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高频波激发低频磁场和离子声波的重整化强湍动理论

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提 要

本文给出了高温等离子体中高频波激发低频磁场和离子声波强湍动过程的重整化理论，以便改善通常的弱非线性处理方法．从 Vlasov-Maxwell 方程组出发，在 Fourier 表象中得到了包含“最发散”和“次发散”效应互相耦合的高频和低频传播子重整化方程组，从而获得了高、低频振荡粒子重整化分布函数和场的耦合关系．

在“最发散”重整化近似下，我们求解了高低频传播子方程组，得到了展开到 ν_e（高频湍动场能密度与等离子体热能密度之比）一次方的近似解和重整化介电函数等表达式．然后，在 Fourier 逆变换下导得了我们所要的时空表象中重整化强湍动方程组．

最后，作为一个说明重整化作用的例子，在一维稳态下求解了孤立子的形式．

在文献[1, 2]中，从微观动力学理论而不是传统的流体动力学框架出发，我们导得了纵、横高频波拍频激发离子声波和低频磁场波，并包含自洽的非线性 Landau 阻尼的强湍动方程组，同时讨论了它们的一些重要特性，从而在较为普遍情况下推广了 Thornhill 和 Haar[3] 以及 Rudakov 和 Tsytovich[4] 等人于1978年提出的 Langmuir 波强湍动理论．

但遗憾的是上述所有结果都是建立在二次高频场拍频为低频波的弱非线性基础上进行．为了改善这种结果，我们曾讨论过四次高频场的拍频修正[5]，表明了四次高频场可能削弱二次非线性场能在局部空间的凝聚作用．为了使强湍动理论包含更高次的非线性效应．本文将从 Vlasov-Maxwell 方程组出发尝试用重整化方法去导得高频湍动场和低频波相互作用相干性的动力学方程组，把 Horton 等人[6]研究 Vlasov-Poisson 方程组的重整化方案推广应用到我们研究的问题上来，即把高次微扰久期项中“最发散”和“次发散”图形收集在一起，得到一组包含湍动碰撞频率的非线性传播子方程．接着把重整化后的粒子分布函数和传播子方程分成高、低频耦合关系，并在“最发散”近似下求解了高、低频传播子方程组，得到了展开到 ν_e 一次方项的高、低频重整化传播子方程近似解和相应介电函数表达式．

对有关方程作 Fourier 逆变换后，便可得到时空表象中高频电场、低频自生磁场、离子声波相互耦合的重整化强湍动方程组．最后还求得了方程组的孤立子形式解．

一、Vlasov-Maxwell 方程组的重整化形式

如果以电场 $\boldsymbol{E}(k)=\boldsymbol{e}_k^\sigma E^\sigma(k)$ 取代文献[6]中 Poisson 场 $\boldsymbol{k}\phi_k$，就可得到我们情况下的重整化传播子 $G_\alpha(k,\boldsymbol{v})$ 所服从的方程

$$G_\alpha(k,\boldsymbol{v})=\{\Theta-\boldsymbol{k}\cdot\boldsymbol{v}+i\Gamma_\alpha(k,\boldsymbol{v})\}^{-1}. \tag{1}$$

这里

$$\Gamma_\alpha(k,\boldsymbol{v})=-i\left(\frac{e_\alpha}{m_\alpha}\right)^2\sum_{k_1}|E^\sigma(k_1)|^2\frac{\partial}{\partial\boldsymbol{v}}\cdot\boldsymbol{e}_{k_1}'^\sigma G_\alpha(k-k_1,\boldsymbol{v})\boldsymbol{e}_{k_1}^\sigma\cdot\frac{\partial}{\partial\boldsymbol{v}} \tag{2}$$

为湍动碰撞频率算子；$k\equiv\{\Theta,\boldsymbol{k}\}$ 为频率 Θ 和波矢 $\boldsymbol{k}$ 的四维矢量；m_α，e_α 分别为粒子质量和电荷，$\alpha=i$，e；$\phi_{\boldsymbol{k}}$ 为静电势；σ 为波型符号；重复角标表示求和.

$$\boldsymbol{e}_k'^\sigma\equiv\left(1-\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\Theta}\right)\boldsymbol{e}_k^\sigma+\frac{\boldsymbol{e}_k^\sigma\cdot\boldsymbol{v}}{\Theta}\boldsymbol{k}, \tag{3}$$

$\boldsymbol{e}_k^\sigma$ 为 σ 波单位偏振矢量. 显然 $\boldsymbol{e}_k'^\sigma\cdot\boldsymbol{v}=\boldsymbol{e}_k^\sigma\cdot\boldsymbol{v}$.

方程(1)只包含了最重要的"最发散"和"次发散"的久期图形求和部分. 有了重整化传播子，就可写出重整化粒子分布函数和场方程. 我们感兴趣具有长波长湍动特性的高频波拍频为低频波过程，这时高频波(包括纵波和横波)的频率都接近电子等离子体频率，于是从 Vlasov-Maxwell 方程组取到二次高频场拍频近似就可得到高、低频耦合的重整化关系式如下:

高频振荡粒子分布函数

$$\begin{aligned}f_\alpha(k,\boldsymbol{v})=&-i\frac{e_\alpha}{m_\alpha}G_\alpha(k,\boldsymbol{v})\Big\{E_h^\sigma(k)\boldsymbol{e}_k'\cdot\partial/\partial\boldsymbol{v}F_{0\alpha}\\&+\int\boldsymbol{e}_1'\cdot\partial/\partial\boldsymbol{v}\delta F_\alpha(q,\boldsymbol{v})E_h^{\sigma_1}(k_1)\ \delta^4(k-k_1-q)\frac{d^4k_1d^4q}{(2\pi)^4}\\&-i\frac{e_\alpha}{m_\alpha}\int\boldsymbol{e}_q'\cdot\frac{\partial}{\partial\boldsymbol{v}}G_\alpha(k_1,\boldsymbol{v})\boldsymbol{e}_1'\cdot\frac{\partial}{\partial\boldsymbol{v}}F_{0\alpha}(\boldsymbol{v})\ E_h^{\sigma_1}(k_1)\\&\times E_d^\beta(q)\delta^4(k-k_1-q)\ \frac{d^4k_1d^4q}{(2\pi)^4}\Big\},\end{aligned} \tag{4}$$

低频振荡粒子分布函数

$$\begin{aligned}\delta F_\alpha(q,\boldsymbol{v})=&-i\frac{e_\alpha}{m_\alpha}G_\alpha(q,\boldsymbol{v})\Big\{E_d^\beta(q)\boldsymbol{e}_q'\cdot\frac{\partial}{\partial\boldsymbol{v}}F_{0\alpha}-i\frac{e_\alpha}{m_\alpha}\\&\times\int_{(d)}\boldsymbol{e}_1'\cdot\frac{\partial}{\partial\boldsymbol{v}}G_\alpha(k_2,\boldsymbol{v})\boldsymbol{e}_2'\cdot\frac{\partial}{\partial\boldsymbol{v}}F_{0\alpha}E_h^{\sigma_1}(k_1)E_h^{\sigma_2}(k_2)\\&\times\delta^4(q-k_1-k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4}\Big\},\end{aligned} \tag{5}$$

高频传播子方程

$$\left\{\omega-\boldsymbol{k}\cdot\boldsymbol{v}-i\frac{\partial}{\partial\boldsymbol{v}}\cdot\mathbf{D}_\alpha(k,\boldsymbol{v})\cdot\frac{\partial}{\partial\boldsymbol{v}}\right\}G_\alpha(k,\boldsymbol{v})=1. \tag{6}$$

这里

$$\mathbf{D}_\alpha(k, \boldsymbol{v}) = i\left(\frac{e_\alpha}{m_\alpha}\right)^2 \int \boldsymbol{e}_1' \boldsymbol{e}_1' |E_{h}^{\sigma_1}(k_1)|^2 G_\alpha(q, \boldsymbol{v}) \delta^4(k - k_1 - q) \frac{d^4 k_1 d^4 q}{(2\pi)^4} \qquad (7)$$

与均匀稳态高频湍动谱有关．(7)式中已用了四维盒子体积趋向∞时 $\sum_k \to \int \cdots \frac{d^4 k}{(2\pi)^4}$ 关系式．

低频传播子方程

$$\left\{\Omega - \boldsymbol{q} \cdot \boldsymbol{v} - i \frac{\partial}{\partial \boldsymbol{v}} \cdot \mathbf{D}_\alpha(q, \boldsymbol{v}) \cdot \frac{\partial}{\partial \boldsymbol{v}}\right\} G_\alpha(q, \boldsymbol{v}) = 1, \qquad (8)$$

而

$$\mathbf{D}_\alpha(q, \boldsymbol{v}) = i\left(\frac{e_\alpha}{m_\alpha}\right)^2 \int_{(d)} \boldsymbol{e}_1' \boldsymbol{e}_1' |E_{h}^{\sigma_1}(k_1)|^2 G_\alpha(k_2, \boldsymbol{v}) \delta^4(q - k_1 - k_2) \frac{d^4 k_1 d^4 k_2}{(2\pi)^4} \qquad (9)$$

同样与均匀稳态高频湍动谱有关．

在方程组(5)—(9)中，$F_{0\alpha}$ 为背景分布函数，以后取为温度为 T_α 的 Maxwell 分布，$\int F_{0\alpha}(\boldsymbol{v}) d^3 v = n_0$，$n_0$ 为背景粒子数密度；$k \equiv \{\omega, \boldsymbol{k}\}$ 和 $q \equiv \{\Omega, \boldsymbol{q}\}$ 分别表示高低频)波的四维波矢，并记 $d^4 k \equiv d\omega d^3 k$，$\boldsymbol{e}_1' \equiv \boldsymbol{e}_{k_1}'^{\sigma}$，$\boldsymbol{e}_q' \equiv \boldsymbol{e}_q'^{\beta}$ 等等．方程组中，$E_h^\sigma(k)$ 和 $E_d^\beta(q)$ 分别表示高低频标量电场；积分号 $\int_{(d)}$ 表示积分只作用于高频波拍频成为低频波过程，即积分只取 $\{-k_1, +k_2\}$ 和 $\{+k_1, -k_2\}$两组情况．

现在通过重整化电流从电磁场 Maxwell 方程组就可得到高频场方程如下：

$$\varepsilon_h^\sigma(k) E_h^\sigma(k) = S_1^\sigma(k) + S_2^\sigma(k), \qquad (10)$$

其中

$$S_1^\sigma(k) = -\sum_\alpha \frac{\omega_\alpha^2}{n_0 \omega} \int \boldsymbol{e}_{\boldsymbol{k}} \cdot \boldsymbol{v} G_\alpha(k, \boldsymbol{v}) \boldsymbol{e}_1' \cdot \frac{\partial}{\partial \boldsymbol{v}} \delta F_\alpha(q, \boldsymbol{v}) E_h^{\sigma_1}(k_1)$$
$$\times \delta^4(k - k_1 - q) \frac{d^4 k_1 d^4 q}{(2\pi)^4} d^3 v, \qquad (11)$$

$$S_2^\sigma(k) = i \sum_\alpha \frac{\omega_\alpha^2}{n_0 \omega} \frac{e_\alpha}{m_\alpha} \int H_\alpha^{\sigma\beta\sigma_1}(k, q, k_1) E_h^{\sigma_1}(k_1) E_d^\beta(q)$$
$$\times \delta^4(k - k_1 - q) \frac{d^4 k_1 d^4 q}{(2\pi)^4}, \qquad (12)$$

这里

$$\omega_\alpha^2 = \frac{4\pi n_0 e_\alpha^2}{m_\alpha},$$

$$H_\alpha^{\sigma\beta\sigma_1}(k, q, k_1) = \int \boldsymbol{e}_{\boldsymbol{k}} \cdot \boldsymbol{v} G_\alpha(k, \boldsymbol{v}) \boldsymbol{e}_q' \cdot \frac{\partial}{\partial \boldsymbol{v}} G_\alpha(k_1, \boldsymbol{v}) \boldsymbol{e}_1'$$
$$\cdot \frac{\partial}{\partial \boldsymbol{v}} F_{0\alpha}(\boldsymbol{v}) d^3 v. \qquad (13)$$

低频电场方程为

$$\varepsilon_d^\beta(q) E_d^\beta(q) = i \sum_\alpha \frac{\omega_\alpha^2}{n_0 \Omega} \frac{e_\alpha}{m_\alpha} \int_{(d)} H_\alpha^{\beta\sigma_1\sigma_2}(q, k_1, k_2) E_h^{\sigma_1}(k_1) E_h^{\sigma_2}(k_2)$$
$$\times \delta^4(q - k_1 - k_2) \frac{d^4 k_1 d^4 k_2}{(2\pi)^4}, \qquad (14)$$

其中

$$H_{\alpha}^{\beta\sigma_1\sigma_2}(q, k_1, k_2) = \frac{1}{2}\int \boldsymbol{e}_{\boldsymbol{q}} \cdot \boldsymbol{v} G_{\alpha}(q, \boldsymbol{v}) \Big[\boldsymbol{e}_1' \cdot \frac{\partial}{\partial \boldsymbol{v}} G_{\alpha}(k_2, \boldsymbol{v}) \boldsymbol{e}_2' \cdot \frac{\partial}{\partial \boldsymbol{v}} F_{0\alpha} + 1 \longleftrightarrow 2 \Big] d^3v. \tag{15}$$

交换下标 1 与 2 互为对称.

在(10)和(14)式中,重整化高低频色散响应函数分别为

$$\varepsilon_h^{\sigma}(k) = 1 - \frac{c^2k^2}{\omega^2}\delta_{\sigma t} + \sum_{\alpha} \frac{\omega_{\alpha}^2}{n_0\omega}\int \boldsymbol{e_k} \cdot \boldsymbol{v} G_{\alpha}(k, \boldsymbol{v})\, \boldsymbol{e}_k' \cdot \frac{\partial}{\partial \boldsymbol{v}} F_{0\alpha}(\boldsymbol{v}) d^3v, \tag{16}$$

$$\varepsilon_d^{\beta}(q) = 1 - \frac{c^2q^2}{\Omega^2}\delta_{\beta t} + \sum_{\alpha} \frac{\omega_{\alpha}^2}{n_0\Omega}\int \boldsymbol{e_q} \cdot \boldsymbol{v} G_{\alpha}(q, \boldsymbol{v}) \boldsymbol{e}_q' \cdot \frac{\partial}{\partial \boldsymbol{v}} F_{0\alpha}(\boldsymbol{v}) d^3v. \tag{17}$$

这里 $\varepsilon_h^{\sigma}(k) + \frac{c^2k^2}{\omega^2}\delta_{\sigma t}$ 为介电函数, $\delta_{\sigma t} = \begin{cases} 1 & \sigma = t \text{(横波)}, \\ 0 & \sigma = l \text{(纵波)}. \end{cases}$

最后,写出低频磁场方程

$$\boldsymbol{B}(q) = \frac{c}{\Omega}\boldsymbol{q} \times \boldsymbol{E}_d^t(q), \tag{18}$$

c 为光速.

(4)—(18)式一起构成了 Fourier 表象中重整化强湍动方程组. 显然, 当 G_{α} 变成自由传播子 $G_{\alpha}^{(0)}$ 时就退化为文献[1]中结果.

二、传播子方程的近似解

与一般的重整化结果不同,这里高低频传播子是互相耦合的非线性方程组,无法严格求解. 为简化问题,下面只研究"最发散"项组成的重整化解,这时张量系数 $\mathbf{D}$ 中的 G_{α} 变成自由传播子,从而(6)和(8)式变成互相独立的线性方程,于是对均匀稳态的高频湍动谱作些近似假定可望获得方程组的近似解.

下面只讨论高频场是 Langmuir 波情况,但是被激发的低频波可以是纵场和横场,后者将导致低频自生磁场的产生. 对于高频横场可仿此讨论.

1. 低频传播子方程的近似解

方程(8)的 Green 函数方程为

$$\left\{\Omega - \boldsymbol{q} \cdot \boldsymbol{v} - i\frac{\partial}{\partial \boldsymbol{v}} \cdot \mathbf{D}_{\alpha}(q, \boldsymbol{v}) \cdot \frac{\partial}{\partial \boldsymbol{v}}\right\} G_{\alpha}(q, \boldsymbol{v}; \boldsymbol{v}') = \delta(\boldsymbol{v} - \boldsymbol{v}'). \tag{19}$$

利用(19)式的解可得算子作用关系式

$$G_{\alpha}(q, \boldsymbol{v})\Phi_{\alpha}(q, \boldsymbol{v}) = \int G_{\alpha}(q, \boldsymbol{v}; \boldsymbol{v}')\Phi_{\alpha}(q, \boldsymbol{v}')d^3v', \tag{20}$$

其中 $\Phi_{\alpha}(q, \boldsymbol{v})$ 表示(5)式右端大括号内量. 在"最发散"重整化条件下,张量扩散系数为

$$\mathbf{D}_{\alpha}(q, \boldsymbol{v}) = i\left(\frac{e_{\alpha}}{m_{\alpha}}\right)^2\int \boldsymbol{E}_+(k_1)\boldsymbol{E}_{\pm}^*(k_1)\left\{\frac{\delta^4(q + k_1 - k_2)}{\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v} + i0_+}\right.$$

$$
- \frac{\delta^4(q - k_1 + k_2)}{\omega_2 - \boldsymbol{k}_2 \cdot \boldsymbol{v} - i0_+} \Big\} \frac{d^4k_1 d^4k_2}{(2\pi)^4}
$$

$$
\simeq - i \left(\frac{e_\alpha}{m_\alpha}\right)^2 \frac{(\Omega - \boldsymbol{q} \cdot \boldsymbol{v})}{\omega_e^2} \int \boldsymbol{e}_1 \boldsymbol{e}_1 |E(k_1)|^2 \frac{d^4k_1}{(2\pi)^4}. \tag{21}
$$

这里 $\Omega = \omega_1 - \omega_2$，$\boldsymbol{q} = \boldsymbol{k}_1 - \boldsymbol{k}_2$，$\boldsymbol{e}'_k \to \boldsymbol{e}_k = \frac{\boldsymbol{k}}{|\boldsymbol{k}|}$；同时 $\boldsymbol{E}(k) = \boldsymbol{E}_+(k) + \boldsymbol{E}_-(k)$，$\boldsymbol{E}_\pm(k) = \boldsymbol{E}(\pm k)$，显然 $\boldsymbol{E}(k)\boldsymbol{E}^*(k) = 2\boldsymbol{E}_+(k)\boldsymbol{E}_+^*(k)$，Langmuir 波场的上、下角标已被省略；(21)式中已用了 $\omega_1 \simeq \omega_e \gg \Omega$，$\boldsymbol{k} \cdot \boldsymbol{v}$ 近似.

注意到长波长区弱湍动高频谱在与粒子作用时所表现出来的各向同性化特性[3,7]，假定由调制不稳定性发展的强湍动早期高频湍动场能谱接近各向同性化分布，这样，

$$
\mathbf{D}_\alpha(q, \boldsymbol{v}) \simeq - i\mathbf{I}(\Omega - \boldsymbol{q} \cdot \boldsymbol{v}) a_\alpha, \tag{22}
$$

其中

$$
a_\alpha = v_\alpha^2 \frac{\omega_\alpha^2}{\omega_e^2} \nu_\alpha, \quad \nu_\alpha = \int |E(k_1)|^2 \frac{d^4k_1}{(2\pi)^4} \Big/ 4\pi n_0 T_\alpha, \tag{23}
$$

$\mathbf{I}$ 为单位张量. 于是，方程(19)简化为

$$
\Big\{\Omega - \boldsymbol{q} \cdot \boldsymbol{v} + a_\alpha \boldsymbol{q} \cdot \frac{\partial}{\partial \boldsymbol{v}} - a_\alpha (\Omega - \boldsymbol{q} \cdot \boldsymbol{v}) \frac{\partial}{\partial \boldsymbol{v}} \cdot \frac{\partial}{\partial \boldsymbol{v}}\Big\} G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') = \delta(\boldsymbol{v} - \boldsymbol{v}'), \tag{24}
$$

(24)式是自伴的，所以有对易关系 $G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') = G_\alpha(q, \boldsymbol{v}'; \boldsymbol{v})$，并满足 $\boldsymbol{v}, \boldsymbol{v}' \to \infty$ 时，$G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') \to 0$ 的条件.

方程(24)的解为

$$
\begin{aligned}
G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') = \frac{-i}{(2\pi)^3 |\boldsymbol{q}|} \int d\xi_\perp \int_{-\infty}^{\infty} d\xi_1 \int_0^\infty d\tau \exp \Big\{ & \frac{i}{|\boldsymbol{q}|} (\Omega - \boldsymbol{q} \cdot \boldsymbol{v}')\tau \\
& + i\xi_\perp(\boldsymbol{v}_\perp - \boldsymbol{v}'_\perp) + i\xi_1 (v_1 - v'_1) - \frac{1}{2} \\
& \times \ln [1 + a_\alpha(\xi_\perp^2 + (\xi_1 + \tau)^2)] - \frac{1}{2} \ln [1 + a_\alpha(\xi_1^2 + \xi_\perp^2)] \Big\}.
\end{aligned} \tag{25}
$$

对于 Maxwell 分布背景等离子体，只有 $|\boldsymbol{v}| \sim v_\alpha$ 的 G_α 对以后计算才是重要的. 由于 $|\boldsymbol{\xi}| \sim 1/v_\alpha$，因此，当 $\nu_\alpha < 1$ 时，主要贡献来自 $a_\alpha \xi^2 < 1$ 项. 展开(25)式的对数部分，取到一级项积分后便得

$$
\begin{aligned}
G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') = \frac{-i}{8|\boldsymbol{q}|(\pi a_\alpha)^{3/2}} \int_0^\infty d\tau \exp \Big\{ & - \frac{a_\alpha}{4} \tau^2 + \frac{i}{|\boldsymbol{q}|} \\
& \times \Big[\Omega - \frac{(\boldsymbol{v} + \boldsymbol{v}')}{2} \cdot \boldsymbol{q} \Big] \tau - \frac{(\boldsymbol{v} - \boldsymbol{v}')^2}{4a_\alpha} \Big\}.
\end{aligned} \tag{26}
$$

显然，$\nu_\alpha \to 0$ 时，自然退化到自由传播子 Green 函数解 $G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') = \frac{\delta(\boldsymbol{v} - \boldsymbol{v}')}{\Omega - \boldsymbol{q} \cdot \boldsymbol{v}'}$.

下面给出几个有用的关系式：

$$
\int G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}') d^3v' = \frac{-i}{|\boldsymbol{q}|} \int_0^\infty d\tau \exp \Big\{ \frac{-a_\alpha}{2} \tau^2 + \frac{i}{|\boldsymbol{q}|} (\Omega - \boldsymbol{q} \cdot \boldsymbol{v})\tau \Big\}
$$

$$\equiv G_{0\alpha}(q,\boldsymbol{v}) = G^{(0)}(q,\boldsymbol{v}) + \frac{a_\alpha q^2}{(\Omega - \boldsymbol{q}\cdot\boldsymbol{v})^2} + o(\nu_\alpha^2), \tag{27}$$

$$\int \boldsymbol{e}_{\boldsymbol{q}}^{l}\cdot\boldsymbol{v}' G_\alpha(q,\boldsymbol{v};\boldsymbol{v}')d^3v' = \frac{1}{|\boldsymbol{q}|}\{\Omega G_{0\alpha}(q,\boldsymbol{v}) - 1\}, \tag{28}$$

$$\int \boldsymbol{e}_{\boldsymbol{q}}^{t}\cdot\boldsymbol{v}' G_\alpha(q,\boldsymbol{v};\boldsymbol{v}')d^3v' = \boldsymbol{e}_{\boldsymbol{q}}^{t}\cdot\boldsymbol{v}G_{0\alpha}(q,\boldsymbol{v}). \tag{29}$$

(28)式可由方程(24)严格求得,而(29)式利用(26)式得到.

于是,(17)式的低频纵波介电函数为 $\varepsilon_d^l(q) = {}^{(0)}\varepsilon_d^l(q) + {}^{(r)}\chi_e^l$, 这里 ${}^{(0)}\varepsilon_d^l$ 为通常没有重整化的部分,而

$$^{(r)}\chi_e^l(q) = a_e\frac{\omega_e^2}{n_0}\int\frac{\boldsymbol{q}\cdot\dfrac{\partial F_{0e}}{\partial\boldsymbol{v}}}{(\Omega-\boldsymbol{q}\cdot\boldsymbol{v})^3}d^3v + o(\nu_e^2) \tag{30}$$

为重整化电子极化部分. 由于 $a_i \ll a_e$, 得到(30)式时离子重整化效应已被略去.

低频横波的色散响应函数为 $\varepsilon_d^t(q) = {}^{(0)}\varepsilon_d^t(q) - \dfrac{c^2q^2}{\Omega^2} + {}^{(r)}\chi_e^t(q)$, ${}^{(0)}\varepsilon_d^t(q)$ 为没有重整化部分,而重整化电子极化部分为

$$\begin{aligned}^{(r)}\chi_e^t(q) &= a_e\frac{q^2\omega_e^2}{n_0\Omega}\int\frac{\boldsymbol{e}_q^t\cdot\boldsymbol{v}\boldsymbol{e}_q'^t\cdot\dfrac{\partial}{\partial\boldsymbol{v}}F_{0e}(\boldsymbol{v})}{(\Omega-\boldsymbol{q}\cdot\boldsymbol{v})^3}d^3v\\ &= -a_e\frac{q^2\omega_e^2}{n_0\Omega}\int_{-\infty}^{\infty}\frac{F_{0e}(v)}{(\Omega-8v)^3}dv + o(\nu_e^2).\end{aligned} \tag{31}$$

这样,在 $v_i \ll \dfrac{\Omega}{|\boldsymbol{q}|} \ll v_e$ 相速度区, Landau 阻尼可忽略,就有

$$\varepsilon_d^l(q) = \frac{1}{q^2\lambda_e^2\Omega^2}[\Omega^2(1-\nu_e) - q^2v_s^2] + o(\nu_e^2), \tag{32}$$

$$\varepsilon_d^t(q) = -\frac{1}{q^2\lambda_e^2\Omega^2}[\Omega^2(1-\nu_e) + q^2v_s^2 + c^2\lambda_e^2q^4] + o(\nu_e^2). \tag{33}$$

这里 $\lambda_\alpha = \left(\dfrac{T_\alpha}{4\pi n_0 e_\alpha^2}\right)^{1/2}$ 为 Debye 长度 $(\alpha = i, e)$, $v_s = \left(\dfrac{T_e}{m_i}\right)^{1/2}$.

2. 高频传播子方程近似解

方程(6)的 Green 函数方程为

$$\left\{\omega - \boldsymbol{k}\cdot\boldsymbol{v} - i\frac{\partial}{\partial\boldsymbol{v}}\cdot\mathbf{D}_\alpha(k,\boldsymbol{v})\cdot\frac{\partial}{\partial\boldsymbol{v}}\right\}G_\alpha(k,\boldsymbol{v};\boldsymbol{v}') = \delta(\boldsymbol{v}-\boldsymbol{v}'). \tag{34}$$

在"最发散"近似下,注意到高频波 $(\boldsymbol{k}\cdot\boldsymbol{v}/\omega)_{1,2} \ll 1$ 及 $\sum\limits_{l(\pm k)}\boldsymbol{E}^\sigma(k)\boldsymbol{E}^{\sigma*}(k) = \boldsymbol{E}(k)\boldsymbol{E}^*(k)$, 即有

$$\begin{aligned}\mathbf{D}_\alpha(k,\boldsymbol{v}) &= i\left(\frac{e_\alpha}{m_\alpha}\right)^2\int\frac{\boldsymbol{e}_1\boldsymbol{e}_1|E(k_1)|^2\delta^4(k-k_1-q)}{\Omega-\boldsymbol{q}\cdot\boldsymbol{v}+i0_+}\frac{d^4k_1d^4q}{(2\pi)^4}\\ &= \mathbf{D}_\alpha^{rc}(k,\boldsymbol{v}) + \mathbf{D}_\alpha^{nr}(k,\boldsymbol{v}),\end{aligned} \tag{35}$$

其中

$$\mathbf{D}_\alpha^{rc}(k, \boldsymbol{v}) = \pi\left(\frac{e_\alpha}{m_\alpha}\right)^2 \int \boldsymbol{e}_1\boldsymbol{e}_1 |E(k_1)|^2 \delta(\omega - \omega_1 - \boldsymbol{k}\cdot\boldsymbol{v} + \boldsymbol{k}_1\cdot\boldsymbol{v}) \frac{d^4k_1}{(2\pi)^4} \tag{36}$$

为共振张量扩散系数，而

$$\mathbf{D}_\alpha^{nr}(k, \boldsymbol{v}) = i\left(\frac{e_\alpha}{m_\alpha}\right)^2 \mathscr{P} \int \frac{\boldsymbol{e}_1\boldsymbol{e}_1 |E(k_1)|^2}{\omega - \omega_1 - (\boldsymbol{k} - \boldsymbol{k}_1)\cdot\boldsymbol{v}} \frac{d^4k_1}{(2\pi)^4} \tag{37}$$

为非共振张量扩散系数，$\mathscr{P}$ 为主值积分符号.

当高频湍动谱接近各向同性时，

$$\mathbf{D}_\alpha^{rc}(k, \boldsymbol{v}) \simeq \frac{\pi}{4v}\int \frac{d^4k_1}{(2\pi)^4} \frac{|E(k_1)|^2}{|\boldsymbol{k}_1|} \{(\hat{\theta}\hat{\theta}) + \hat{\varphi}\hat{\varphi})(1 - \zeta^2) + \hat{v}\hat{v}2\zeta^2\}, \tag{38}$$

$$\mathbf{D}_\alpha^{nr}(k, \boldsymbol{v}) = \frac{i}{4v}\left(\frac{e_\alpha}{m_\alpha}\right)^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{|E(k_1)|^2}{|\boldsymbol{k}_1|} \mathscr{P}\int_{-1}^{1} [(\hat{\theta}\hat{\theta} + \hat{\varphi}\hat{\varphi})(1 - \mu'^2) + 2\hat{v}\hat{v}\mu'^2](\mu' - \zeta)^{-1} d\mu', \tag{39}$$

式中 $\zeta = \dfrac{\boldsymbol{k}\cdot\boldsymbol{v} - \Omega}{|\boldsymbol{k}_1| v}$，$\Omega = \omega - \omega_1$；$\hat{v}$，$\hat{\theta}$，$\hat{\varphi}$ 表示球坐标三个轴的单位矢量. 注意到

$$\mathscr{P}\int_{-1}^{1} \frac{x^2 dx}{x - \zeta} = 2\zeta + \zeta^2\left(\int_{-1}^{\zeta-\Delta} \frac{dx}{x - \zeta} + \int_{\zeta+\Delta}^{1} \frac{dx}{x - \zeta}\right)$$
$$= 2\zeta + \zeta^2 \ln \frac{|1 - \zeta|}{|1 + \zeta|}, \tag{40}$$

其中Δ为奇点附近半圆的半径，△→0.

在粒子与高频场有明显作用情况下，$\mu = \dfrac{\boldsymbol{k}\cdot\boldsymbol{v}}{|\boldsymbol{k}| v} < 1$，$\zeta < 1$，于是

$$(1 - \zeta^2)\ln \frac{|1 - \zeta|}{|1 + \zeta|} - 2\zeta \simeq -4\zeta.$$

同时 $\mathbf{D}_\alpha^{nr}$ 及 $\mathbf{D}_\alpha^{rc}$ 中 $\hat{v}\hat{v}$ 方向分量可忽略. 这样，对方位角 φ 平均后，方程(34)近似为

$$\left\{\omega - |\boldsymbol{k}| v\mu - i\frac{\partial}{\partial\mu}\frac{(1 - \mu^2)}{v^2}(D_\alpha^{rc} + D_\alpha^{nr})\frac{\partial}{\partial\mu}\right\} G_\alpha(k, \mu; \mu') = \delta(\mu - \mu'), \tag{41}$$

其中

$$G_\alpha(k, \boldsymbol{v}; \boldsymbol{v}') = G_\alpha(k, \mu; \mu')\frac{\delta(v - v')}{2\pi v'^2}, \tag{42}$$

$$\frac{1}{v^2}(D_\alpha^{nr} + D_\alpha^{rc}) = \Lambda_\alpha - i\frac{4}{\pi}\Lambda_{1\alpha}\mu, \tag{43}$$

$$\begin{pmatrix}\Lambda_\alpha \\ \Lambda_{1\alpha}\end{pmatrix} = \frac{\pi}{4v^3}\left(\frac{e_\alpha}{m_\alpha}\right)^2 \int \frac{|E(k_1)|^2}{|\boldsymbol{k}_1|} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{k}}{|\boldsymbol{k}_1|}\end{pmatrix} \frac{d^4k_1}{(2\pi)^4}. \tag{44}$$

令(44)式中 $|\boldsymbol{k}_1| \sim |\boldsymbol{k}|$，即有 $\Lambda_\alpha = \Lambda_{1\alpha} \simeq \dfrac{\pi}{4}\dfrac{\omega_\alpha^2}{|\boldsymbol{k}|\lambda_\alpha}\left(\dfrac{v_\alpha}{v}\right)^3 \nu_\alpha$. (45)

记 $x = |\boldsymbol{k}| v\mu$ 及 $b_\alpha = \Lambda_\alpha^2 k^2 v^2$，便可写(41)式为

$$\left\{\omega - x - ib_\alpha\frac{\partial^2}{\partial x^2}\right\} G_\alpha(k, x; x') = |\boldsymbol{k}| v\delta(x - x'). \tag{46}$$

(46)式是自伴的，其解为

$$G_{\alpha}(k, x; x') = \frac{-i|\boldsymbol{k}|v}{2\sqrt{\pi b_\alpha}}\int_0^\infty \frac{d\tau}{\sqrt{\tau}}\exp\left\{i\left(\omega - \frac{x+x'}{2}\right)\tau - \frac{b_\alpha}{12}\tau^3 - \frac{(x-x')^2}{4b_\alpha\tau}\right\}. \tag{47}$$

显然，交换 x 与 x' 是互为对称的.

下面给出与高频 Green 函数积分量有关的几个式子：

$$\int_{-1}^{1} G_\alpha(k, x; x')d\mu' \simeq -i\int_0^\infty d\tau \exp\left\{i(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\tau - \frac{b_\alpha}{3}\tau^3\right\} \equiv G_{0\alpha}(k, \boldsymbol{v}). \tag{48}$$

得到上式时仍设 $\nu_e < 1$，并取积分 $\int_{\sigma_-}^{\sigma_+} e^{-\sigma^2}d\sigma \simeq \sqrt{\pi}$，这里 $\sigma_\pm = \frac{1}{2\sqrt{b_\alpha\tau}}(\pm kv - x + ib_\alpha\tau^2)$.

$$\int_{-1}^{1} \mu' G_\alpha(k, x; x')d\mu' = \frac{\omega}{|\boldsymbol{k}|v}\left\{G_{0\alpha}(k, \boldsymbol{v}) - \frac{1}{\omega}\right\}. \tag{49}$$

$b_\alpha \to 0$ 时，(48)式即为自由传播子 $G^{(0)}(k,\boldsymbol{v}) = 1/(\omega - \boldsymbol{k}\cdot\boldsymbol{v})$，一般情况下，

$$G_{0\alpha}(k, \boldsymbol{v}) = G^{(0)}(k, \boldsymbol{v}) + i\frac{c_\alpha \nu_\alpha}{v(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^4} + o(\nu_e^2). \tag{50}$$

这里 $c_\alpha = \frac{\pi}{2}|\boldsymbol{k}|\lambda_\alpha\omega_\alpha^3\nu_\alpha$.

现在写出 Langmuir 波的重整化介电函数具体形式. 由(16)式，$\varepsilon_h^l(k) = {}^{(0)}\varepsilon_h^l(k) + {}^{(r)}\chi_e^l$，其中无重整化时的介电函数为

$${}^{(0)}\varepsilon_h^l(k) \simeq 1 - \frac{\omega_e^2}{\omega^2} - 3k^2\lambda_e^2 + i\sqrt{\frac{\pi}{2}}\frac{1}{|\boldsymbol{k}|\lambda_e}e^{-\frac{1}{2k^2\lambda_e^2}}, \tag{51}$$

而重整化电子极化部分为

$${}^{(r)}\chi_e^l(k) = i\frac{\pi}{2}\frac{\omega_e^3 v_e^3}{|\boldsymbol{k}|\lambda_e n_0}\nu_e\int\frac{\boldsymbol{k}\cdot\frac{\partial F_{0e}}{\partial \boldsymbol{v}}d^3v}{v(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^4} + o(\nu_e^2). \tag{52}$$

因为 $\omega \gg |\boldsymbol{k}|v_e$，立刻得到

$${}^{(r)}\chi_e^l(k) \simeq -i\frac{4}{3}\sqrt{2\pi}\left(\frac{\omega_e}{\omega}\right)^5|\boldsymbol{k}|\lambda_e\nu_e + o\left(\nu_e^2, \left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)^2\right). \tag{53}$$

由于高频传播子湍动碰撞频率中只考虑了共振项作用，所以重整化结果相当于阻尼效应. 当 $\frac{8}{3}k^2\lambda_e^2\nu_e > \exp\left(-\frac{1}{2k^2\lambda_e^2}\right)$ 时，重整化效应超过了线性 Landau 阻尼，因为 $|\boldsymbol{k}|\lambda_e \ll 1$，所以这一条件在 $\nu_e \ll 1$ 时也很好满足.

三、Langmuir 湍动场激发低频自生磁场和离子声波的重整化方程组

1. 高、低频重整化非线性电流跃迁系数的计算

在背景等离子体粒子为 Maxwell 分布情况下重整化高频传播子(50)式可近似地写为

$G_{0e}(k, \boldsymbol{v}) \simeq \frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}}\left[1 + i\frac{\pi}{2}k\lambda_e\nu_e\right] + o(\nu_e^2)$，(27)式比较低频传播子 $G_{0e}(q, \boldsymbol{v}) \simeq \frac{1}{\Omega - \boldsymbol{q}\cdot\boldsymbol{v}}\left[1 + \frac{q^2v_e^2}{(\Omega - \boldsymbol{q}\cdot\boldsymbol{v})^2}\nu_e\right] + o(\nu_e^2)$ 可知，前者重整化部分远较后者小，故在计算低频重整化非线性电流跃迁系数时(15)式中可取 $G_{0e}(k, \boldsymbol{v}) \simeq \frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}}$，由此得

$$H_e^{\beta ll}(q, k_1, k_2) = \int \boldsymbol{e}_q^\beta \cdot \boldsymbol{v} G_e(q, \boldsymbol{v}; \boldsymbol{v}')\mathscr{L}_e(k_1, k_2, \boldsymbol{v}')d^3v'd^3v, \tag{54}$$

其中

$$\mathscr{L}_e(k_1, k_2, \boldsymbol{v}) = \frac{1}{2}\left[\boldsymbol{e}_1\cdot\frac{\partial}{\partial\boldsymbol{v}}\frac{1}{\omega_2 - \boldsymbol{k}_2\cdot\boldsymbol{v}}\boldsymbol{e}_2\cdot\frac{\partial}{\partial\boldsymbol{v}}F_{0e}(\boldsymbol{v}) + 1\longleftrightarrow 2\right]. \tag{55}$$

对 $\beta = l$，　$H_e^{lll}(q, k_1, k_2) = {}^{(0)}H_e^{lll}(q, k_1, k_2) + {}^{(r)}H_e^{lll}(q, k_1, k_2)$. (56)

这里[1]，

$$\begin{aligned} {}^{(0)}H_e^{lll}(q, k_1, k_2) &= \frac{\Omega}{\omega_2}\,{}^{(0)}H_e^{lll}(k_2, q, k_1) \\ &= \frac{\Omega|\boldsymbol{q}|n_0}{2\omega_e^2\omega_2^2}\boldsymbol{e}_1\cdot\boldsymbol{e}_2[{}^{(0)}\varepsilon_e^l(q) - 1] + o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)_{1,2} \end{aligned} \tag{57}$$

为非重整化部分，${}^{(0)}\varepsilon_e^l(q) = 1 + \frac{\omega_e^2}{n_0q^2}\int\frac{\boldsymbol{q}\cdot\frac{\partial F_{0e}}{\partial\boldsymbol{v}}}{\Omega - \boldsymbol{q}\cdot\boldsymbol{v}}d^3v$，而重整化部分

$$\begin{aligned} {}^{(r)}H_e^{lll}(q, k_1, k_2) &= \nu_eq^2v_e^2\frac{\Omega}{2|\boldsymbol{q}|}\int\left\{\frac{(\Omega - \boldsymbol{q}\cdot\boldsymbol{v})}{\omega_1\omega_2}\boldsymbol{e}_1\cdot\frac{\partial}{\partial\boldsymbol{v}}\boldsymbol{e}_2\right. \\ &\left.\cdot\frac{\partial}{\partial\boldsymbol{v}}F_{0e} + \frac{\boldsymbol{e}_1\cdot\boldsymbol{e}_2}{\omega_2^2}\boldsymbol{q}\cdot\frac{\partial F_{0e}}{\partial\boldsymbol{v}}\right\}(\Omega - \boldsymbol{q}\cdot\boldsymbol{v})^{-3}d^3v \\ &+ o\left(\nu_e^2, \left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)_{1,2}^2\right) = \frac{\nu_e}{2}\frac{\Omega n_0}{\omega_e^4q^2\lambda_e^2|\boldsymbol{q}|}\{2\boldsymbol{e}_1\cdot\boldsymbol{q}\boldsymbol{e}_2\cdot\boldsymbol{q} - \boldsymbol{e}_1\cdot\boldsymbol{e}_2q^2\} \\ &+ o(\nu_e^2), \end{aligned} \tag{58}$$

后一等式是在 $v_i \ll \Omega/|\boldsymbol{q}| \ll v_e$ 条件下得到．　由于这时 ${}^{(0)}\varepsilon_e^l(\boldsymbol{q}) - 1 \simeq 1/q^2\lambda_e^2$，比较(57)和(58)式可知，重整化修正约为一个 ν_e 因子．

对 $\beta = t$，利用(29)后(54)式可写成

$$H_e^{tll}(q, k_1, k_2) = {}^{(0)}H_e^{tll}(q, k_1, k_2) + {}^{(r)}H_e^{tll}(q, k_1, k_2), \tag{59}$$

其中

$${}^{(0)}H_e^{tll}(q, k_1, k_2) = \frac{n_0}{2\omega_1\omega_2^2}\boldsymbol{e}_q^t\cdot[\boldsymbol{q}\times(\boldsymbol{e}_1\times\boldsymbol{e}_2)] + o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)_{1,2}^{[1]} \tag{60}$$

是导致自生磁场产生的低频非线性电流激发系数；而重整化系数 ${}^{(r)}H_e^{tll}(q, k_1, k_2) = \nu_eq^2v_e^2\int\frac{\boldsymbol{e}_q^t\cdot\boldsymbol{v}}{(\Omega - \boldsymbol{q}\cdot\boldsymbol{v})^3}\mathscr{L}_e(k_1, k_2, \boldsymbol{v})d^3v + o(\nu_e^2) = -\frac{\nu_e\Omega n_0}{\omega_e^2q^2v_e^2}\boldsymbol{e}_q^t\cdot[\boldsymbol{e}_1\boldsymbol{e}_2\cdot\boldsymbol{q} + 1\longleftrightarrow 2] + o(\nu_e^2)$. (61)

得到(61)式后一等式时同样用了 $v_i \ll \Omega/|\boldsymbol{q}| \ll v_e$ 条件．比较(60)与(61)式可见，后者约小一个因子 $\frac{\Omega\omega_e}{q^2v_e^2}\nu_e$.

下面给出重整化高频跃迁系数的计算．由(13)式得

$$H_e^{l\beta l}(k, q, k_1) = {}^{(0)}H_e^{l\beta l}(k, q, k_1) + {}^{(r)}H_e^{l\beta l}(k, q, k_1), \tag{62}$$

其中

$$
{}^{(0)}H_e^{l\beta l}(k,q,k_1)=\int\frac{\boldsymbol{e}_k\cdot\boldsymbol{v}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\left[\left(1-\frac{\boldsymbol{q}\cdot\boldsymbol{v}}{\Omega}\right)\boldsymbol{e}_q^\beta+\frac{\boldsymbol{e}_q^\beta\cdot\boldsymbol{v}}{\Omega}\boldsymbol{q}\right]
$$

$$
\cdot\frac{\partial}{\partial\boldsymbol{v}}\cdot\frac{\boldsymbol{e}_1\cdot\dfrac{\partial F_{0e}}{\partial\boldsymbol{v}}}{\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v}}d^3v=\frac{n_0}{\omega\omega_1\Omega}\boldsymbol{e}_k\cdot[\boldsymbol{e}_1\times(\boldsymbol{q}\times\boldsymbol{e}_q^\beta)]
$$

$$
+o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right). \tag{63}
$$

显然，${}^{(0)}H_e^{lll}(k,q,k_1)=0$. 重整化部分

$$
{}^{(r)}H_e^{l\beta l}(k,q,k_1)=-i\frac{\omega_1}{|\boldsymbol{k}_1|v_e}c_e(k_1)\int\frac{\boldsymbol{e}_k\cdot\boldsymbol{v}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\boldsymbol{e}_q'
$$

$$
\cdot\frac{\partial}{\partial\boldsymbol{v}}\frac{F_{0e}(\boldsymbol{v})}{v(\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v})^4}d^3v+i\frac{\omega}{|\boldsymbol{k}|}c_e(k)v_e\int\frac{1}{v(\omega-\boldsymbol{k}\cdot\boldsymbol{v})^4}\boldsymbol{e}_q'
$$

$$
\cdot\frac{\partial}{\partial\boldsymbol{v}}\frac{\boldsymbol{e}_1\cdot\dfrac{\partial}{\partial\boldsymbol{v}}F_{0e}}{(\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v})}d^3v+o(\nu_e^2). \tag{64}
$$

于是有

$$
{}^{(r)}H_e^{lll}(k,q,k_1)=i\sqrt{\frac{\pi}{2}}\frac{n_0\nu_e}{\omega_e^2v_e}\left[\boldsymbol{e}_q^l\cdot\boldsymbol{e}_k-\frac{1}{3}\cos\theta_0\right]+o\left(\nu_e^2,\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right), \tag{65}
$$

$$
{}^{(r)}H_e^{ltl}(k,q,k_1)=i\sqrt{\frac{\pi}{2}}\frac{n_0\nu_e}{\omega_e^2v_e}\left[\boldsymbol{e}_q^t\cdot\boldsymbol{e}_k-\frac{2}{3}\sin\theta_0\right]+o\left(\nu_e^2,\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right), \tag{66}
$$

这里 $\boldsymbol{q}=\boldsymbol{k}-\boldsymbol{k}_1$, $\cos\theta_0=(q^2+k_1^2-k^2)/2|\boldsymbol{k}_1||\boldsymbol{q}|$. (64)式中 $c_e(k)$ 见(50)式. 比较(63)和(66)式可见,高频跃迁系数重整化修正比低频的小一个因子 $\dfrac{\Omega}{|\boldsymbol{q}|v_e}\nu_e\ll1$.

2. Langmuir 波激发低频磁场和离子声强湍动过程的重整化方程组

首先给出高频场方程形式. 由于 $\nu_e<1$ 时高频跃迁系数的重整化修正较小，因此在计算高频非线性电流时略去了它的贡献,只在高频线性介电函数中保留了重整化效应.这样,略去了离子非线性电流后,(11)式成为

$$
S_1(k)=-\frac{\omega_e^2}{n_0\omega}\int\boldsymbol{e}_k\cdot\boldsymbol{v}G_e(k,\boldsymbol{v};\boldsymbol{v}')\boldsymbol{e}_1\cdot\frac{\partial}{\partial\boldsymbol{v}'}\delta F_e(q,\boldsymbol{v}')E(k_1)\delta^4(k-k_1-q)
$$

$$
\times d^3vd^3v'\frac{d^4k_1d^4q}{(2\pi)^4}\simeq\frac{\omega_e^2}{\omega^2}\int\frac{n_e(q)}{n_0}\boldsymbol{e}_k\cdot\boldsymbol{E}(k_1)\delta^4(k-k_1-q)
$$

$$
\times\frac{d^4k_1d^4q}{(2\pi)^4}+o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right), \tag{67}
$$

其中 $n_e(q)=\int\delta F_e(q,\boldsymbol{v})d^3v$ 为低频振荡电子密度.

注意到(63)式后,(12)式成为

$$
S_2(k)=\frac{-i\omega_ee}{\omega^2m_e\Omega}\sum_\beta\int\boldsymbol{e}_k\cdot\{\boldsymbol{E}(k_1)\times(\boldsymbol{q}\times\boldsymbol{E}_d^\beta(q))\}\delta^4(k-k_1-q)
$$

$$\times \frac{d^4k_1d^4q}{(2\pi)^4} + o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right) = \frac{-i\omega_e e}{\omega^2 m_e c}\int \boldsymbol{e}_k \cdot \{\boldsymbol{E}(k_1)\times\boldsymbol{B}(q)\}$$

$$\times \delta^4(k-k_1-q)\frac{d^4k_1d^4q}{(2\pi)^4} + o\left(\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right). \tag{68}$$

应用(51)和(53)式,忽略线性 Landau 阻尼,在(10)式两边乘上 $\boldsymbol{e}_k\cdot\boldsymbol{e}_k=1$ 后,得到高频场的方程如下:

$$\left\{\omega^2-\omega_e^2-3k^2v_e^2-i\frac{4}{3}\sqrt{2\pi}|\boldsymbol{k}|\lambda_e\nu_e\omega_e^2\right\}\boldsymbol{E}(k)$$

$$=\omega_e^2\int\left\{\frac{n_e(q)}{n_0}\boldsymbol{E}(k_1)-\frac{ie}{m_e\omega_e c}\boldsymbol{E}(k_1)\times\boldsymbol{B}(q)\right\}\delta^4(k-k_1-q)$$

$$\times\frac{d^4k_1d^4q}{(2\pi)^4}+o\left(\nu_e,\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right). \tag{69}$$

上式右端已忽略了重整化小项.

为了使(69)式闭合,尚需给出 $n_e(q)$ 和 $\boldsymbol{B}(q)$ 的方程，为此，先研究低频电场方程(14). 略去离子非线性电流项,注意到(57),(58)式及 $H_e^{\beta ll}(q,k_1,k_2)$ 对下标 1,2 的对称性,我们得到

$$E_d^\beta(q)=\frac{i2\omega_e^2 e}{n_0m_e\Omega\varepsilon_d^\beta(q)}\int H_e^{\beta ll}(q,k_1,-k_2)E_+(k_1)E_-^*(k_2)\delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4}. \tag{70}$$

这里用了关系式 $\boldsymbol{e}_{-k}=-\boldsymbol{e}_k$, $E(-k)=-E_-^*(k)$.

对低频纵波，$\beta=l$，应用(57)和(58)式并在(70)式两边作用 $\boldsymbol{e}_q^l=\frac{\boldsymbol{q}}{|\boldsymbol{q}|}$，便有

$$\boldsymbol{E}_d^l(q)=\frac{-ie\boldsymbol{q}}{m_e\omega_e^2\varepsilon_d^l(q)}\int\left\{(\varepsilon_e^l(q)-1)\boldsymbol{E}_+(k_1)\cdot\boldsymbol{E}_-^*(k_2)\right.$$

$$\left.+2\frac{v_e^2}{q^2\lambda_e^2q^2}\boldsymbol{q}\cdot\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2)\right\}\delta^4(q-k_1+k_2)$$

$$\times\frac{d^4k_1d^4k_2}{(2\pi)^4}+o(\nu_e^2), \tag{71}$$

其中 $\varepsilon_e^\beta(q)={}^{(0)}\varepsilon_e^\beta(q)+{}^{(r)}\chi_e^\beta(q)$. (71)式也可写成

$$\boldsymbol{E}_d^l(q)=\frac{-ie\boldsymbol{q}}{m_e\omega_e^2\varepsilon_d^l(q)}\int\left\{({}^{(0)}\varepsilon_e^l(q)-{}^{(r)}\chi_e^l(q)-1)\boldsymbol{E}_+(k_1)\cdot\boldsymbol{E}_-^*(k_2)\right.$$

$$\left.+2\frac{v_e}{q^2\lambda_e^2q^2}\boldsymbol{q}\cdot[(\boldsymbol{q}\times\boldsymbol{E}_+(k_1))\times\boldsymbol{E}_-^*(k_2)]\right\}$$

$$\times\delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4}. \tag{72}$$

对低频横波，$\beta=t$，(70)式两边作用 $\boldsymbol{e}_q^t\cdot\boldsymbol{e}_q^t=1$ 后,应用(60)和(61)式便有

$$\boldsymbol{E}_d^t(q)=\frac{-ie}{m_e\omega_e\Omega\varepsilon_d^t(q)}\int\left\{\boldsymbol{q}\times(\boldsymbol{E}_+(k_1)\times\boldsymbol{E}_-^*(k_2))-\frac{2\nu_e\Omega\omega_e}{q^2v_e^2}\right.$$

$$\left.\times[\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2)+\boldsymbol{E}_-^*(k_2)\boldsymbol{q}\cdot\boldsymbol{E}_+(k_1)]\right\}$$

$$\times\delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4}+o\left(\nu_e^2,\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right). \tag{73}$$

于是自生磁场便为

$$
\begin{aligned}
\boldsymbol{B}(q) = \frac{-iec}{m_e\omega_e\Omega^2\varepsilon_d^t(q)}\int \boldsymbol{q} \times \Big\{ & \boldsymbol{q} \times (\boldsymbol{E}_+(k_1) \times \boldsymbol{E}_-^*(k_2)) \\
& - \frac{2v_e\Omega\omega_e}{q^2v_e^2}[\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2) + \boldsymbol{E}_-^*(k_2)\boldsymbol{q}\cdot\boldsymbol{E}_+(k_1)]\Big\} \\
& \times \delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4} + o\left(v_e^2, \frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right).
\end{aligned} \tag{74}
$$

现在给出低频振荡电子密度的表达式．从(5)式得

$$
\begin{aligned}
n_e(q) = \int \delta F_e(q, \boldsymbol{v})d^3v = & \, i\frac{e}{m_e}E_d^\beta(q)\int G_e(q, \boldsymbol{v}; \boldsymbol{v}')\boldsymbol{e}_q'^\beta \\
& \cdot \frac{\partial}{\partial \boldsymbol{v}}F_{0e}(\boldsymbol{v}')d^3v'd^3v - \frac{e^2}{m_e^2}\int_{(d)} I_e(q, k_1, k_2)E(k_1)E(k_2) \\
& \times \delta^4(q-k_1-k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4},
\end{aligned} \tag{75}
$$

其中

$$
\begin{aligned}
I_\alpha(q, k_1, k_2) = & \int G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}')\boldsymbol{e}_1\cdot\frac{\partial}{\partial \boldsymbol{v}'}G_\alpha(k_2, \boldsymbol{v}'; \boldsymbol{v}'')\boldsymbol{e}_2 \\
& \cdot\frac{\partial}{\partial \boldsymbol{v}''}F_{0\alpha}(\boldsymbol{v}'')d^3v''d^3v'd^3v = \int G_{0\alpha}(q, \boldsymbol{v})\boldsymbol{e}_1 \\
& \cdot\frac{\partial}{\partial \boldsymbol{v}}G_\alpha(k_2, \boldsymbol{v}; \boldsymbol{v}')\boldsymbol{e}_2\cdot\frac{\partial}{\partial \boldsymbol{v}'}F_{0\alpha}(\boldsymbol{v}')d^3v'd^3v.
\end{aligned} \tag{76}
$$

注意到(28)式，即有

$$
I_\alpha(q, k_1, k_2) = \frac{|\boldsymbol{q}|}{\Omega}H_\alpha^{lll}(q, k_1, k_2), \tag{77}
$$

同时由于

$$
\begin{aligned}
& \int G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}')\boldsymbol{e}_q'^\beta\cdot\frac{\partial}{\partial \boldsymbol{v}'}F_{0\alpha}(\boldsymbol{v}')d^3v'd^3v = \frac{|\boldsymbol{q}|}{\Omega}\int \boldsymbol{e}_q^l\cdot\boldsymbol{v}G_\alpha(q, \boldsymbol{v}; \boldsymbol{v}')\boldsymbol{e}_q'^\beta \\
& \cdot\frac{\partial}{\partial \boldsymbol{v}'}F_{0\alpha}(\boldsymbol{v}')d^3vd^3v' \\
& = \begin{cases} \dfrac{n_0|\boldsymbol{q}|}{\omega_e^2}(\varepsilon_\alpha^l(q)-1) & 对\ \beta = l, \\ 0 & 对\ \beta = t, \end{cases}
\end{aligned} \tag{78}
$$

代入(75)式后便得到

$$
\begin{aligned}
n_e(q) = & \, 2\frac{e^2|\boldsymbol{q}|}{m_e^2\Omega}\frac{{}^{(0)}\varepsilon_i^l(q)}{\varepsilon_\alpha^l(q)}\int H_e^{lll}(q, k_1, -k_2)E_+(k_1)E_-^*(k_2) \\
& \times \delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4} = -\frac{q^2}{4\pi m_e\omega_e^2}\frac{{}^{(0)}\varepsilon_i^l}{\varepsilon_d^l(q)} \\
& \times \int\Big\{(\varepsilon_e^l(q)-1)\boldsymbol{E}_+(k_1)\cdot\boldsymbol{E}_-^*(k_2) + \frac{2v_e}{q^2\lambda_e^2q^2}\boldsymbol{q} \\
& \cdot\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2)\Big\}\delta^4(q-k_1+k_2)\frac{d^4k_1d^4k_2}{(2\pi)^4}.
\end{aligned} \tag{79}
$$

从上式可知，低频横波或自生磁场不直接影响低频振荡粒子密度的变化，它只通过对高频场的反作用产生间接效应.

现在对(69)式进行逆 Fourier 变换，应用关系式 $-i\Omega \to \frac{\partial}{\partial t}$ 及 $i\boldsymbol{q} \to \nabla$，便得到高频场方程

$$\left(\frac{\partial^2}{\partial t^2} - 3v_e^2\nabla^2\right)\boldsymbol{E}(\boldsymbol{x}, t) - i\,\frac{4}{3}\,(2\pi)^{-3/2}\lambda_e\nu_e\omega_e^2\nabla^2\int\frac{E(\boldsymbol{x}, t)}{|\boldsymbol{x}-\boldsymbol{x}'|^2}\,d^3x'$$
$$= -\omega_e^2\left(1+\frac{n_e(\boldsymbol{x}, t)}{n_0}\right)\boldsymbol{E}(\boldsymbol{x}, t) + \frac{i\omega_e e}{m_e c}\,\boldsymbol{E}\times\boldsymbol{B}(\boldsymbol{x}, t). \tag{80}$$

这里重整化产生的修正项，表示了湍动碰撞对高频场的作用.

注意到(32)式及相应相速度范围内 ${}^{(0)}\varepsilon_i^l(q) = -\omega_i^2/\Omega^2$, $\varepsilon_e^l(q) - 1 \doteq \frac{1}{q^2\lambda_e^2}(1-\nu_e)$, (79)式两边乘上 $(\Omega^2 - q^2v_s^2 - \Omega^2\nu_e)$ 因子，逆变换后便可得 $n_e(q)$ 方程，这时出现了

$$\int\boldsymbol{q}\cdot\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2)\delta^4(q-k_1+k_2)\,e^{i(\boldsymbol{q}\cdot\boldsymbol{x}-\Omega t)}\,\frac{d^4k_1d^4k_2d^4q}{(2\pi)^8}$$
$$= -\{\boldsymbol{E}_-^*(x,t)\cdot\nabla(\nabla\cdot\boldsymbol{E}_+) + \boldsymbol{E}_+(\boldsymbol{x},t)\cdot\nabla(\nabla\cdot\boldsymbol{E}_-^*)$$
$$+\nabla\cdot\boldsymbol{E}_+\nabla\cdot\boldsymbol{E}_-^* + \nabla\boldsymbol{E}_+:\nabla\boldsymbol{E}_-^*\}$$
$$= -\frac{1}{2}\{\nabla^2|\boldsymbol{E}|^2 + |\nabla\cdot\boldsymbol{E}|^2 - \nabla\boldsymbol{E}:\nabla\boldsymbol{E}^*\}. \tag{81}$$

得到上式时已用了 Langmuir 波是有势场的性质以及关系式 $\boldsymbol{E}(k)\,\boldsymbol{E}^*(k) = 2\boldsymbol{E}_+(k)\,\boldsymbol{E}_-^*(k)$. 于是时空表象中 n_e 的方程为

$$\left[(1-\nu_e)\frac{\partial^2}{\partial t^2} - v_s^2\nabla^2\right]n_e(\boldsymbol{x}, t) = \frac{(1+\nu_e)}{8\pi m_i}\nabla^2|\boldsymbol{E}(\boldsymbol{x}, t)|^2$$
$$+\frac{\nu_e}{4\pi m_i}[|\nabla\cdot\boldsymbol{E}|^2 - \nabla\boldsymbol{E}:\nabla\boldsymbol{E}^*]. \tag{82}$$

同样，(74)式两边乘以(33)式，注意到

$$\int\boldsymbol{q}\times\{\boldsymbol{E}_+(k_1)\boldsymbol{q}\cdot\boldsymbol{E}_-^*(k_2) + \boldsymbol{E}_-^*(k_2)\boldsymbol{q}\cdot\boldsymbol{E}_+(k_1)\}e^{i(\boldsymbol{q}\cdot\boldsymbol{x}-\Omega t)}$$
$$\times\,\delta^4(q-k_1+k_2)\,\frac{d^4k_1d^4k_2d^4q}{(2\pi)^8} = \frac{1}{2}\{\boldsymbol{E}^*\times\nabla(\nabla\cdot\boldsymbol{E}) + c.c.\}, \tag{83}$$

便得自生磁场方程

$$\left[(1-\nu_e)\frac{\partial^2}{\partial t^2} - (\lambda_e^2c^2\nabla^2 - v_s^2)\nabla^2\right]\boldsymbol{B}(\boldsymbol{x}, t)$$
$$= -\frac{ce}{2m_e\omega_e}\Big\{i\lambda_e^2\nabla^2\nabla\times[\nabla\times(\boldsymbol{E}\times\boldsymbol{E}^*)]$$
$$+\frac{2\nu_e}{\omega_e}\frac{\partial}{\partial t}[\boldsymbol{E}\times\nabla(\nabla\cdot\boldsymbol{E}^*) + c.c.]\Big\}. \tag{84}$$

通常，条件 $|\boldsymbol{q}|\lambda_e \gg v_s/c$ 及 $|\boldsymbol{q}|\lambda_e \gg \Omega/c|\boldsymbol{q}| = \frac{\Omega}{|\boldsymbol{q}|v_e}\beta_e$, $\beta_e = v_e/c$ 很容易成立，所以(84)式又可写成

$$\nabla^4 \boldsymbol{B}(\boldsymbol{x}, t) = \frac{e}{2\lambda_e^2 c m_e \omega_e}\left\{ i\lambda_e^2 \nabla^2 \nabla \times [\nabla \times (\boldsymbol{E} \times \boldsymbol{E}^*)] + \frac{2\nu_e}{\omega_e}\frac{\partial}{\partial t}[\boldsymbol{E} \times \nabla(\nabla \cdot \boldsymbol{E}^*) + c.c.]\right\}. \tag{85}$$

方程 (80), (82) 和 (84) 组成了时空表象中描述强湍动过程闭合的重整化方程组. 当$\nu_e \to 0$ 时,上述方程组自然退化到文献[1]中导得的无 Landau 阻尼时的形式. 为了包含 Landau 阻尼效应,处理是简单的,完全可以搬用文献[1]的结果.

在 $\Omega \ll |\boldsymbol{q}|v_i$ 静态近似下,也容易得到重整化方程组的形式. 这时

$$\varepsilon_d^l(q) = {}^{(0)}\varepsilon_d^l(q) + {}^{(r)}\chi_e^l(q) \simeq \frac{1}{q^2\lambda_e^2}\left[\frac{T_e + T_i}{T_i} - \nu_e\right] \qquad (\nu_e \gg q^2\lambda_e^2), \tag{86}$$

$$\frac{{}^{(0)}\varepsilon_i^l(q)(\varepsilon_e^l(q) - 1)}{\varepsilon_d^l(q)} \simeq \frac{1}{q^2\lambda_i^2}\frac{(1 - \nu_e)}{\left(\frac{T_e + T_i}{T_i} - \nu_e\right)}, \tag{87}$$

$$\varepsilon_d^t(q) = {}^{(0)}\varepsilon_d^t(q) + {}^{(r)}\chi_e^t(q) - \frac{c^2q^2}{\Omega^2} \simeq -\frac{1}{q^2\lambda_e^2\Omega^2} \times \left[c^2q^4\lambda_e^2 + \left(\frac{T_e + T_i}{T_i} - \nu_e\right)\Omega^2\right]. \tag{88}$$

于是时空表象中,低频振荡粒子密度和自生磁场方程分别为

$$-\nabla^2 n_e(\boldsymbol{x}, t) = \frac{(1 + \nu_e)}{8\pi(T_e + T_i)}\nabla^2|\boldsymbol{E}(\boldsymbol{x}, t)|^2 + \frac{\nu_e}{4\pi(T_e + T_i)} \times [|\nabla \cdot \boldsymbol{E}|^2 - \nabla\boldsymbol{E}:\nabla\boldsymbol{E}^*], \tag{89}$$

$$\left(\frac{T_e + T_i}{T_i} - \nu_e\right)\frac{\partial^2}{\partial t^2}\boldsymbol{B}(\boldsymbol{x}, t) - \lambda_e^2 c^2 \nabla^4 \boldsymbol{B}(\boldsymbol{x}, t) = \frac{-ce}{2m_e\omega_e}\left\{ i\lambda_e^2\nabla^2\nabla \times [\nabla \times (\boldsymbol{E} \times \boldsymbol{E}^*)] + \frac{2\nu_e}{\omega_e}\frac{\partial}{\partial t} \times [\boldsymbol{E} \times \nabla(\nabla \cdot \boldsymbol{E}^*) + c.c.]\right\}. \tag{90}$$

当获得(85)式的类似条件满足时,(90)式便成为(85)式, 这时 $n_e(\boldsymbol{x}, t)$ 和 $\boldsymbol{B}(\boldsymbol{x}, t)$ 中的时间变量均以隐参量形式出现.

四、一维稳态解

为了求解方便,需要把方程组(80)、(82) 和 (84) 中的双时标关系变换到同一时间尺度. 分离出高频场中的快振荡因子,就可得到慢时标的振幅方程,令

$$\boldsymbol{E} = \frac{1}{\sqrt{2}}(\boldsymbol{\varepsilon}' e^{-i\omega_e t} + \boldsymbol{\varepsilon}'^* e^{i\omega_e t}), \tag{91}$$

略去 $\boldsymbol{\varepsilon}'$ 的二次时间导数项,(80)式即为

$$\left(i\frac{\partial}{\partial t} + \frac{3v_e^2}{2\omega_e}\nabla^2\right)\boldsymbol{\varepsilon}' + i\frac{4}{3}(2\pi)^{-3/2}\nu_e v_e \nabla^2 \int \frac{\boldsymbol{\varepsilon}'(\boldsymbol{x}', t)}{|\boldsymbol{x} - \boldsymbol{x}'|^2} d^3x'$$

$$= \frac{\omega_e}{2}\frac{n_e}{n_0}\boldsymbol{\varepsilon}' - \frac{ie}{2m_e c}\boldsymbol{\varepsilon}' \times \boldsymbol{B}. \tag{92}$$

对(92),(82)和(84)式引入如下无量纲变换:

$$\frac{2}{3}\omega_e\frac{m_e}{m_i}t \to t, \quad \frac{2}{3}\sqrt{\frac{m_e}{m_i}}\frac{\boldsymbol{x}}{\lambda_e} \to \boldsymbol{x}, \quad \frac{3}{4}\frac{m_i}{m_e}\frac{n_e}{n_0} \to n,$$

$$\frac{\sqrt{3m_i}\,\boldsymbol{\varepsilon}'}{(64\pi m_e n_0 T_e)^{1/2}} \to \boldsymbol{\varepsilon}, \quad \frac{3m_i\boldsymbol{B}}{(64\pi n_0 m_e^3 c^2)^{\frac{1}{2}}} \to \boldsymbol{B}, \tag{93}$$

上述方程就成为如下无量纲形式:

$$i\frac{\partial}{\partial t}\boldsymbol{\varepsilon} + \nabla^2\boldsymbol{\varepsilon} + i\gamma'\nabla^2\int\frac{\boldsymbol{\varepsilon}(\boldsymbol{x}',t)}{|\boldsymbol{x}-\boldsymbol{x}'|^2}d^3x' = n\boldsymbol{\varepsilon} - i\boldsymbol{\varepsilon}\times\boldsymbol{B}, \tag{94}$$

$$\left[(1-\nu_e)\frac{\partial^2}{\partial t^2} - \nabla^2\right]n = (1+\nu_e)\nabla^2|\boldsymbol{\varepsilon}|^2 + 2\nu_e[|\nabla\cdot\boldsymbol{\varepsilon}|^2 - \nabla\boldsymbol{\varepsilon}:\nabla\boldsymbol{\varepsilon}^*], \tag{95}$$

$$\left\{(1-\nu_e)\frac{\partial^2}{\partial t^2} - \left[\left(\frac{2}{3\beta_e}\right)^2\nabla^2 - 1\right]\nabla^2\right\}\boldsymbol{B} = -i\frac{4}{9}\nabla^2\nabla\times[\nabla\times(\boldsymbol{\varepsilon}\times\boldsymbol{\varepsilon}^*)]$$
$$-\frac{4}{3}\nu_e\frac{\partial}{\partial t}[\boldsymbol{\varepsilon}\times\nabla(\nabla\cdot\boldsymbol{\varepsilon}^*) + c.c.]. \tag{96}$$

这里 $\gamma'_e = \frac{4}{3(2\pi)^2}\sqrt{\frac{2\pi m_i}{m_e}}\nu_e$. 与(85)式相对应,(96)式还可由下式很好代替,即

$$\nabla^4\boldsymbol{B} = i\beta_e^2\nabla^2\nabla\times[\nabla\times(\boldsymbol{\varepsilon}\times\boldsymbol{\varepsilon}^*)] + 3\beta_e^2\nu_e\frac{\partial}{\partial t}[\boldsymbol{\varepsilon}\times\nabla(\nabla\cdot\boldsymbol{\varepsilon}^*) + c.c.]. \tag{97}$$

在一维平板几何下,方程组成为

$$i\frac{\partial}{\partial t}\varepsilon + \frac{\partial^2}{\partial x^2}\varepsilon - \gamma_e\frac{\partial}{\partial x}\varepsilon = n\varepsilon, \tag{98}$$

$$\left[(1-\nu_e)\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right]n = (1+\nu_e)\frac{\partial^2}{\partial x^2}|\boldsymbol{\varepsilon}|^2, \tag{99}$$

其中 $\gamma_e = \frac{2}{3}\sqrt{\frac{2\pi m_i}{m_e}}\nu_e$. 得到(98)式时已应用了三维变为一维的关系式,即

$$\nabla^2\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|^2} \to i\frac{(2\pi)^2}{2}\delta(y-y')\delta(z-z')\frac{\partial}{\partial x}\delta(x-x').$$

引入行波变量 $\xi = x - ut$ 及函数关系 $n = n(\xi)$, $\boldsymbol{\varepsilon} = \varepsilon_0(\xi)\exp(iqx - i\Omega t)$, 这里 u, q 和 Ω 均为参量,于是由(99)式得

$$n(\xi) = -\frac{(1+\nu_e)\varepsilon_0^2(\xi)}{1-(1-\nu_e)u^2}. \tag{100}$$

由于 ν_e 是正定的,所以与无重整化时一样,方程组 (98)—(99) 具有孤立子解只能在 $u < \sqrt{1/(1-\nu_e)}$ 亚声条件下,这时从(98)式得到

$$\frac{d^2\varepsilon_0}{d\xi^2} - \alpha_0^2\varepsilon_0 + \beta_0^2\varepsilon_0^3 = 0, \tag{101}$$

其中要求 $\alpha_0^2=\frac{\gamma_e^2 q}{2q-u}+q^2-\Omega>0$; $\beta_0^2=\frac{1+\nu_e}{1-(1-\nu_e)u^2}>0$. (101) 式解为熟知的形式 $\varepsilon_0(\xi)=\varepsilon_{00}\operatorname{sech}\alpha_0\xi$, (102)

振幅

$$\varepsilon_{00}=\sqrt{2}\,\frac{\alpha_0}{\beta_0}=\left\{2\left[\frac{\gamma_e^2 q+(q^2-\Omega)(2q-u)}{2q-u}\right]\right.$$
$$\left.\times\left[\frac{1-(1-\nu_e)u^2}{1+\nu_e}\right]\right\}^{\frac{1}{2}}. \tag{103}$$

满足 $\alpha_0^2>0$ 和 $\beta_0^2>0$ 条件的解(100)和(102)式就是重整化后方程的孤立子解. 当 $\nu_e\to 0$ 时,它们退化为通常 Zakharov 方程的双参量孤立子形式: $q=\frac{u}{2}$, $\alpha_0^2\to q^2-\Omega$, $\beta_0^2\to 1/(1-u^2)$. 这样重整化作用使孤立子的宽度和高度发生了明显的变化.

在静态近似下,方程组(80),(85)和(89)的无量纲替换基本上同(93)式,只把 $\boldsymbol{\varepsilon}'$ 的变换改为

$\frac{\sqrt{3m_i}\,\boldsymbol{\varepsilon}'}{(64\pi m_e n_0(T_e+T_i))^{\frac{1}{2}}}\to\boldsymbol{\varepsilon}$, 这样,(80)式形式上仍为(94)式而密度和磁场方程分别为

$$-\nabla^2 n=(1+\nu_e)\nabla^2|\boldsymbol{\varepsilon}|^2+2\nu_e[|\nabla\cdot\boldsymbol{\varepsilon}|^2-\nabla\boldsymbol{\varepsilon}:\nabla\boldsymbol{\varepsilon}^*], \tag{104}$$

$$\nabla^4\boldsymbol{B}=i\beta_e'^2\nabla^2\nabla\times[\nabla\times(\boldsymbol{\varepsilon}\times\boldsymbol{\varepsilon}^*)]+3\beta_e'^2\nu_e\frac{\partial}{\partial t}[\boldsymbol{\varepsilon}\times\nabla(\nabla\cdot\boldsymbol{\varepsilon}^*)+c.c.], \tag{105}$$

其中 $\beta_e'^2=(T_e+T_i)/m_e c^2$.

这些方程组同样有孤立子解,且当 $\nu_e\to 0$ 时,它们就成为通常的非线性 Schrödinger 方程结果.

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THE RENORMALIZED STRONG TURBULENCE THEORY FOR THE LOW-FREQUENCY MAGNETIC FIELD AND THE ION ACOUSTIC WAVE EXCITED BY HIGH-FREQUENCY WAVE

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ABSTRACT

In this paper, the renormalized turbulence theory for the lowfrequency magnetic field and the ion acoustic wave in the high temperature plasma is developed in order to improve the usual weak nonlinear-approach. From Vlasov-Maxwell equations, the coupled renormalized equations of the high and low frequency propagator, with the "most divergence" and "secondary divergence" effects included, are derived in the Fourier representation. Thus, we obtain the coupled relation of the renormalized partical distribution function and field for high and low-frequency oscillation.

Under "most divergence" renormalization approximation, the propagator equations for high and low-frequency are solved. Expanding to the order of ν_e—the ratio of the energy density of the high-frequency turbulence field to the thermal energy density of the plasma particle, the approximate solutions for the propagator and the expressions for the renormalized dielectric function are obtained. Then, by performing Fourier inverse transformation, the renormalized strong turbulence equations are derived in the spacetime representation.

Finally, as an example which shows the renormalized effects, under onedimensional and stationary approximation, the analytical form for the soliton is solved.