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聚焦-偏转复合系统的相对论五级几何象差方程

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提　要

本文以复数内积及形式导数为工具，并利用变分原理，导出了电-磁的聚焦-偏转重迭场复合系统的相对论五级几何象差方程.

一、引　言

文献[1]给出了聚焦系统的非相对论五级几何象差．近来又有一些文献[2—4]对五级几何象差表现兴趣．本文将复数内积[5,6]及形式导数[7,8]相结合，作为数学工具，来研究电-磁的聚焦-偏转重迭场复合系统，并利用变分原理，导出了这系统的、以电子轨迹初条件表达的相对论五级几何象差方程．在此基础上，我们将另文导出这系统的相对论五级几何象差及象差系数．

二、变分函数

按照文献[5]，复数 $u=u_r+iu_i$ 及 $v=v_r+iv_i$ 的内积定义为

$$\langle u, v\rangle = u_r v_r + u_i v_i. \tag{1}$$

于是在圆柱坐标系 (ρ, θ, z) 中，静电势 φ 及磁标势 ϕ 可表示为[5,6]

$$\varphi(\rho,\theta,z)=\sum_{m=0}^{\infty}\sum_{\mu=0}^{\infty}(-1)^{\mu}\frac{m!}{4^{\mu}\mu!(m+\mu)!}\rho^{m+2\mu}\langle\Phi_m^{(2\mu)}(z), e^{im\theta}\rangle,$$

$$\sqrt{\frac{e}{2m_0}}\,\phi(\rho,\theta,z)=\sum_{m=0}^{\infty}\sum_{\mu=0}^{\infty}(-1)^{\mu}\frac{m!}{4^{\mu}\mu!(m+\mu)!}\rho^{m+2\mu}\langle\Psi_m^{(2\mu)}(z), e^{im\theta}\rangle, \tag{2}$$

其中 e, m_0 分别为电子的电荷绝对值及静止质量．以下令 c_0 为真空中的光速，并令

$$\Phi=\Phi_0, \quad B_0=-\Psi_0', \quad {}'=\frac{d}{dz},$$

$$V=\Phi(1+\varepsilon\Phi), \quad \sigma=1+2\varepsilon\Phi, \quad \varepsilon=\frac{e}{2m_0c_0^2}. \tag{3}$$

对比本文(1)式及文献[5,6]中的(1)式，可知本文中的 B_0 及 $\Psi_m(m=0,1,2,\cdots)$ 分别等于文献[5, 6]中的 $\sqrt{\frac{e}{2m_0}}B_0$ 及 $\sqrt{\frac{e}{2m_0}}\Psi_m(m=0,1,2,\cdots)$.

在固定圆柱坐标系 (ρ,θ,z) 中，电子位置以 (μ,z) 表示，其中

$$u=\rho e^{i\theta}. \tag{4}$$

在旋转圆柱坐标系中，电子位置以 (v,z) 表示，其中

$$v=ue^{-i\Theta},\quad \Theta=\frac{1}{2}\int_{z_0}^{z}\frac{B_0}{V^{1/2}}dz. \tag{5}$$

今把 $u,u',\Phi_1^{(\mu)},\Psi_1^{(\mu)}(\mu=0,1,2,3)$, $\Phi_3^{(\nu)},\Psi_3^{(\nu)}(\nu=0,1,2)$, Φ_5 及 Ψ_5 都看作一级小量，则聚焦-偏转复合系统的变分函数 F 可展开如下:

$$F=F_0+F_2+F_4+F_6+\cdots. \tag{6}$$

这里

$$F_0=V^{1/2}, \tag{7}$$

$$F_2=L^*\langle v',v'\rangle-M\langle v,v\rangle+\langle\alpha_1,v\rangle, \tag{8}$$

$$F_4=F_{4C}+F_{4D}, \tag{9}$$

$$\begin{aligned}-F_{4C}=&\frac{L}{4}\langle v,v\rangle^2+\frac{M}{2}\langle v,v\rangle\langle v',v'\rangle+\frac{L^*}{4}\langle v',v'\rangle^2\\&+V^{1/2}[N\langle v,v\rangle+N^*\langle v',v'\rangle]\langle iv,v'\rangle+VK\langle iv,v'\rangle^2,\end{aligned} \tag{10}$$

$$\begin{aligned}-F_{4D}=&-\langle\alpha_3,v^3\rangle+\langle\lambda,v\rangle\langle v',v'\rangle+\langle\mu,v\rangle\langle v,v\rangle\\&+\langle\nu,v'\rangle\langle v,v\rangle+2\langle\nu,v\rangle\langle v,v'\rangle\\&+2V^{1/2}\langle\kappa,v\rangle\langle iv,v'\rangle+\langle\tau,v\rangle^2,\end{aligned} \tag{11}$$

$$F_6=F_{6C}+F_{6D}, \tag{12}$$

$$\begin{aligned}-F_{6C}=&\frac{P}{6}\langle v,v\rangle^3+\frac{P^*}{6}\langle v',v'\rangle^3+\frac{Q}{2}\langle v,v\rangle^2\langle v',v'\rangle+\frac{Q^*}{2}\langle v',v'\rangle^2\langle v,v\rangle\\&+V^{1/2}[R\langle v,v\rangle^2+S\langle v,v\rangle\langle v',v'\rangle+R^*\langle v',v'\rangle^2]\langle iv,v'\rangle\\&-2V[T\langle v,v\rangle+T^*\langle v',v'\rangle]\langle iv,v'\rangle^2-4V^{3/2}U\langle iv,v'\rangle^3,\end{aligned} \tag{13}$$

$$\begin{aligned}-F_{6D}=&-\langle\alpha_5,v^5\rangle+\langle\eta_1,v\rangle\langle v,v\rangle^2+\langle\eta_1^*,v\rangle\langle v',v'\rangle^2\\&+2\langle\eta_2,v\rangle\langle v,v\rangle\langle v,v'\rangle+\langle\eta_3,v\rangle\langle v,v\rangle\langle v',v'\rangle\\&+2V^{1/2}[\langle\eta_4,v\rangle\langle v,v\rangle+\langle\eta_4^*,v\rangle\langle v',v'\rangle]\langle iv,v'\rangle\\&+4V\langle\eta_5,v\rangle\langle iv,v'\rangle^2+\langle\eta_6,v^3\rangle\langle v,v\rangle+\langle\eta_6^*,v^3\rangle\langle v',v'\rangle\\&+2V^{1/2}\langle\eta_7,v^3\rangle\langle iv,v'\rangle+2\langle\eta_8,v^3\rangle\langle\eta,v\rangle\\&+2[\langle\eta_9,v\rangle\langle v,v\rangle+\langle\eta_9^*,v\rangle\langle v',v'\rangle]\langle\eta,v\rangle\\&+4c_1V^{1/2}\langle\eta,v\rangle^2\langle iv,v'\rangle-4c_2\langle\eta,v\rangle^3,\end{aligned} \tag{14}$$

其中

$$\alpha_m=\left(\frac{\sigma\Phi_m}{2V^{1/2}}+i\Psi_m\right)e^{-im\Theta}\qquad m=1,3,5, \tag{15}$$

$$L=\frac{1}{32}\left[\frac{\Phi''^2+2\sigma\Phi''B_0^2+B_0^4}{V^{3/2}}-\frac{\sigma\Phi^{(4)}+4B_0B_0''}{V^{1/2}}\right],$$

$$M=\frac{\sigma\Phi''+B_0^2}{8V^{1/2}},\qquad L^*=\frac{V^{1/2}}{2},$$

$$N=\frac{1}{16}\left[\frac{(\sigma\Phi''+B_0^2)B_0}{V^{3/2}}-\frac{B_0''}{V^{1/2}}\right],$$

$$N^*=\frac{B_0}{4V^{1/2}},\qquad K=\frac{B_0^2}{8V^{3/2}},\tag{16}$$

$$\kappa=-\left(\frac{\sigma B_0\Phi_1}{8V^{3/2}}+\frac{\Psi_1'}{6V^{1/2}}\right)e^{-i\theta},\quad \lambda=-\frac{\sigma\Phi_1}{4V^{1/2}}e^{-i\theta},$$

$$\mu=-\left[\frac{(\Phi''+\sigma B_0^2)\Phi_1}{16V^{3/2}}-\frac{1}{32}\left(\frac{\Phi'}{V^{3/2}}+\frac{i\sigma B_0}{V}\right)\Phi_1'+\frac{3B_0\Psi_1'}{16V^{1/2}}\right]e^{-i\theta},$$

$$\nu=-\left(\frac{\sigma\Phi_1'}{16V^{1/2}}+\frac{i\Psi_1'}{24}\right)e^{-i\theta},\qquad \tau=\frac{\Phi_1}{(4V)^{3/4}}e^{-i\theta},\tag{17}$$

$$P=\frac{1}{512}\left[\frac{3(\sigma\Phi''^3+\Phi''^2B_0^2-\sigma\Phi''B_0^4-B_0^6)}{V^{5/2}}\right.$$
$$\left.-\frac{3(\Phi''\Phi^{(4)}+\sigma\Phi^{(4)}B_0^2)}{V^{3/2}}+\frac{(2/3)\sigma\Phi^{(6)}+4B_0B_0^{(4)}}{V^{1/2}}\right],$$

$$Q=\frac{1}{128}\left[\frac{\Phi''^2-2\sigma\Phi''B_0^2-3B_0^4}{V^{3/2}}-\frac{\sigma\Phi^{(4)}}{V^{1/2}}\right],$$

$$P^*=-\frac{3V^{1/2}}{8},\qquad Q^*=-\frac{\sigma\Phi''+3B_0^2}{32V^{1/2}},$$

$$R=\frac{1}{256}\left[\frac{\Phi''^2B_0-2\sigma\Phi''B_0^3-3B_0^5}{V^{5/2}}-\frac{\sigma\Phi^{(4)}B_0}{V^{3/2}}+\frac{2B_0^{(4)}}{3V^{1/2}}\right],$$

$$S=-\frac{\sigma\Phi''B_0+3B_0^3}{32V^{3/2}},\qquad R^*=-\frac{3B_0}{16V^{1/2}},$$

$$T=\frac{\sigma\Phi''B_0^2+3B_0^4}{128V^{5/2}},\quad T^*=\frac{3B_0^2}{32V^{3/2}},\quad U=\frac{B_0^3}{64V^{5/2}},\tag{18}$$

$$\eta_1=\frac{1}{256}\left[\left(\frac{-3\sigma\Phi''^2-2\Phi''B_0^2+\sigma B_0^4}{V^{5/2}}+\frac{\Phi^{(4)}}{V^{3/2}}\right)\Phi_1+\frac{2(\Phi''+\sigma B_0^2)\Phi_1''}{V^{3/2}}\right.$$
$$\left.-\frac{1}{3}\left(\frac{\Phi'}{V^{3/2}}+\frac{i\sigma B_0}{V}\right)\Phi_1'''+\frac{10B_0\Psi_1'''}{3V^{1/2}}\right]e^{-i\theta},$$

$$\eta_1^*=\frac{\sigma\Phi_1}{16V^{1/2}}e^{-i\theta},\qquad \eta_2=\frac{1}{384}\left(\frac{5\sigma\Phi_1'''}{2V^{1/2}}+i\Psi_1'''\right)e^{-i\theta},$$

$$\eta_3=\frac{1}{32}\left[\frac{(-\Phi''+\sigma B_0^2)\Phi_1}{V^{3/2}}+\frac{\sigma\Phi_1''}{V^{1/2}}\right]e^{-i\theta},$$

$$\eta_4=\frac{1}{64}\left[\frac{(-\Phi''+\sigma B_0^2)B_0\Phi_1}{V^{5/2}}+\frac{\sigma B_0\Phi_1''}{V^{3/2}}-\frac{i\sigma\Phi_1'''}{12V}+\frac{5\Psi_1'''}{6V^{1/2}}\right]e^{-i\theta},$$

$$\eta_4^*=\frac{\sigma B_0\Phi_1}{16V^{3/2}}e^{-i\theta},\qquad \eta_5=\frac{\sigma B_0^2\Phi_1}{64V^{5/2}}e^{-i\theta},$$

$$\eta_6=\frac{1}{16}\left[-\frac{(\Phi''+\sigma B_0^2)\Phi_3}{V^{3/2}}+\frac{\sigma\Phi_3''}{2V^{1/2}}-\frac{8B_0\Psi_3'}{5V^{1/2}}+\frac{3i\Psi_3''}{5}\right]e^{-i3\Theta},$$

$$\eta_6^*=-\frac{\sigma\Phi_3}{4V^{1/2}}e^{-i3\Theta},\qquad \eta_7=-\left(\frac{\sigma B_0\Phi_3}{8V^{3/2}}+\frac{\Psi_3'}{10V^{1/2}}\right)e^{-i3\Theta},$$

$$\eta_8=\frac{\Phi_3}{8V^{3/2}}e^{-i3\Theta},\qquad \eta_9=\frac{1}{128}\left[\frac{(3\sigma\Phi''+B_0^2)\Phi_1}{V^{3/2}}-\frac{2\Phi_1''}{V^{3/2}}\right]e^{-i\Theta},$$

$$\eta_9^*=\frac{\Phi_1}{32V^{3/2}}e^{-i\Theta},\qquad \eta=\Phi_1e^{-i\Theta},$$

$$c_1=\frac{B_0}{64V^{5/2}},\qquad c_2=\frac{\sigma}{64V^{5/2}}. \tag{19}$$

三、高斯轨迹及象差方程

电子轨迹方程为

$$(\nabla_{v'}F)'-\nabla_vF=0. \tag{20}$$

故由(6)—(8)及(16)式得

$$(V^{1/2}v')'+2Mv-\alpha_1=\Delta_v(F_4+F_6+\cdots)-[\nabla_{v'}(F_4+F_6+\cdots)]'. \tag{21}$$

于是高斯轨迹方程为

$$(V^{1/2}v')'+2Mv=\alpha_1. \tag{22}$$

由上式的对应齐次线性方程得

$$V^{1/2}(st'-s't)=\text{const}=V_a^{1/2}, \tag{23}$$

其中 s, t, V_a 的定义见文献[5, 6]. 同时可知满足初条件

$$v(z_a)=v_a,\quad v'(z)=v_a' \tag{24}$$

的高斯轨迹及其斜率为[6]

$$v_g=\gamma s+\omega t,\qquad v_g'=\gamma s'+\omega t', \tag{25}$$

其中

$$\gamma=v_a+\delta,\qquad \omega=v_a'+\beta. \tag{26}$$

这里 δ 及 β 即为文献 [5, 6] 的 $\mathscr{D}$ 及 $\mathscr{B}$.

现在将 v, $\nabla_v(F_4+F_6+\cdots)$ 及 $\Delta_{v'}(F_4+F_6+\cdots)$ 在高斯值附近展开后代入(21)式,即得三级小量方程及五级小量方程

$$(V^{1/2}\Delta v_3')'+2M\Delta v_3=\nabla_vF_{4g}-(\nabla_{v'}F_{4g})', \tag{27}$$

$$(V^{1/2}\Delta v_5')'+2M\Delta v_5=\nabla_vF_{6g}-(\nabla_{v'}F_{6g})'+(\langle\Delta v_3,\nabla_v\rangle+\langle\Delta v_3',\nabla_{v'}\rangle)\nabla_vF_{4g}$$
$$-[(\langle\Delta v_3,\nabla_v\rangle+\langle\Delta v_3',\nabla_{v'}\rangle)\nabla_{v'}F_{4g}]', \tag{28}$$

其中 Δv_3 及 Δv_5 为三级象差及五级象差. 于是由常数变易法及附录，即可知道(27)式及(28)式的解分别为

$$\Delta v_3=\frac{1}{V^{1/2}}\left\{-s\int_{z_a}^{z}\nabla_\omega F_{4g}dz+t\int_{z_a}^{z}\nabla_\gamma F_{4g}dz+t(\nabla_\omega F_{4g})_{z_a}\right\}, \tag{29}$$

$$\Delta v_5=\frac{1}{V_a^{1/2}}\left\{-s\int_{z_a}^{z}\nabla_\omega F_{6g}dz+t\int_{z_a}^{z}\nabla_\gamma F_{6g}dz\right\}$$

$$+\frac{1}{V_a}\left\{-s\int_{z_a}^{z}\left[\left\langle\int_{z_a}^{z}\nabla_\gamma F_{4g}dz-\left(\frac{V}{V_a}\right)^{1/2}s(s\nabla_\omega-t\nabla_\gamma)F_{4g}+(\nabla_\omega F_{4g})_{z_a},\nabla_\omega\right\rangle\right.\right.$$

$$\left.-\left\langle\int_{z_a}^{z}\nabla_\omega F_{4g}dz-\left(\frac{V}{V_a}\right)^{1/2}t(s\nabla_\omega-t\nabla_\gamma)F_{4g},\nabla_\gamma\right\rangle\right]\nabla_\omega F_{4g}dz$$

$$+t\int_{z_a}^{z}\left[\left\langle\int_{z_a}^{z}\nabla_\gamma F_{4g}dz-\left(\frac{V}{V_a}\right)^{1/2}s(s\nabla_\omega-t\nabla_\gamma)F_{4g}+(\nabla_\omega F_{4g})_{z_a},\nabla_\omega\right\rangle\right.$$

$$\left.-\left\langle\int_{z_a}^{z}\nabla_\omega F_{4g}dz-\left(\frac{V}{V_a}\right)^{1/2}t(s\nabla_\omega-t\nabla_\gamma)F_{4g},\nabla_\gamma\right\rangle\right]\nabla_\gamma F_{4g}dz$$

$$\left.+t\left[\nabla_\omega F_{6g}+\left(\left\langle\int_{z_a}^{z}\nabla_\gamma F_{4g}dz,\nabla_\omega\right\rangle-\left\langle\int_{z_a}^{z}\nabla_\omega F_{4g}dz,\nabla_\gamma\right\rangle\right)\nabla_\omega F_{4g}\right]_{z_a}\right\}. \tag{30}$$

因为在象平面 $z=z_b$ 处有 $t(z_b)=0$，并且横向放大率 $m=s(z_b)$，故 $z=z_b$ 处的相对论五级几何象差为

$$\Delta v_{5b}=\frac{m}{V_a^{1/2}}\int_{z_a}^{z_b}\nabla_\omega(-F_{6g})\,dz$$

$$-\frac{m}{V_a}\int_{z_a}^{z_b}\left[\left\langle\int_{z_a}^{z}\nabla_\gamma(-F_{4g})dz-\left(\frac{V}{V_a}\right)^{1/2}s(s\nabla_\omega-t\nabla_\gamma)(-F_{4g})\right.\right.$$

$$\left.+(\nabla_\omega(-F_{4g}))_{z_a},\nabla_\omega\right\rangle-\left\langle\int_{z_a}^{z}\nabla_\omega(-F_{4g})dz\right.$$

$$\left.\left.-\left(\frac{V}{V_a}\right)^{1/2}t(s\nabla_\omega-t\nabla_\gamma)(-F_{4g}),\nabla_\gamma\right\rangle\right]\nabla_\omega(-F_{4g})dz. \tag{31}$$

由 (9)—(14), (23), (25) 式及附录可以知道上述 F_{4g} 及 F_{6g} 为

$$F_{4g}=F_{4Cg}+F_{4Dg}, \tag{32}$$

$$-F_{4Cg}=A\langle\gamma,\omega\rangle^2+B\langle\gamma,\omega\rangle\langle\gamma,\gamma\rangle+B^+\langle\gamma,\omega\rangle\langle\omega,\omega\rangle+\frac{C}{4}\langle\gamma,\gamma\rangle^2$$
$$+\frac{C^+}{4}\langle\omega,\omega\rangle^2+\frac{D}{2}\langle\gamma,\gamma\rangle\langle\omega,\omega\rangle+a\langle i\gamma,\omega\rangle\langle\gamma,\omega\rangle$$
$$+b\langle i\gamma,\omega\rangle\langle\gamma,\gamma\rangle+b^+\langle i\gamma,\omega\rangle\langle\omega,\omega\rangle, \tag{33}$$

$$-F_{4Dg}=\langle\gamma,\gamma\rangle[\langle a_1,\gamma\rangle+\langle a_2^+,\omega\rangle]+\langle\omega,\omega\rangle[\langle a_2,\gamma\rangle+\langle a_1^+,\omega\rangle]$$
$$+2\langle\gamma,\omega\rangle[\langle a_3,\gamma\rangle+\langle a_3^+,\omega\rangle]+2\langle i\gamma,\omega\rangle[\langle a_4,\gamma\rangle$$
$$+\langle a_4^+,\omega\rangle]-\langle\alpha_3,(\gamma s+\omega t)^3\rangle+\langle\tau,\gamma s+\omega t\rangle^2. \tag{34}$$

$$F_{6g}=F_{6Cg}+F_{6Dg}, \tag{35}$$

$$-F_{6Cg}=\frac{4E}{3}\langle\gamma,\omega\rangle^3+2F\langle\gamma,\omega\rangle^2\langle\gamma,\gamma\rangle+2F^+\langle\gamma,\omega\rangle^2\langle\omega,\omega\rangle$$
$$+G\langle\gamma,\omega\rangle\langle\gamma,\gamma\rangle\langle\omega,\omega\rangle+H\langle\gamma,\omega\rangle\langle\gamma,\gamma\rangle^2$$
$$+H^+\langle\gamma,\omega\rangle\langle\omega,\omega\rangle^2+\frac{I}{2}\langle\gamma,\gamma\rangle^2\langle\omega,\omega\rangle$$

$$+ \frac{I^+}{2}\langle\omega,\omega\rangle^2\langle\gamma,\gamma\rangle + \frac{J}{6}\langle\gamma,\gamma\rangle^3 + \frac{J^+}{6}\langle\omega,\omega\rangle^3$$
$$+ 4e\langle i\gamma,\omega\rangle\langle\gamma,\omega\rangle^2 + 2f\langle i\gamma,\omega\rangle\langle\gamma,\omega\rangle\langle\gamma,\gamma\rangle$$
$$+ 2f^+\langle i\gamma,\omega\rangle\langle\gamma,\omega\rangle\langle\omega,\omega\rangle + g\langle i\gamma,\omega\rangle\langle\gamma,\gamma\rangle\langle\omega,\omega\rangle$$
$$+ h\langle i\gamma,\omega\rangle\langle\gamma,\gamma\rangle^2 + h^+\langle i\gamma,\omega\rangle\langle\omega,\omega\rangle^2, \tag{36}$$

$$-F_{6Dg} = \langle\gamma,\gamma\rangle^2\langle b_1,\gamma s+\omega t\rangle + \langle\omega,\omega\rangle^2,\langle b_1^+,\gamma s+\omega t\rangle$$
$$+ \langle\gamma,\gamma\rangle\langle\omega,\omega\rangle\langle b_2,\gamma s+\omega t\rangle + 4\langle\gamma,\omega\rangle^2\langle b_3,\gamma s+\omega t\rangle$$
$$+ 2\langle\gamma,\omega\rangle[\langle\gamma,\gamma\rangle\langle b_4,\gamma s+\omega t\rangle + \langle\omega,\omega\rangle\langle b_4^+,\gamma s+\omega t\rangle]$$
$$+ 2\langle i\gamma,\omega\rangle[\langle\gamma,\gamma\rangle\langle b_5,\gamma s+\omega t\rangle + \langle\omega,\omega\rangle\langle b_5^+,\gamma s+\omega t\rangle]$$
$$+ 4\langle i\gamma,\omega\rangle\langle\gamma,\omega\rangle\langle b_6,\gamma s+\omega t\rangle - \langle\alpha_5,(\gamma s+\omega t)^5\rangle$$
$$+ \langle\gamma,\gamma\rangle\langle b_7,(\gamma s+\omega t)^3\rangle + \langle\omega,\omega\rangle\langle b_7^+,(\gamma s+\omega t)^3\rangle$$
$$+ 2\langle\gamma,\omega\rangle\langle b_8,(\gamma s+\omega t))^3\rangle + 2V_a^{1/2}\langle i\gamma,\omega\rangle\langle\eta_7,(\gamma s+\omega t)^3\rangle$$
$$+ 2\langle\eta_8,(\gamma s+\omega t)^3\rangle\langle\eta,\gamma s+\omega t\rangle$$
$$+ 2[\langle\gamma,\gamma\rangle\langle b_9,\gamma s+\omega t\rangle + \langle\omega,\omega\rangle\langle b_9^+,\gamma s+\omega t\rangle]\langle\eta,\gamma s+\omega t\rangle$$
$$+ 4\langle\gamma,\omega\rangle\langle b_{10},\gamma s+\omega t\rangle\langle\eta,\gamma s+\omega t\rangle$$
$$+ 4c_1V_a^{1/2}\langle i\gamma,\omega\rangle\langle\eta,\gamma s+\omega t\rangle^2 - 4c_2\langle\eta,\gamma s+\omega t\rangle^3, \tag{37}$$

其中

$$A = Ls^2t^2 + 2Msts't' + L^*s'^2t'^2 - V_aK,$$
$$B = Ls^3t + Mss'(st)' + L^*s'^3t', \quad B^+ = Lt^3s + Mtt'(ts)' + L^*t'^3s',$$
$$C = Ls^4 + 2Ms^2s'^2 + L^*s'^4, \quad C^+ = Lt^4 + 2Mt^2t'^2 + L^*t'^4,$$
$$D = Ls^2t^2 + M(s^2t'^2 + s'^2t^2) + L^*s'^2t'^2 + 2V_aK,$$
$$a = 2V_a^{1/2}(Nst + N^*s't'),$$
$$b = V_a^{1/2}(Ns^2 + N^*s'^2), \quad b^+ = V_a^{1/2}(Nt^2 + N^*t'^2); \tag{38}$$
$$a_1 = \lambda ss'^2 + \mu s^3 + \nu(s^3)', \quad a_1^+ = \lambda tt'^2 + \mu t^3 + \nu(t^3)',$$
$$a_2 = \lambda st'^2 + \mu st^2 + \nu(st^2)', \quad a_2^+ = \lambda ts'^2 + \mu ts^2 + \nu(ts^2)',$$
$$a_3 = \lambda ss't' + \mu s^2t + \nu(s^2t)', \quad a_3^+ = \lambda tt's' + \mu t^2s + \nu(t^2s)',$$
$$a_4 = V_a^{1/2}\kappa s, \qquad a_4^+ = V_a^{1/2}\kappa t, \tag{39}$$
$$E = Ps^3t^3 + P^*s'^3t'^3 + 3(Qs^2s't^2t' + Q^*s'^2st'^2t) + 3V_a(Tst + T^*s't'),$$
$$F = Ps^4t^2 + P^*s'^4t'^2 + Q(s^2s'^2t^2 + 2s^3s'tt')$$
$$+ Q^*(s'^2s^2t'^2 + 2s'^3st't) + V_a(Ts^2 + T^*s'^2),$$
$$F^+ = Pt^4s^2 + P^*t'^4s'^2 + Q(t^2t'^2s^2 + 2t^3t'ss')$$
$$+ Q^*(t'^2t^2s'^2 + 2t'^3ts's) + V_a(Tt^2 + T^*t'^2),$$
$$G = 2[Ps^3t^3 + P^*s'^3t'^3 + Q(ss'^2t^3 + s^2s't^2t' + s^3tt'^2)$$
$$+ Q^*(s's^2t'^3 + s'^2st'^2t' + s'^3t't^2) - 2V_a(Tst + T^*s't')],$$
$$H = Ps^5t + P^*s'^5t' + Q(s^4s't' + 2s^3s'^2t) + Q^*(s'^4st + 2s'^3s^2t'),$$
$$H^+ = Pt^5s + P^*t'^5s' + Q(t^4t's' + 2t^3t'^2s) + Q^*(t'^4ts + 2t'^3t^2s'),$$
$$I = Ps^4t^2 + P^*s'^4t'^2 + Q(s^4t'^2 + 2s^2s'^2t^2)$$
$$+ Q^*(s'^4t^2 + 2s'^2s^2t'^2) - 4V_a(Ts^2 + T^*s'^2),$$

$$
\begin{aligned}
I^{+} &= Pt^4s^2 + P^*t'^4s'^2 + Q(t^4s'^2 + 2t^2t'^2s^2) \\
&\quad + Q^*(t'^4s^2 + 2t'^2t^2s'^2) - 4V_a(Tt^2 + T^*t'^2), \\
J &= Ps^6 + P^*s'^6 + 3(Qs^4s'^2 + Q^*s'^4s^2), \\
J^{+} &= Pt^6 + P^*t'^6 + 3(Qt^4t'^2 + Q^*t'^4t^2), \\
e &= V_a^{1/2}(Rs^2t^2 + Sss'tt' + R^*s'^2t'^2) + V_a^{3/2}U, \\
f &= V_a^{1/2}[2Rs^3t + S(s^2s't' + s'^2st) + 2R^*s'^3t'], \\
f^{+} &= V_a^{1/2}[2Rt^3s + S(t^2t's' + t'^2ts) + 2R^*t'^3s'], \\
g &= V_a^{1/2}[2Rs^2t^2 + S(s^2t'^2 + s'^2t^2) + 2R^*s'^2t'^2] - 4V_a^{3/2}U, \\
h &= V_a^{1/2}[Rs^4 + Ss^2s'^2 + R^*s'^4], \\
h^{+} &= V_a^{1/2}[Rt^4 + St^2t'^2 + R^*t'^4];
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
b_1 &= \eta_1s^4 + \eta_1^*s'^4 + 2\eta_2s^3s' + \eta_3s^2s'^2, \\
b_1^{+} &= \eta_1t^4 + \eta_1^*t'^4 + 2\eta_2t^3t' + \eta_3t^2t'^2, \\
b_2 &= 2(\eta_1s^2t^2 + \eta_1^*s'^2t'^2) + 2\eta_2(s^2tt' + t^2ss') + \eta_3(s^2t'^2 + s'^2t^2) + 4V_a\eta_5, \\
b_3 &= \eta_1s^2t^2 + \eta_1^*s'^2t'^2 + \eta_2(s^2tt' + t^2ss') + \eta_3ss'tt' - V_a\eta_5, \\
b_4 &= 2(\eta_1s^3t + \eta_1^*s'^3t') + \eta_2(s^3t' + 3s^2s't) + \eta_3(s^2s't' + s'^2st), \\
b_4^{+} &= 2(\eta_1t^3s + \eta_1^*t'^3s') + \eta_2(t^3s' + 3t^2t's) + \eta_3(t^2t's' + t'^2ts), \\
b_5 &= V_a^{1/2}(\eta_4s^2 + \eta_4^*s'^2), \qquad b_5^{+} = V_a^{1/2}(\eta_4t^2 + \eta_4^*t'^2), \\
b_6 &= V_a^{1/2}(\eta_4st + \eta_4^*s't'), \qquad b_7 = \eta_6s^2 + \eta_6^*s'^2, \\
b_7^{+} &= \eta_6t^2 + \eta_6^*t'^2, \qquad b_8 = \eta_6st + \eta_6^*s't', \\
b_9 &= \eta_9s^2 + \eta_9^*s'^2, \qquad b_9^{+} = \eta_9t^2 + \eta_9^*t'^2, \\
b_{10} &= \eta_9st + \eta_9^*s't'.
\end{aligned} \tag{41}
$$

四、结 论

本文导出了聚焦场与偏转场重迭时复合系统的,以轨迹初条件表示的,相对论五级几何象差方程. 由这方程出发,即可进一步导出这系统的五级几何象差及其系数.

附 录

我们知道[7,8],如果 f 是 $2m$ 个实变量 x_k, y_k $(k=1,\cdots,m)$ 的实值或复值的可微函数, 则 f 作为 $2m$ 个独立复变量 $w_k = x_k + iy_k$, $\bar{w} = x_k - iy_k (k=1,\cdots,m)$ 的函数,可以运用形式导数

$$\frac{\partial f}{\partial w_k} = \frac{1}{2}\left(\frac{\partial f}{\partial x_k} - i\frac{\partial f}{\partial y_k}\right), \qquad \frac{\partial f}{\partial \bar{w}_k} = \frac{1}{2}\left(\frac{\partial f}{\partial x_k} + i\frac{\partial f}{\partial y_k}\right). \tag{A.1}$$

因而可证明下述等式成立:

$$
\begin{aligned}
\frac{\partial(f_1f_2)}{\partial w_k} &= f_1\frac{\partial f_2}{\partial w_k} + \frac{\partial f_1}{\partial w_k}f_2, \\
\frac{\partial(f_1f_2)}{\partial \bar{w}_k} &= f_1\frac{\partial f_2}{\partial \bar{w}_k} + \frac{\partial f_1}{\partial \bar{w}_k}f_2.
\end{aligned} \tag{A.2}
$$

此外,如果 x_k, $y_k(k=1,\cdots,m)$ 是 $2n$ 个实变量 $\xi_j,\eta_j(j=1,\cdots,n)$ 的可微函数,则当 w_k, $\bar{w}_k(k=1,\cdots,m)$ 作为 $2n$ 个独立复变量 $t_j = \xi_j + i\eta_j$, $\bar{t}_j = \xi_j - i\eta_j(j=1,\cdots,n)$ 的函数时,我们有

$$\frac{\partial f}{\partial t_j}=\sum_{k=1}^{n}\left(\frac{\partial w_k}{\partial t_j}\frac{\partial}{\partial w_k}+\frac{\partial \bar{w}_k}{\partial t_j}\frac{\partial}{\partial \bar{w}_k}\right)f,\qquad \frac{\partial f}{\partial \bar{t}_j}=\sum_{k=1}^{n}\left(\frac{\partial w_k}{\partial \bar{t}_j}\frac{\partial}{\partial w_k}+\frac{\partial \bar{w}_k}{\partial \bar{t}_j}\frac{\partial}{\partial \bar{w}_k}\right)f. \tag{A.3}$$

由(1) (A.1) 式及文献[5,6]定义的算符 ∇_v 和 $\langle u,\nabla_v\rangle$ 可知，对任意复变量 u, v, p, q 有

$$\langle iu, v\rangle\langle ip, q\rangle=\langle u, p\rangle\langle v, q\rangle-\langle u, q\rangle\langle v, p\rangle, \langle pu, v\rangle=\langle u, \bar{p}v\rangle, \langle u, v\rangle=\frac{1}{2}(u\bar{v}+\bar{u}v),$$

$$\nabla_v=2\frac{\partial}{\partial \bar{v}},\ \langle u, \nabla_v\rangle=u\frac{\partial}{\partial v}+\bar{v}\frac{\partial}{\partial \bar{u}}, \tag{A.4}$$

其中，外文字母上加"—"表示共轭复数．于是可得

$$\nabla_v(pq)=p\nabla_v q+q\nabla_v p,$$

$$\langle u, \nabla_v\rangle(pq)=p\langle u, \nabla_v\rangle q+q\langle u, \nabla_v\rangle p,$$

$$\langle u, \nabla_v\rangle\langle p, q\rangle=\left\langle\left(u\frac{\partial}{\partial v}+\bar{u}\frac{\partial}{\delta \bar{v}}\right)p, q\right\rangle+\left\langle p, \left(u\frac{\partial}{\partial v}+\bar{u}\frac{\partial}{\partial \bar{v}}\right)q\right\rangle,$$

$$\nabla_v\langle p, q\rangle=\frac{\partial}{\partial \bar{v}}(p\bar{q}+\bar{p}q). \tag{A.5}$$

由 (23) 及 (25) 式得

$$\gamma=\left(\frac{V}{V_a}\right)^{1/2}(t'v_g-tv_g'),\qquad \omega=\left(\frac{V}{V_a}\right)^{1/2}(-s'v_g+sv_g') \tag{A.6}$$

故由 (A.3),(A.4) 及 (25) 式知道，当 v 及 v' 取高斯值时，有

$$\nabla_v=\left(\frac{V}{V_a}\right)^{1/2}(-s'\nabla_\omega+t'\nabla_\gamma),\qquad \nabla_{v'}=\left(\frac{V}{V_a}\right)^{1/2}(s\nabla_\omega-t\nabla_\gamma), \tag{A.7}$$

$$\nabla_\gamma=s\nabla_v+s'\nabla_{v'},\qquad \nabla_\omega=t\nabla_v+t'\nabla_{v'}. \tag{A.8}$$

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ON THE RELATIVISTIC FIFTH ORDER GEOMETRICAL ABERRATION EQUATION OF A COMBINED FOCUSING-DEFLECTION SYSTEM

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ABSTRACT

In this paper, the relativistic fifth order geometrical aberration equation of a combined electric-magnetic focusing-deflection system with superimposed fields is derived by using variational principle and taking symbolic derivative and inner product of complex numbers as mathematical tools