i\theta_0},\ \boldsymbol{r}_0\cdot\boldsymbol{r}_0 = |\boldsymbol{r}_0|^2 = r_0^2,$$

$$\dot{\boldsymbol{r}}_0 = \sqrt{2\eta\varepsilon_\perp}\, e^{i\beta_0},\ \underset{\smile}{\dot{\boldsymbol{r}}_0} = i\sqrt{2\eta\varepsilon_\perp}\, e^{i\beta_0},\ \dot{\boldsymbol{r}}_0\cdot\dot{\boldsymbol{r}}_0 = |\dot{\boldsymbol{r}}_0|^2 = 2\eta\varepsilon_\perp,$$

$$(\boldsymbol{r}_0\cdot\dot{\boldsymbol{r}}_0) = r_0\sqrt{2\eta\varepsilon_\perp}\cos(\beta_0-\theta_0),\ (\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0)\boldsymbol{k} = r_0\sqrt{2\eta\varepsilon_\perp}\sin(\beta_0-\theta_0). \tag{39}$$

利用上述复数表示，并将(28)和(37)式代入(38)式，则得到复数形式的广义 w, v 矩阵表示的象差为

$$\Delta\boldsymbol{u}_3(z_i) = \begin{bmatrix} \varepsilon_\perp/V_i \\ 2r_0\sqrt{\varepsilon_\perp/V_i}\cos(\beta_0-\theta_0) \\ 2r_0\sqrt{\varepsilon_\perp/V_i}\sin(\beta_0-\theta_0) \\ r_0^2 \end{bmatrix}^* \left\{ w(z_i)\begin{bmatrix} B_w & 0 & G_w-F_w & g_w-f_w \\ F_w & 0 & C_w & c_w/2 \\ f_w & 0 & c_w/2 & 0 \\ D_w-C_w & 0 & E_w & e_w \end{bmatrix} + v(z_i)\begin{bmatrix} B_v & b_v & G_v-F_v & g_v-f_v \\ F_v & 0 & C_v & (c_v+d_v)/2 \\ f_v & 0 & (c_v-d_v)/2 & 0 \\ D_v-C_v & 0 & E_v & e_v \end{bmatrix}\right\}\begin{bmatrix} \sqrt{\varepsilon_\perp/V_i}\, e^{i\beta_0} \\ i\sqrt{\varepsilon_\perp/V_i}\, e^{i\beta_0} \\ r_0 e^{i\theta_0} \\ i r_0 e^{i\theta_0} \end{bmatrix}, \tag{40}$$

式中 $V_i = V(z_i)$. $B_w, \cdots, e_w, B_v, \cdots, e_v$ 是文献[3]所采用的象差系数(但是该文仅考虑 $V_0''=0$ 的情况)。将(40)式与(28),(37),(38)诸式比较，可以得到以下结果：

$$\begin{aligned}
&B_w = (2\eta V_i)^{3/2}(B - C_d),\ G_w - F_w = (2\eta V_i)(F - C_m),\\
&g_w - f_w = (2\eta V_i)(f - \dot{\chi}_0 B - C_r + \dot{\chi}_0 C_d),\ F_w = (2\eta V_i)F,\\
&C_w = (2\eta V_i)^{1/2}(C + 2\dot{\chi}_0 f - \dot{\chi}_0^2 B),\ c_w/2 = (2\eta V_i)^{1/2}(c - \dot{\chi}_0 F),\\
&f_w = (2\eta V_i)(f - \dot{\chi}_0 B),\\
&D_w - C_w = (2\eta V_i)^{1/2}\{D - 6\dot{\chi}_0 f + 3\dot{\chi}_0^2 B - (2\eta V_0''/4)C_d\},\\
&E_w = E - 2\dot{\chi}_0 c + \dot{\chi}_0^2 F - (2\eta V_0''/4)C_m,\\
&e_w = e - \dot{\chi}_0 D + 3\dot{\chi}_0^2 f - \dot{\chi}_0^3 B - (2\eta V_0''/4)(C_r - \dot{\chi}_0 C_d).
\end{aligned} \tag{41}$$

$$\begin{aligned}
&B_v = 2\eta V_i^{3/2}(-F + C_m),\ b_v = 2\eta V_i^{3/2}(f - C_r),\\
&G_v - F_v = (2\eta)^{1/2}V_i(-D + 3\dot{\chi}_0 f + C_e - \dot{\chi}_0 C_r),\\
&g_v - f_v = (2\eta)^{1/2}V_i(2c + \dot{\chi}_0 F - \dot{\chi}_0 C_m),\\
&F_v = (2\eta)^{1/2}V_i(-C - \dot{\chi}_0 f),\ C_v = V_i^{1/2}(-E + \dot{\chi}_0^2 F),\\
&(c_v + d_v)/2 = V_i^{1/2}\{(e/2) + \dot{\chi}_0 C + \dot{\chi}_0^2 f/2 - (\eta/4)V_0'' C_r\},\\
&f_v = (2\eta)^{1/2}V_i(-2c + \dot{\chi}_0 F),\\
&(c_v - d_v)/2 = V_i^{1/2}\{-(3/2)(e + \dot{\chi}_0^2 f) + \dot{\chi}_0 D + (\eta/4)V_0'' C_r\},\\
&D_v - C_v = V_i^{1/2}\{-E + 4\dot{\chi}_0 c - 3\dot{\chi}_0^2 F + (2\eta V_0''/4)C_m\},
\end{aligned}$$

$$E_v=(2\eta)^{-1/2}\{-A+3\dot{\chi}_0e-\dot{\chi}_0^2D+\dot{\chi}_0^3f+(2\eta V_0''/4)(C_a-\dot{\chi}_0C_r)\},$$
$$e_v=(2\eta)^{-1/2}\{\dot{\chi}_0E-2\dot{\chi}_0^2c+\dot{\chi}_0^3F-\dot{\chi}_0(2\eta V_0''/4)C_m+\mathscr{H}_0''/16\}. \quad (42)$$

以上(41),(42)式给出了文献[3]的象差系数与 Glaser[4] 象差系数的关系. 同样可以将这些象差系数与文献[1]的象差系数联系起来. 为了节省篇幅,不再详述.

将(40)式的广义 w, v 矩阵进行乘法运算,即得

$$\begin{aligned}\Delta\boldsymbol{u}_3(z_i)=&\ r_0^3e^{i\theta_0}[w(z_i)(E_w+ie_w)+v(z_i)(E_v+ie_v)] \quad (\text{畸变})\\&+r_0^2\sqrt{\varepsilon_\perp/V_i}\,e^{i\beta_0}[w(z_i)D_w+v(z_i)(D_v+id_v)] \quad (\text{场曲})\\&+r_0^2\sqrt{\varepsilon_\perp/V_i}\,e^{i(2\theta_0-\beta_0)}[w(z_i)(C_w+ic_w)+v(z_i)(C_v+ic_v)] \quad (\text{象散})\\&+r_0(\varepsilon_\perp/V_i)e^{i\theta_0}[w(z_i)(G_w+ig_w)+v(z_i)(G_v+ig_v)] \quad (\text{彗差长度})\\&+r_0(\varepsilon_\perp/V_i)e^{i(2\beta_0-\theta_0)}[w(z_i)(F_w-if_w)+v(z_i)(F_v-if_v)] \quad (\text{彗差半径})\\&+(\varepsilon_\perp/V_i)^{3/2}e^{i\beta_0}[w(z_i)B_w+v(z_i)(B_v+ib_v)] \quad (\text{色球差}). \quad (43)\end{aligned}$$

可见,各种不同的象差系数对应于不同的象差散射图形. 但是存在磁场的情况下,在 $v(z_i,\varepsilon_z)\neq 0$ 的平面中还会出现各向异性的球差 b_v. 在 $v(z_i,\varepsilon_z)=0$ 的象平面中,上式中含有 $w(z_i)$ 的象差与文献[10]中圆形透镜的象差是互相对应的.

最后应该指出,上述三级象差是在旋转坐标系中算出的. 而在固定坐标系中,则由(6)式有

$$\begin{aligned}\boldsymbol{r}(z_i)=&\ e^{i\chi(z_i)}\boldsymbol{u}(z_i)=e^{i\chi(z_i)}[\boldsymbol{u}_g(z_i)+\Delta\boldsymbol{u}_3(z_i)]\\=&\ e^{i\chi(z_i)}\{\sqrt{\varepsilon_\perp}\,e^{i\beta_0}v(z_i)+r_0e^{i\theta_0}[w(z_i)-i\dot{\chi}_0v(z_i)(2\eta)^{-1/2}]\\&+\Delta\boldsymbol{u}_3(z_i)\}. \quad (44)\end{aligned}$$

上式中 $\chi(z_i)$ 见(7)式, $\boldsymbol{u}_g$ 是近轴轨迹见(5)式, $\Delta\boldsymbol{u}_3(z_i)$ 是三级象差见(43)式.

四、结 论

1. 本文运用变分原理讨论了复合阴极透镜的三级象差. 利用矢量描写轨迹, 运用矩阵表示象差. 由此得到任意象平面中全部的三级象差,见(28),(37),(41)和(42)式,只需用 Glaser[4] 导出的 9 个几何象差系数和 4 个色差系数的线性组合来表征. 我们得到的复合阴极透镜的象差系数一则形式简洁,再则它们不论 ε_z, $\mathscr{H}_0$, V_0'', $v(z_i)$ 是否为零都不会发散. 因而这套象差系数公式适合于计算机数值计算.

2. 我们研究阴极透镜象差的变分方法, 物理概念简单明瞭, 推导步骤亦较简单, 并与轨迹法的结果[1-3]一致. 本文所得到的任意象平面中的象差, 考虑到阴极面上 $\mathscr{H}_0\neq 0$, 且横向电场不为零,即 $V_0''\neq 0$ 的情况. 由此可见,我们的方法是正确的,结果是普遍的.

3. 本文将变分方法运用到阴极透镜上,而且所得的象差系数形式上与 Glaser 的结果一致. 这样便把细束电子光学与宽束电子光学建立起联系. 这在理论上和实际上都很有意义.

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VARIATIONAL THEORY OF ABERRATIONS IN CATHODE LENSES

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ABSTRACT

In this paper, the aberration theory for cathode lenses with combined electrostatic and magnetic focusing is discussed in detail. The more general case, when cathode is situated in both magnetic and transversal electric fields, has also been considered. Based on the variational principle, we have derived the general formulae of aberration coefficients of cathode lenses on any image plane, and transformed them into the linear combination of the well-known aberration coefficients given by Glaser. It follows that the aberrations for both the wide electron beam system (cathode lens) and the narrow electron beam system (conventional electron lens) can be treated by the universal variational theory. Hence the relationship between these two systems is also established.

The paraxial electron trajectories of cathode lenses are described in the vector form. The aberrations are expressed in the matrix form, and they are suitable for computer calculation.