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Boltzmann 方程的奇异扰动解法 (I)

正 规 解

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提 要

本文用奇异扰动方法讨论了具有小 Knudsen 数的 Boltzmann 方程的正规解，证明把 Hilbert 展开和 Enskog 展开加以改进可以消除久期项.

一、引 言

Boltzmann 方程是稀薄气体动力学的基本方程，它无论在理论上或实际上都有重要的意义[1-3]. 因此，自从 1876 年 Boltzmann 提出这个方程以来，人们一直在探讨这个方程的导出、解法和应用. 一百多年来的艰苦努力表明求解 Boltzmann 方程是极其困难的. 但是，人们已经发现[4,1]，在小 Knudsen 数的情况下，有一类所谓正规解比较容易讨论. 这里所说的正规解，是指除去初始层、边界层和激波层之外的时间和空间中的解. 在初始层、边界层和激波层之中，方程的解比较复杂，而正规解相对说来比较简单，但它却在绝大部分时间及空间之中有效；即使在初始层、边界层和激波层之中，也常常需要以正规解为基础来讨论非正规解. 因此，弄清 Boltzmann 方程的正规解是十分重要的.

从物理上说，正规解是一种从偏离平衡不远的状态出发的解. 但是并非在平衡附近的任意一种状态都可以作为正规解的初态. 事实上，对正规解的初条件限制是十分苛刻的. 我们在讨论正规解时，不必去考虑给定的初条件能否被满足，相反地，可以仔细地选择初条件以满足正规解的要求. 在求得正规解之后，再添加一个初始层解，以满足给定的初条件.

求正规解时遇到的主要困难是 $t\to\infty$ 时展开级数中可能出现一些无界的项. 从奇异扰动的观点来说就是存在久期项[5,6]. 事实上，久期项是由于展开方法不当造成的. 只要找到了正确的展开方式，就可以避免久期项.

利用通常的记号，对于散射截面与二分子相撞速率成反比的单原子 Maxwell 分子气体，无外场时的 Boltzmann 方程为[1,2]

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}}=\frac{1}{\varepsilon}\int d\boldsymbol{w}d\hat{\boldsymbol{u}}'g_{\mathrm{M}}(\hat{\boldsymbol{u}}\cdot\hat{\boldsymbol{u}}')[f(\boldsymbol{v}')f(\boldsymbol{w}')-f(\boldsymbol{v})f(\boldsymbol{w})], \tag{1}$$

其中 f 为单粒子分布函数，$\boldsymbol{u}=\boldsymbol{v}-\boldsymbol{w}$，$\boldsymbol{u}'=\boldsymbol{v}'-\boldsymbol{w}'$，$\hat{}$ 表示单位矢量，$g_{\mathrm{M}}(\hat{\boldsymbol{u}}\cdot\hat{\boldsymbol{u}}')$是

散射截面与相撞速率之积，$\boldsymbol{r}$, t, $\boldsymbol{v}$ 都是无量纲化了的量，时间和长度的单位在后面选定，ε 为 Knudsen 数:

$$\varepsilon = l_r/l, \tag{2}$$

其中 l_r 为分子平均自由程，l 为系统的特征长度，也就是我们选用的单位长度．所谓正规解都是就 $\varepsilon \ll 1$ 而言的．也就是说，气体中碰撞相当频繁，但气体仍然是稀薄的，三体碰撞可以忽略．f 满足归一化条件

$$\frac{1}{V}\int f d\boldsymbol{v} d\boldsymbol{r} = 1. \tag{3}$$

其中 V 为容器的体积．如果器壁是镜反射的，那么气体与环境没有能量交换，还可以选用单位使能量守恒关系写成

$$\frac{1}{V}\int v^2 f d\boldsymbol{v} d\boldsymbol{r} = 3. \tag{4}$$

如果对(1)式实行 Fourier 变换

$$\varphi(\boldsymbol{r}, \boldsymbol{k}, t) = \int e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} f(\boldsymbol{r}, \boldsymbol{v}, t) d\boldsymbol{v}, \tag{5}$$

那么(1)式就可写成[7,8]

$$\frac{\partial\varphi}{\partial t} + i\frac{\partial^2\varphi}{\partial\boldsymbol{k}\cdot\partial\boldsymbol{r}} = \frac{1}{\varepsilon} J_{\mathrm{M}}(\varphi, \varphi), \tag{6}$$

其中

$$J_{\mathrm{M}}(\varphi, \varphi) = \int d\hat{\boldsymbol{u}}' g_{\mathrm{M}}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{u}}')\left\{\varphi\left[\frac{k}{2}(\hat{\boldsymbol{k}}+\hat{\boldsymbol{u}}')\right]\varphi\left[\frac{k}{2}(\hat{\boldsymbol{k}}-\hat{\boldsymbol{u}}')\right] - \varphi(\boldsymbol{k})\varphi(0)\right\}. \tag{7}$$

如果仅限于讨论平面对称问题，那么

$$\varphi = \varphi(x, k, \mu, t), \tag{8}$$

其中 x 是空间的一个笛卡儿坐标，已假定 φ 与另二维空间坐标 y, z 无关; $\mu = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{e}}_x = \cos\chi$, χ 是 $\boldsymbol{k}$ 与 x 轴的夹角．这时，(6)式简化成

$$\frac{\partial\varphi}{\partial t} + i\mu\frac{\partial^2\varphi}{\partial k\partial x} + \frac{i(1-\mu^2)}{k}\frac{\partial^2\varphi}{\partial\mu\partial x} = \frac{1}{\varepsilon} J_{\mathrm{M}}(\varphi, \varphi). \tag{9}$$

若系统与环境无能量交换，那么定态就是

$$\varphi_e = e^{-\frac{k^2}{2}}. \tag{10}$$

本文仅讨论 Maxwell 分子．对于非 Maxwell 分子，将在其他文章中讨论．

二、初级近似

设(9)式的正规解可以写成

$$\varphi_n = \rho(x, t) e^{-\frac{k^2}{2}\theta(x,t) - ik\mu c(x,t)}[1 + \xi(x, k, \mu, t)], \tag{11}$$

将(11)式代人(9)式，得到

$$I_{\mathrm{M}}(\xi) = -J_{\mathrm{M}}(\xi, \xi) + \frac{\varepsilon}{\rho}\left[\frac{\partial\xi}{\partial t} + D_0 + D_0\xi + D_1(\xi) + D_2(\xi)\right], \tag{12}$$

其中

$$I_{\mathrm{M}}(\xi)=J_{\mathrm{M}}(1,\xi)+J_{\mathrm{M}}(\xi,1), \tag{13}$$

$$D_0=\left\{\frac{1}{\rho}\frac{\partial\rho}{\partial t}+\frac{1}{\rho}\frac{\partial(\rho c)}{\partial x}\right\}+\left\{\frac{\partial c}{\partial t}+c\frac{\partial c}{\partial x}+\frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x}\right\}(-ik\mu)$$
$$+\left\{\frac{\partial\theta}{\partial t}+c\frac{\partial\theta}{\partial x}+\frac{2}{3}\theta\frac{\partial c}{\partial x}\right\}\left(-\frac{k^2}{2}\right)+\frac{4}{3}\theta\frac{\partial c}{\partial x}\left\{-\frac{k^2}{2}P_2(\mu)\right\}+3\theta\frac{\partial\theta}{\partial x}\left(\frac{ik^3}{6}\mu\right), \tag{14}$$

$$D_1(\xi)=(c-\theta ik\mu)\frac{\partial\xi}{\partial x}+i\mu\frac{\partial^2\xi}{\partial k\partial x}+\frac{i(1-\mu^2)}{k}\frac{\partial^2\xi}{\partial\mu\partial x}, \tag{15}$$

$$D_2(\xi)=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial x}-\frac{k^2}{2}\frac{\partial\theta}{\partial x}-ik\mu\frac{\partial c}{\partial x}\right)\left[i\mu\frac{\partial\xi}{\partial k}+\frac{i(1-\mu^2)}{k}\frac{\partial\xi}{\partial\mu}\right]. \tag{16}$$

记

$$e_{nl}=\frac{(-ik)^n}{n!}P_l(\mu)\qquad n=0,1,2,\cdots;\ l=n,n-2,\cdots,1\text{ 或 }0, \tag{17}$$

其中 $P_l(\mu)$ 是 l 阶 Legendre 多项式．可以证明[7-9]

$$I_{\mathrm{M}}(e_{nl})=-\lambda_{nl}e_{nl}, \tag{18}$$

其中

$$\lambda_{nl}=2\pi\int_{-1}^{1}d\nu g_{\mathrm{M}}(\nu)\left[1+\delta_{n,0}\delta_{l,0}-\left(\frac{1+\nu}{2}\right)^{\frac{n}{2}}P_l\left(\sqrt{\frac{1+\nu}{2}}\right)\right.$$
$$\left.-\left(\frac{1-\nu}{2}\right)^{\frac{n}{2}}P_l\left(\sqrt{\frac{1-\nu}{2}}\right)\right]. \tag{19}$$

这个结果首先是由王承书和 Uhlenbeck 得到的[9]，但当时表示为速度空间的函数． 值得注意的是

$$\lambda_{00}=\lambda_{11}=\lambda_{20}=0, \tag{20}$$

它对应着碰撞过程中质量、动量和能量三个守恒定律．对于 Maxwell 分子，可以规定

$$\lambda_{40}=1, \tag{21}$$

并由(4)和(21)式规定时间和长度的单位．

令

$$\xi=\sum_{j=0}^{\infty}\varepsilon^j\xi^{(j)}, \tag{22}$$

$$\xi^{(j)}=\sum_{n\,l}a_{nl}^{(j)}(x,t)e_{nl}. \tag{23}$$

对 n, l 的求和包括 $n=0,1,2,\cdots$; $l=n,n-2,\cdots,1$ 或 0．限制 $n+l=$ 偶数的理由是 $\varphi(x,k,\mu,t)=\varphi(x,-k,-\mu,t)$ 应当成立．不失一般性，可以假定

$$a_{00}^{(0)}=a_{11}^{(0)}=a_{20}^{(0)}=0. \tag{24}$$

将 (22) 式代入 (12) 式中，ε^0 级方程为

$$I_{\mathrm{M}}(\xi^{(0)})=-J_{\mathrm{M}}(\xi^{(0)},\xi^{(0)}), \tag{25}$$

它有解(而且是条件(24)式下的唯一解)：

$$\xi^{(0)}=0. \tag{26}$$

三、一级近似

假定

$$\frac{\partial\rho}{\partial t}=\sum_{i=0}^{\infty}\varepsilon^i p_i,\quad \frac{\partial c}{\partial t}=\sum_{i=0}^{\infty}\varepsilon^i q_i,\quad \frac{\partial\theta}{\partial t}=\sum_{i=0}^{\infty}\varepsilon^i s_i,\tag{27}$$

其中 p_i, q_i, s_i 都是待定的函数,它们只与 ρ, c, θ 及其对 x 的导数有关,以保证(27)式成为封闭的方程组. 由(27),(22),(26)式可知(12)式的 ε 级为

$$I_M(\xi^{(1)})=\frac{1}{\rho}\left\{\left[\frac{p_0}{\rho}+\frac{1}{\rho}\frac{\partial(\rho c)}{\partial x}\right]+e_{11}\left[q_0+c\frac{\partial c}{\partial x}+\frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x}\right]\right.$$
$$\left.+e_{20}\left[s_0+c\frac{\partial\theta}{\partial x}+\frac{2}{3}\theta\frac{\partial c}{\partial x}\right]+e_{22}\frac{4}{3}\theta\frac{\partial c}{\partial x}+e_{31}3\theta\frac{\partial\theta}{\partial x}\right\}.\tag{28}$$

因此得到

$$p_0=-\frac{\partial(\rho c)}{\partial x},\quad q_0=-c\frac{\partial c}{\partial x}-\frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x},\quad s_0=-c\frac{\partial\theta}{\partial x}-\frac{2}{3}\theta\frac{\partial c}{\partial x},\tag{29}$$

$$\xi^{(1)}=a_{00}^{(1)}e_{00}+a_{11}^{(1)}e_{11}+a_{20}^{(1)}e_{20}-\frac{4\theta}{3\lambda_{22}\rho}\frac{\partial c}{\partial x}e_{22}-\frac{3\theta}{\lambda_{31}\rho}\frac{\partial\theta}{\partial x}e_{31},\tag{30}$$

其中 $a_{00}^{(1)}$, $a_{11}^{(1)}$ 和 $a_{20}^{(1)}$ 是 x 和 t 的任意函数.

四、二级近似

假定

$$\frac{\partial a_{nl}^{(j)}}{\partial t}=\sum_{i=0}^{\infty}\varepsilon^i\alpha_{nl}^{(j,i)}\qquad (n,l)=(0,0),\ (1,1),\ (2,0),\tag{31}$$

其中 $\alpha_{nl}^{(j,i)}$ 仅含 ρ, c, θ, $a_{00}^{(1)}$, $a_{11}^{(1)}$, $a_{20}^{(1)}$, $a_{00}^{(2)}$, $\cdots$, $a_{20}^{(j)}$ 及其对 x 的导数. 将(30)式对 t 求导,利用(27),(31)式按 ε 幂展开:

$$\frac{\partial\xi^{(1)}}{\partial t}=\sum_{i=0}^{\infty}\varepsilon^i\left(\frac{\partial\xi^{(1)}}{\partial t}\right)_i.\tag{32}$$

利用(27),(29)和(17)式,可以把(14)式写成

$$D_0=\sum_{i=0}^{\infty}\varepsilon^i D_{0i},\tag{33}$$

$$D_{00}=\frac{4}{3}\theta\frac{\partial c}{\partial x}e_{22}+3\theta\frac{\partial\theta}{\partial x}e_{31},\tag{34}$$

$$D_{0j}=\frac{1}{\rho}p_j+e_{11}q_j+e_{20}s_j\qquad j=1,2,\cdots.\tag{35}$$

于是(12)式的 ε^2 级为

$$I_M(\xi^{(2)})=-J_M(\xi^{(1)},\xi^{(1)})+\frac{1}{\rho}\left[D_{00}\xi^{(1)}+D_{01}+D_1(\xi^{(1)})+D_2(\xi^{(1)})+\left(\frac{\partial\xi^{(1)}}{\partial t}\right)_0\right].\tag{36}$$

比较上式两边 e_{00}, e_{11}, e_{20} 的系数,得到

$$\frac{1}{\rho}p_1 + \alpha_{00}^{(1,0)} + c\frac{\partial a_{00}^{(1)}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(1)}) = 0,$$

$$q_1 + \alpha_{11}^{(1,0)} + c\frac{\partial a_{11}^{(1)}}{\partial x} + \theta\frac{\partial a_{00}^{(1)}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(1)}) + a_{11}^{(1)}\frac{\partial c}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(1)}) = 0,$$

$$s_1 + \alpha_{20}^{(1,0)} + c\frac{\partial a_{20}^{(1)}}{\partial x} + a_{11}^{(1)}\frac{\partial\theta}{\partial x} + \frac{2}{3}\theta\frac{\partial a_{11}^{(1)}}{\partial x} + \frac{2}{3}a_{20}^{(1)}\frac{\partial c}{\partial x} + \frac{2}{3}a_{22}^{(1)}\frac{\partial c}{\partial x} + \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(1)}) = 0, \tag{37}$$

其中 $a_{22}^{(1)}$ 及 $a_{31}^{(1)}$ 已由(30)式给出．在(37)式的三个方程中，每个方程都有两个可供选择的量(左边前两项)，因此选择的方式不是唯一的．我们的标准是消去久期项．

首先指出，取

$$p_1 = q_1 = s_1 = 0 \tag{38}$$

是不行的．事实上(38)式相当于 Hilbert 的方法[4]，由于(38)式，精确到 $O(\varepsilon^2)$ 时(27)式成为 Euler 方程

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho c)}{\partial x} = 0,\quad \frac{\partial c}{\partial t} + c\frac{\partial c}{\partial x} + \frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x} = 0,$$
$$\frac{\partial\theta}{\partial t} + c\frac{\partial\theta}{\partial x} + \frac{2}{3}\theta\frac{\partial c}{\partial x} = 0. \tag{39}$$

考虑平衡态附近的小扰动

$$\rho = 1 + \rho',\quad \theta = 1 + \theta',\quad c = c', \tag{40}$$

其中 ρ', c', θ' 是小量．(39)式保留到一级小量成为

$$\frac{\partial\rho'}{\partial t} + \frac{\partial c'}{\partial x} = 0,\quad \frac{\partial c'}{\partial t} + \frac{\partial}{\partial x}(\rho' + \theta') = 0,\quad 3\frac{\partial\theta'}{\partial t} + 2\frac{\partial c'}{\partial x} = 0. \tag{41}$$

讨论小振动

$$\rho' = \rho_1 e^{\lambda t + i\kappa x},\quad \theta' = \theta_1 e^{\lambda t + i\kappa x},\quad c' = c_1 e^{\lambda t + i\kappa x}, \tag{42}$$

其中 κ 是实数．把(42)式代入(41)式，得到关于 ρ_1, c_1, θ_1 的线性代数方程组，它有非零解的条件为

$$Q_{\kappa^2}(\lambda) = \lambda(3\lambda^2 + 5\kappa^2) = 0. \tag{43}$$

显然 $\mathrm{Re}\lambda = 0$．由(37)，(38)及(31)式得到关于 $a_{00}^{(1)}$, $a_{11}^{(1)}$, $a_{20}^{(1)}$ 的方程(精确到 $O(\varepsilon)$)，并做类似(40)式的代换，得到的方程线性化之后与(41)式类似，但带有由 ρ', c', θ' 组成的非齐次项，因此

$$a_{nl}^{(1)} \sim te^{\lambda t + i\kappa x}\qquad (n, l) = (0, 0), (1, 1)\ (2, 0). \tag{44}$$

显然，在 $\mathrm{Re}\lambda = 0$ 时，这是久期项．

如果不取(38)式而令

$$p_1 = 0,\quad q_1 = -\frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(1)}),\ s_1 = -\frac{2}{3}a_{22}^{(1)}\frac{\partial c}{\partial x} - \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(1)}), \tag{45}$$

就可以避免出现(44)式那样的久期项．事实上，由(45)式，在精确到 $O(\varepsilon^2)$ 时，(27)式成为下面的 Navier-Stokes 方程：

$$\frac{\partial\rho}{\partial t} = -\frac{\partial(\rho c)}{\partial x},\quad \frac{\partial c}{\partial t} = -c\frac{\partial c}{\partial x} - \frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x} + \frac{4\varepsilon}{3\lambda_{22}\rho}\frac{\partial}{\partial x}\left(\theta\frac{\partial c}{\partial x}\right),$$

$$\frac{\partial\theta}{\partial t}=-c\frac{\partial\theta}{\partial x}-\frac{2}{3}\theta\frac{\partial c}{\partial x}+\frac{8\varepsilon\theta}{9\lambda_{22}\rho}\left(\frac{\partial c}{\partial x}\right)^2+\frac{5\varepsilon}{3\lambda_{31}\rho}\frac{\partial}{\partial x}\left(\theta\frac{\partial\theta}{\partial x}\right). \tag{46}$$

作代换(40),(42)式之后,得到关于 ρ_1, c_1, θ_1 的线性代数方程组,它有非零解的条件为

$$Q_{\kappa^2}(\lambda)=\lambda^3+\lambda^2(\eta+\omega)\kappa^2+\lambda\left(\eta\omega\kappa^2+\frac{5}{3}\right)\kappa^2+\omega\kappa^4=0, \tag{47}$$

其中 $\eta=4\varepsilon/3\lambda_{22}>0$, $\omega=5\varepsilon/3\lambda_{31}>0$. 不难证明,对任何 $\kappa^2>0$, (47)式的根 λ 的实部总是负的. 这是因为

1) 方程(47)的实根必定是负的,由于方程的所有系数包括常数项都是正的;

2) (47) 式至少有一个实根 λ_1, 它满足 $-(\eta+\omega)\kappa^2<\lambda_1<0$; 否则, 若 $\lambda_1\leqslant-(\eta+\omega)\kappa^2$, 那么

$$Q_{\kappa^2}(\lambda_1)\leqslant\lambda_1\left(\eta\omega\kappa^2+\frac{5}{3}\right)\kappa^2+\omega\kappa^4<0,$$

这是不行的;

3) (47)式的另外两个根 λ_2, λ_3 满足 $\lambda_2+\lambda_3=(\eta+\omega)\kappa^2-\lambda_1<0$, 如果它们是实根,那么由1)知它们都是负的,如果它们是共轭复根,那么它们的实部为负.

由此可知,(46)式的解是稳定的. 由(37)和(45)式可知:

$$\alpha_{00}^{(1,0)}=-c\frac{\partial a_{00}^{(1)}}{\partial x}-\frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(1)}),$$

$$\alpha_{11}^{(1,0)}=-\frac{\partial}{\partial x}(ca_{11}^{(1)})-\theta\frac{a_{00}^{(1)}}{\partial x}-\frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(1)}),$$

$$\alpha_{20}^{(1,0)}=-c\frac{\partial a_{20}^{(1)}}{\partial x}-\frac{2}{3}\theta\frac{\partial a_{11}^{(1)}}{\partial x}-\frac{2}{3}a_{20}^{(1)}\frac{\partial c}{\partial x}-a_{11}^{(1)}\frac{\partial\theta}{\partial x}. \tag{48}$$

比较(36)式两边所有 e_{nl} 的系数,可以知道 $\xi^{(2)}$ 包含直到 e_{62} 以前的诸矩,其中

$$a_{22}^{(2)}=\frac{-1}{\lambda_{22}}\left[\frac{1}{\rho}\left(\frac{\partial a_{22}^{(1)}}{\partial t}\right)_0+\frac{4}{3\rho}\frac{\partial c}{\partial x}(a_{20}^{(1)}+a_{22}^{(1)})+\frac{4}{9\rho^2}\frac{\partial}{\partial x}(\rho a_{31}^{(1)})\right.$$
$$\left.+\frac{4\theta}{3\rho}\frac{\partial a_{11}^{(1)}}{\partial x}+\frac{c}{\rho}\frac{\partial a_{22}^{(1)}}{\partial x}-\lambda_{40}(a_{11}^{(1)})^2\right], \tag{49}$$

$$a_{31}^{(2)}=\frac{-1}{\lambda_{31}}\left[\frac{1}{\rho}\left(\frac{\partial a_{31}^{(1)}}{\partial t}\right)_0+\frac{3}{\rho}\frac{\partial\theta}{\partial x}(a_{20}^{(1)}+a_{22}^{(1)})+\frac{11}{5\rho}\frac{\partial c}{\partial x}a_{31}^{(1)}\right.$$
$$+\frac{6\theta}{5\rho}\frac{\partial a_{22}^{(1)}}{\partial x}+\frac{3\theta}{\rho}\frac{\partial a_{20}^{(1)}}{\partial x}+\frac{c}{\rho}\frac{\partial a_{31}^{(1)}}{\partial x}$$
$$\left.-\frac{6}{5}\lambda_{22}a_{11}^{(1)}a_{22}^{(1)}-3\lambda_{40}a_{11}^{(1)}a_{20}^{(1)}+\frac{21}{10}\lambda_{40}a_{11}^{(1)}a_{22}^{(1)}\right]. \tag{50}$$

这里及以下 $(\partial a_{22}^{(1)}/\partial t)_0$ 之类记号意义类似(32)式,不再赘述.

五、三 级 近 似

(12)式的 ε^3 级为

$$I_M(\xi^{(3)})=\frac{1}{\rho}\left[D_{00}\xi^{(2)}+D_{01}\xi^{(1)}+D_{02}+D_1(\xi^{(2)})+D_2(\xi^{(2)})+\left(\frac{\partial\xi^{(1)}}{\partial t}\right)_1\right.$$

$$+\left(\frac{\partial\xi^{(2)}}{\partial t}\right)_0\Big] - J_M(\xi^{(1)}, \xi^{(2)}) - J_M(\xi^{(2)}, \xi^{(1)}). \tag{51}$$

比较上式两边 e_{00}, e_{11}, e_{20} 的系数,得

$$\frac{1}{\rho}p_2 + \alpha_{00}^{(1,1)} + \alpha_{00}^{(2,0)} + c\frac{\partial a_{00}^{(2)}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(2)}) = 0,$$

$$q_2+\alpha_{11}^{(1,1)}+\alpha_{11}^{(2,0)} + a_{00}^{(1)}q_1 + c\frac{\partial a_{11}^{(2)}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}[\rho(a_{20}^{(2)} + a_{22}^{(2)})] + \theta\frac{\partial a_{00}^{(2)}}{\partial x} + a_{11}^{(2)}\frac{\partial c}{\partial x} = 0,$$

$$s_2 + \alpha_{20}^{(1,1)} + \alpha_{20}^{(2,0)} + \frac{2}{3}\alpha_{11}^{(1)}q_1 + a_{00}^{(1)}s_1 + c\frac{\partial a_2^{(2)}}{\partial x} + \frac{2}{3}\theta\frac{\partial a_{11}^{(2)}}{\partial x} + \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(2)})$$
$$+\frac{2}{3}\frac{\partial c}{\partial x}(a_{20}^{(2)} + a_{22}^{(2)}) + a_{11}^{(2)}\frac{\partial\theta}{\partial x} = 0, \tag{52}$$

其中共有九个可供调整的量.

我们指出,选择

$$a_{nl}^{(1)} = a_{nl}^{(2)} = 0, \quad \alpha_{nl}^{(1,1)} = \alpha_{nl}^{(2,0)} = 0 \qquad (n, l) = (0, 0), (1, 1), (2, 0) \tag{53}$$

是不行的. 事实上,这相当于 Enskog 的方法[1]. 由于(53)式,可以简化(49),(50)式, 从而得到(27)式精确到 $O(\varepsilon^3)$ 的表达式为

$$\rho_t + (\rho c)_x = 0, \quad \rho(c_t + cc_x) + (\rho\theta)_x + \sigma_x = 0,$$

$$\theta_t + c\theta_x + \frac{2}{3}\theta c_x + \frac{5}{9\rho}\delta_x + \frac{2}{3\rho}c_x\sigma = 0,$$

$$\sigma_t = -\frac{4\bar{\eta}}{3}\theta c_x + \frac{2\bar{\eta}^2}{9\rho}\left[4\theta c_x^2 + 9(\theta\theta_x)_x - 6\theta\left\{\frac{1}{\rho}(\rho\theta)_x\right\}_x\right],$$

$$\delta = -\frac{9}{2}\bar{\eta}\theta\theta_x + \frac{3\bar{\eta}^2\theta}{20\rho^2}[95\rho\theta_x c_x - 16\rho_x c_x\theta - 14\rho\theta c_{xx}], \tag{54}$$

其中下标表示对 t 或 x 求导,而 $\bar{\eta} = \varepsilon/\lambda_{22} = 2\varepsilon/(3\lambda_{31})$, $\delta = \rho\varepsilon a_{31}^{(1)} + \rho\varepsilon^2 a_{31}^{(2)}$, $\sigma = \rho\varepsilon a_{22}^{(1)} + \rho\varepsilon^2 a_{22}^{(2)}$.

为了讨论(54)式的稳定性,采用 Bobylev 的作法[10], 记 $x/\bar{\eta}$ 和 $t/\bar{\eta}$ 为 x 和 t, 然后再用代换(40)及(42)式,得到关于 ρ_1, c_1, θ_1 的线性代数方程组,它有非零解的条件为

$$Q_{\kappa^2}(\lambda) = 18\lambda^3 + 69\lambda^2\kappa^2 + \lambda\kappa^2(30 + 97\kappa^2 - 14\kappa^4) + 15\kappa^4(3 + 4\kappa^2) = 0. \tag{55}$$

可以断言,当 κ^2 充分大时(55)式有正根. 这是因为

1) 对任意给定的 $\kappa^2 > 0$, 有 $Q_{\kappa^2}(0) > 0$, $Q_{\kappa^2}(+\infty) > 0$;

2) 对于充分大的 $\kappa^2 > 0$, 有 $Q_{\kappa^2}(\kappa^2) < 0$. 因此,对于充分大的 κ^2, 方程(55)有两个正根. 所以,(53)式的选择造成了久期项.

假如我们取 $p_2 = q_2 = s_2 = \alpha_{00}^{(1,1)} = \alpha_{11}^{(1,1)} = \alpha_{20}^{(1,1)} = 0$, 那么重复第四节的讨论,可以知道仍然存在久期项.

我们指出,下面的取法不会造成久期项:

$$p_2 = q_2 = s_2 = 0, \ \alpha_{00}^{(1,1)} = 0, \ \alpha_{00}^{(2,0)} = -c\frac{\partial a_{00}^{(2)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(2)}),$$

$$\alpha_{11}^{(1,1)} = -a_{00}^{(1)}q_1 - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(2)}),$$

$$\alpha_{11}^{(2,0)} = -\frac{\partial}{\partial x}(c a_{11}^{(2)}) - \theta\frac{\partial a_{00}^{(2)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(2)}),$$

$$\alpha_{20}^{(1,1)} = -\frac{2}{3}a_{11}^{(1)}q_1 - a_{00}^{(1)}s_1 - \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(2)}) - \frac{2}{3}\frac{\partial c}{\partial x}a_{22}^{(2)},$$

$$\alpha_{20}^{(2,0)} = -c\frac{\partial a_{20}^{(2)}}{\partial x} - \frac{2}{3}\theta\frac{\partial a_{11}^{(2)}}{\partial x} - \frac{2}{3}\frac{\partial c}{\partial x}a_{20}^{(2)} - a_{11}^{(2)}\frac{\partial\theta}{\partial x}. \tag{56}$$

事实上,现在关于 ρ, c, θ 的方程精确到 $O(\varepsilon^3)$ 仍然是(46)式，所以 $t\to\infty$ 时将达到平衡值 $\rho=1$，$c=0$，$\theta=1$．而关于 $a_{00}^{(1)}$，$a_{11}^{(1)}$ 和 $a_{20}^{(1)}$ 的精确到 $O(\varepsilon^2)$ 的方程为

$$\frac{\partial a_{00}^{(1)}}{\partial t} = -c\frac{\partial a_{00}^{(1)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(1)}),$$

$$\frac{\partial a_{11}^{(1)}}{\partial t} = -\frac{\partial}{\partial x}(c a_{11}^{(1)}) - \theta\frac{\partial a_{00}^{(1)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(1)}) - \varepsilon\left[a_{00}^{(1)}q_1 + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(2)})\right],$$

$$\frac{\partial a_{20}^{(1)}}{\partial t} = -c\frac{\partial a_{20}^{(1)}}{\partial x} - \frac{2}{3}\theta\frac{\partial a_{11}^{(1)}}{\partial x} - \frac{2}{3}\frac{\partial c}{\partial x}a_{20}^{(1)} - a_{11}^{(1)}\frac{\partial\theta}{\partial x}$$

$$-\varepsilon\left[\frac{2}{3}a_{11}^{(1)}q_1 + a_{00}^{(1)}s_1 + \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(2)}) + \frac{2}{3}\frac{\partial c}{\partial x}a_{22}^{(2)}\right]. \tag{57}$$

这组方程也有稳定解,这可以证明如下：经过充分长的时间之后，$\rho=1$，$c=0, \theta=1$，故

$$a_{nl}^{(1)} = 0 \qquad (n, l) = (2, 2),\ (3, 1),\ (3, 3), \cdots;\ q_1 = s_1 = 0. \tag{58}$$

这里用到(30)和(45)式．所以(49)和(50)式化为

$$a_{22}^{(2)} = \frac{1}{\lambda_{22}}\left[\lambda_{40}(a_{11}^{(1)})^2 - \frac{4}{3}\frac{\partial a_{11}^{(1)}}{\partial x}\right], \quad a_{31}^{(2)} = \frac{1}{\lambda_{31}}\left[3\lambda_{40}a_{11}^{(1)}a_{20}^{(1)} - 3\frac{\partial a_{20}^{(1)}}{\partial x}\right]. \tag{59}$$

这时(57)式改写成

$$\frac{\partial a_{00}^{(1)}}{\partial t} = -\frac{\partial a_{11}^{(1)}}{\partial x}, \quad \frac{\partial a_{11}^{(1)}}{\partial t} = -\frac{\partial a_{00}^{(1)}}{\partial x} - \frac{\partial a_{20}^{(1)}}{\partial x} + \frac{\varepsilon}{\lambda_{22}}\frac{\partial}{\partial x}\left[\frac{4}{3}\frac{\partial a_{11}^{(1)}}{\partial x} - \lambda_{40}(a_{11}^{(1)})^2\right],$$

$$\frac{\partial a_{20}^{(1)}}{\partial t} = -\frac{2}{3}\frac{\partial a_{11}^{(1)}}{\partial x} + \frac{\varepsilon}{\lambda_{31}}\frac{\partial}{\partial x}\left[\frac{5}{3}\frac{\partial a_{20}^{(1)}}{\partial x} - \frac{5}{3}\lambda_{40}a_{11}^{(1)}a_{20}^{(1)}\right]. \tag{60}$$

它有定态解 $a_{00}^{(1)} = a_{11}^{(1)} = a_{20}^{(1)} = 0$．用讨论(46)式的方法可以知道，这个解也是稳定的．

六、更高级近似

我们猜测,在(12)式的以后各级方程中选择

$$p_j = q_j = s_j = 0 \quad \alpha_{nl}^{(j-k,k)} = 0, \qquad j \geqslant 3, \quad k = 2, 3, \cdots, j, \tag{61}$$

以及

$$\alpha_{00}^{(j-1,1)} = 0, \quad \alpha_{00}^{(j,0)} = -c\frac{\partial a_{00}^{(j)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(j)}),$$

$$\alpha_{11}^{(j-1,1)} = -a_{00}^{(j-1)}q_1 - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(j)}),$$

$$\alpha_{11}^{(j,0)} = -\frac{\partial}{\partial x}(c a_{11}^{(j)}) - \theta\frac{\partial a_{00}^{(j)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(j)}),$$

$$a_{20}^{(j-1,1)} = -\frac{2}{3}a_{11}^{(j-1)}q_1 - a_{00}^{(j-1)}s_1 - \frac{2}{3}\frac{\partial c}{\partial x}a_{22}^{(j)} - \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(j)}),$$

$$a_{20}^{(j,0)} = -c\frac{\partial a_{20}^{(j)}}{\partial x} - \frac{2}{3}\theta\frac{\partial a_{11}^{(j)}}{\partial x} - \frac{2}{3}\frac{\partial c}{\partial x}a_{20}^{(j)} - \frac{\partial\theta}{\partial x}a_{11}^{(j)} \qquad (j \geqslant 3), \tag{62}$$

就可以保证各级方程的解都是稳定的. 我们可以用数学归纳法证明这个猜测是正确的. 归纳法证明的第一步在前几节已经完成了.

假定在 ε^j 级方程的讨论中已经确定(61), (62)式的选择可以得到稳定解,就是说,在充分长的时间之后,有

$$\begin{aligned}
&\rho = 1,\ \theta = 1,\ c = 0,\ a_{nl}^{(j-2)} = a_{nl}^{(j-3)} = \cdots = a_{nl}^{(1)} = 0 \\
&(n, l) = (0, 0),\ (1, 1), \cdots, \\
&a_{nl}^{(j-1)} = 0 \qquad (n, l) = (2, 2),\ (3, 1), \cdots, \\
&a_{22}^{(j)} = -\frac{4}{3\lambda_{22}}\frac{\partial a_{11}^{(j-1)}}{\partial x}, \quad a_{31}^{(j)} = -\frac{3}{\lambda_{31}}\frac{\partial a_{20}^{(j-1)}}{\partial x}, \\
&a_{nl}^{(j)} = 0 \qquad (n, l) = (3, 3), (4, 0), \cdots,
\end{aligned} \tag{63}$$

现在,(12)式的 ε^{j+1} 级方程为(在选择(61)式之后)

$$I_M(\xi^{(j+1)}) = \frac{1}{\rho}[D_1(\xi^{(j)}) + D_2(\xi^{(j)}) + D_{00}\xi^{(j)} + D_{01}\xi^{(j-1)}] - \sum_{i=1}^{j}\left[J_M(\xi^{(i)},\ \xi^{(j+1-i)}) - \frac{1}{\rho}\left(\frac{\partial\xi^{(i)}}{\partial t}\right)_{j-i}\right]. \tag{64}$$

显然,(61)和(62)式的选择与(64)式不相冲突,就是说, (64)式两边 e_{00}, e_{11}, e_{20} 的系数相等. 我们得到关于 $a_{00}^{(j-1)}$, $a_{11}^{(j-1)}$ 和 $a_{20}^{(j-1)}$ 的精确方程为

$$\frac{\partial a_{00}^{(j-1)}}{\partial t} = -c\frac{\partial a_{00}^{(j-1)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{11}^{(j-1)}),$$

$$\frac{\partial a_{11}^{(j-1)}}{\partial t} = -\frac{\partial}{\partial x}(c a_{11}^{(j-1)}) - \theta\frac{\partial a_{00}^{(j-1)}}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{20}^{(j-1)}) - \varepsilon\left[a_{00}^{(j-1)}q_1 + \frac{1}{\rho}\frac{\partial}{\partial x}(\rho a_{22}^{(j)})\right],$$

$$\frac{\partial a_{20}^{(j-1)}}{\partial t} = -c\frac{\partial a_{20}^{(j-1)}}{\partial x} - \frac{2}{3}\theta\frac{\partial a_{11}^{(j-1)}}{\partial x} - \frac{2}{3}\frac{\partial c}{\partial x}a_{20}^{(j-1)} - \frac{\partial\theta}{\partial x}a_{11}^{(j-1)} - \varepsilon\left[\frac{2}{3}a_{11}^{(j-1)}q_1 + a_{00}^{(j-1)}s_1 + \frac{2}{3}\frac{\partial c}{\partial x}a_{22}^{(j)} + \frac{5}{9\rho}\frac{\partial}{\partial x}(\rho a_{31}^{(j)})\right]. \tag{65}$$

按照(63)式,可将(65)式改为

$$\frac{\partial a_{00}^{(j-1)}}{\partial t} = -\frac{\partial a_{10}^{(j-1)}}{\partial x}, \quad \frac{\partial a_{11}^{(j-1)}}{\partial t} = -\frac{\partial a_{00}^{(j-1)}}{\partial x} - \frac{\partial a_{20}^{(j-1)}}{\partial x} + \frac{4\varepsilon}{3\lambda_{22}}\frac{\partial^2 a_{11}^{(j-1)}}{\partial x^2},$$

$$\frac{\partial a_{20}^{(j-1)}}{\partial t} = -\frac{2}{3}\frac{\partial a_{11}^{(j-1)}}{\partial x} + \frac{5\varepsilon}{3\lambda_{31}}\frac{\partial^2 a_{20}^{(j-1)}}{\partial x^2}. \tag{66}$$

这个方程组的解是稳定的,在充分长的时间之后有

$$a_{00}^{(j-1)} = a_{11}^{(j-1)} = a_{20}^{(j-1)} = 0, \tag{67}$$

代回(63)式之中,可知

$$a_{22}^{(j)} = a_{31}^{(j)} = 0. \tag{68}$$

最后,(64)式成为

$$l_M(\xi^{(j+1)}) = \frac{4}{3}\frac{\partial a_{11}^{(j)}}{\partial x} e_{22} + 3\frac{\partial a_{20}^{(j)}}{\partial x} e_{31}. \tag{69}$$

于是

$$a_{22}^{(j+1)} = -\frac{4}{3\lambda_{22}}\frac{\partial a_{11}^{(j)}}{\partial x}, \quad a_{31}^{(j+1)} = \frac{-3}{\lambda_{31}}\frac{\partial a_{20}^{(j)}}{\partial x},$$
$$a_{nl}^{(j+1)} = 0 \quad (n, l) = (3, 3), (4, 0), \cdots. \tag{70}$$

至此,我们完成了归纳法证明.

要提起注意的是,(46),(57)及(65)式都是精确的方程式.尽管(46)式的线性化与(57)式的线性化相比较,后者是非齐次方程组而前者正是相应的齐次方程组,但也不会出现像(44)式那样的久期项.事实上,对于我们所讨论的小振动而言,(46)式的解 ρ', c', $\theta' \sim e^{\lambda t + i\kappa x}$,其中 λ 的实部为负,而且是 $O(\varepsilon)$ 数量级的,所以(57)式的解是 $a_{00}^{(1)}$, $a_{11}^{(1)}$, $a_{20}^{(1)} \sim \varepsilon t e^{\lambda t + i\kappa x}$,它不会超过 $O(1)$ 的数量级.以此类推,

$$a_{00}^{(j)},\ a_{11}^{(j)},\ a_{20}^{(j)} \sim \frac{(\varepsilon t)^j}{j!} e^{\lambda t + i\kappa x},$$

它也不会超过 $O(1)$ 的数量级,所以不出现久期项.

七、结 论

Grad 曾经正确地指出,Hilbert 和 Enskog 展开都是渐近展开,它们只在有限的时间范围之内有效[11],也就是说展开式中都存在久期项.我们现在得到的正规解却没有久期项,当初条件满足正规解的要求时,除边界层和激波层之外,这种奇异扰动展开是对所有 t 一致有效的.有些作者从宏观信息的暗示出发,假定[12,2]

$$\frac{\partial f}{\partial t} = \frac{\partial_0 f}{\partial t} + \varepsilon\frac{\partial_1 f}{\partial t} \tag{71}$$

使结果与宏观现象符合.而我们进行的讨论完全从 Boltzmann 方程本身出发,没有引进任何先验的前提.

正规解所要求的初条件是怎样的呢?显然,ρ, c, θ 可以任意给定初条件,而在

$$\xi(t=0) = \sum_{j=0}^{\infty}\sum_{n\,l} a_{nl}^{(j)}(x, 0) l_{nl} \tag{72}$$

之中,不仅已限定

$$a_{nl}^{(0)}(x, 0) = 0 \qquad (n, l) = (0, 0), (1, 1), \cdots, \tag{73}$$

而且所有 $a_{nl}^{(j)}(x, 0)[(n, l) = (2, 2), (3, 1), \cdots;\ j = 1, 2, \cdots]$ 也已经被完全限定.只有 $a_{nl}^{(j)}(x, 0)[(n, l) = (0, 0), (1, 1), (2, 0);\ j = 1, 2, \cdots]$ 可以任意给出;在讨论初始层解时将看到,这些初条件将在另一种意义上限定.但如果只限于讨论正规解,不妨设这些初值为零.然而,这并不会回到 Enskog 展开,因为当 $t > 0$ 时,这些 $a_{nl}^{(j)}(x, t)$ 不一定是零.

由(5),(11),(17),(22),(23)式可以得到宏观物理量的表达式

密度　$n_0 = \rho(1 + a_{00})$, (74)

流体速度　$u_i = \left(c + \frac{a_{11}}{1 + a_{00}}\right)\delta_{1i}$, (75)

胁强张量　$P_{ij} = \rho\left\{\left[\frac{3}{2}a_{22} - \frac{a_{11}^2}{1 + a_{00}}\right]\delta_{1i}\delta_{1j} + \left[(1 + a_{00})\theta + a_{20} - \frac{a_{22}}{2}\right]\delta_{ij}\right\}$, (76)

热流矢量　$\bar{q}_i = \rho\left[\frac{5}{6}a_{31} + \frac{a_{11}^3}{(1 + a_{00})^2} - \frac{5}{2}\frac{a_{11}a_{20}}{1 + a_{00}} - \frac{a_{11}a_{22}}{1 + a_{00}}\right]\delta_{1i}$, (77)

其中

$$a_{nl} = \sum_{j=0}^{\infty} \varepsilon^j a_{nl}^{(j)}.$$

从这些式子看出，a_{nl} 分别的物理意义并不明显．

本文所采用的奇异扰动方法不是双时标度法[5,6]．我们只是把一种复杂的奇异扰动问题(求 Boltzmann 方程的正规解)归结为另一种较简单的(其实仍然是相当复杂的)奇异扰动问题（求解 Navier-Stokes 方程及其他类似的方程)．曾有些作者用双时标度方法讨论过，并得到一些结果[13]，但只是就极特殊的前提条件才能得到．当然，双时标度方法是一种极其有用的奇异扰动方法．在讨论初始层解时将看到这种方法是十分有效的．

本文主要结果已在文献[14]中报道．

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参　考　文　献

[1] S. Chapman and T. G. Cowling, The Mathematical Theory of Non-Uniform Gases, Cambridge University Press, 3-rd ed. (1970).
[2] C. Cercignani, Theory and Application of the Boltzmann Equation, Scottish Academic Press, Edinburgh, (1975).
[3] C. Cercignani, Mathematical Methods in Kinetic Theory, Plenum Press, New York, (1969).
[4] D. Hilbert, *Math. Ann.*, **72** (1912), 562.
[5] A. H. Nayfeh, Perturbation Method, Wiley, New York, (1973).
[6] 钱伟长主编，奇异摄动理论及其在力学中的应用，科学出版社，第一版，(1981).
[7] А. В. Бобылев, *ДАН*, **225**(1975), 1041.
[8] E. H. Hauge and E. Praestagaard, *I. Stat. Phys.*, **24** (1981), **21**.
[9] C. S. Wang Chang and G. E. Uhlenbeck, On the Propagation of Sound in Monoatomic Gases, University of Michigan Press, Project M999. Ann Arbor, Michigan, (1952).
[10] А. В. Бобылев, *ДАН*, **262** (1982), 71.
[11] H. Grad, *Phys. Fluids*, **6** (1963), 147.
[12] L. Trilling, *Phys. Fluids*, **7** (1964), 1681;
H. Grad, in "Transport Theory," Bellman *et al.*, eds. S1AM-AMS Proceedings, Vol. I, AMS (Providence 1968). p. 269.
[13] T. F. Morse, *Phys. Fluids*, **7** (1964), 1691.
[14] 丁鄂江、黄祖洽，物理学报，**33**(1984)，722.

ON THE SINGULAR PERTURBATION SOLUTION OF BOLTZMANN EQUATION (I)

NORMAL SOLUTION

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ABSTRACT

In this paper, we use a kind of singular perturbation method to find the normal solution of the Boltzmann equation with small Knudsen number. It is proved that the secular terms may be removed by improving the Hilbert expansion and Enskog expansion.