

激光加速器中电子能量 增益的广义协变推导*

朱 蔚 通¹⁾

(中国科学院上海光学精密机械研究所)

沈 文 达

(上海科学技术大学物理系)

邓 锡 铭 王 之 江

(中国科学院上海光学精密机械研究所)

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本文通过广义协变的坐标变换导出了激光电子加速器中能量增益的表达式,指出即使不存在外加的引力场,电子也会因背景介质的影响产生一个附加的引力加速,在真空情况,所得结果与用洛伦兹变换得到的结果一致。

一、引 言

激光电子加速器近来引起了相当的关注。一个电磁辐射的波包注入一个低密度的等离子体中,在光子后面激起了一个静电波动。这个静电等离子体波俘获电子到静电势阱中加速电子获得大的能量。以前对这问题的处理都是在平直时空中进行的,实验室坐标系和固定在等离子体波架中的电子能量由洛伦兹变换相联系^[1]。然而,我们知道,由等离子体(或其他运动介质)占据的时空区域在光学度规意义上一般是非闵可夫斯基的。等离子体波不是匀速直线运动的,其相速度也不一定是常数。因而与之相连的参考系统不是惯性的。洛伦兹变换是从一个惯性系统到另一个惯性系统的变换。表示自然定律的各种方程只是对于由一个惯性系统到另一个惯性系统的时间与空间的各种变换来说是不变的。如果一个参考系与运动介质(非惯性系)相联系,另一个参考系与静止真空(惯性系)相联系。那么在这两个参考系中无论质点还是电磁场,其运动规律都是不相同的。其差别在于与介质相联系的参考系必须考虑介质的影响。因而,与介质相联系的参考系和真空参考系之间的变换一般不是惯性系之间的变换,不能用洛伦兹变换来讨论,必须用广义坐标变换来讨论。

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1) 中国高等科学技术中心(世界实验室)理论物理分中心。

不幸的是,当在等离子体中存在着不均匀性或者所处的时空是弯曲的时候,用平面波解描述波在平直时空等离子体中传播的标准处理^[2]不再可以应用. 在物理上,其原因是平面波失去了它的意义. 在数学上,平面波是 $x-t$ 空间中原来场的傅里叶分量,而傅里叶分解的量是在阿贝耳变换群的空间上定义的. 然而,当波长比等离子体不均匀性的尺寸小,或者表征时空弯曲的尺度比黑洞的 Schwarzschild 半径小时,平面波公式将仍在局部上可以应用. 文献[3—6]已给出了处理弯曲时空等离子体中高频电磁波的一系列方程. 本文借助于“光学度规”的概念,实现了两个参考系的广义协变的坐标变换. 结果表明,由于波架与运动等离子体相连,即使忽略激光场的引力效应,其“光学时空度规”也是非闵可夫斯基的,有一附加的加速. 这现象是背景粒子对其作用的结果. 当电子密度趋于零(真空)时,所得的公式与洛伦兹变换得到的相同.

二、理论推导

假设一个运动等离子体占据的时空区域为 M , 运动等离子体以外的真空时空区域为 N . 我们选择与等离子体波一起运动的粒子作为背景等离子体局部静止架的原点. $x^a \equiv (ct, x, y, z)$ 为该局部静止架的四维坐标. 在这点附近的时空度规是平直的

$$d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \quad (1)$$

在静止的真空时空区域 N 中选取与之相连的实验室坐标系 $x'^a \equiv (ct', x', y', z')$. 在这实验室坐标系中时空区域 M 的光学时空度规为

$$d\tau^2 = \bar{g}_{ab} dx'^a dx'^b, \quad (2)$$

时空区域 N 的时空度规为

$$d\tau'^2 = -c^2 dt'^2 + dx'^2 + dy'^2 + dz'^2, \quad (3)$$

式中

$$\bar{g}_{ab} = g_{ab} + \left(1 - \frac{1}{\epsilon\mu}\right) u_a u_b, \quad (4)$$

$$u^0 = dx'^0/d\tau = (1 - v_x^2/c^2)^{-1/2}, \quad (5)$$

$$u^1 = \frac{dx'}{d\tau} = u^0 \frac{v_x}{c}, \quad (6)$$

$$u^2 = u^3 = 0, \quad (7)$$

$$v_x = \frac{dx'^1}{dt'} = \frac{dx'}{dt'}, \quad (8)$$

$$\frac{dy'}{dt'} = \frac{dz'}{dt'} = 0. \quad (9)$$

式中 v_x 为运动等离子体坐标原点在实验室坐标系中沿 x' 方向的运动速度; g_{ab} 为表征引力场和背景激光场产生的引力效应的度规; $\bar{g}_{ab}$ 为包含了运动介质影响的光学时空度规系数; ϵ 为标量介电常数; μ 为磁化率; u^a 为四维速度.

我们知道弯曲时空几何的测地线与局部洛伦兹几何的直线是一致的^[7]. 也就是说,对于同一个物理对象(检验粒子的世界线),在弯曲时空和局部洛伦兹时空的描述是等价的.

因此, 激光电子加速器中电子的运动既可以用方程 (1), 也可以用方程 (2) 描述。两个参考系用广义协变的坐标变换而不是用洛伦兹变换相联系。下面我们来寻求这种联系, 并导出能量增益的关系式。

设 $t' = t'(x, t)$, $x' = x'(x, t)$, 由 (1) 和 (2) 式, 得到

$$-\left(\frac{\partial t}{\partial t'}\right)^2 + \left(\frac{\partial x}{c \partial t'}\right)^2 = g_{00}, \quad (10)$$

$$-c^2 \left(\frac{\partial x}{\partial x'}\right)^2 + \left(\frac{\partial x}{\partial t'}\right)^2 = g_{11}, \quad (11)$$

$$-c \left(\frac{\partial t}{\partial t'}\right) \left(\frac{\partial x}{\partial x'}\right) + \frac{\partial x}{c \partial t'} \frac{\partial x}{\partial x'} = g_{01}. \quad (12)$$

利用 (ct, x, y, z) 坐标系相对于自身速度为零和相对于实验室坐标系 (ct', x', y', z') 速度为 v_x 的性质, 即

$$\frac{dx}{dt} = \frac{\frac{\partial x}{\partial t'} dt' + \frac{\partial x}{\partial x'} dx'}{c \frac{\partial t}{\partial t'} dt' + c \frac{\partial t}{\partial x'} dx'} = 0, \quad (13)$$

和

$$\frac{dx'}{dt'} = v_x, \quad (14)$$

得到

$$\frac{\partial x}{\partial t'} = -v_x \frac{\partial x}{\partial x'}. \quad (15)$$

由 (10)–(12) 和 (15) 式, 得到

$$\left(\frac{\partial t}{\partial x'}\right) = \pm \left[g_{01}^2 + \frac{v_x^2}{c^2} g_{11}^2 + 2 \frac{v_x}{c} g_{01} g_{11} \right]^{1/2} (-c^2 g_{00} - 2c v_x g_{01} - v_x^2 g_{11})^{-1/2}, \quad (16)$$

$$\begin{aligned} \left(\frac{\partial t}{\partial x'}\right) &= \pm \left[v_x^2 \left(\frac{\partial t}{\partial x'}\right)^2 - g_{00} + \frac{v_x^2}{c^2} g_{11} \right]^{1/2} \\ &= \pm [c^2 g_{00}^2 + 2v_x c g_{00} g_{01} + v_x^2 g_{01}^2]^{1/2} (-c^2 g_{00} - 2c v_x g_{01} - v_x^2 g_{11})^{-1/2}, \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial x}{\partial x'} &= \pm \frac{c}{v_x} \left[g_{00} + \left(\frac{\partial t}{\partial x'}\right)^2 \right]^{1/2} \\ &= \pm \frac{(-g_{00} g_{11} + g_{01}^2)^{1/2}}{\left(-g_{00} - 2 \frac{v_x}{c} g_{01} - \frac{v_x^2}{c^2} g_{11}\right)^{1/2}}, \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial x}{\partial t'} &= -v_x \left(\frac{\partial x}{\partial x'}\right) \\ &= \mp \frac{v_x (-g_{00} g_{11} + g_{01}^2)^{1/2}}{\left(-g_{00} - 2 \frac{v_x}{c} g_{01} - \frac{v_x^2}{c^2} g_{11}\right)^{1/2}}. \end{aligned} \quad (19)$$

如果我们认为运动等离子体波的相速度 $v_\phi = v_x$, 那么被这等离子体波俘获的粒子可以随这波一起运动或有一个相对加速, 在运动等离子体中的小的能量增益将导致在实验室

坐标系中的大的能量增益。

假设电子在实验室坐标系中的四维动量为 $P^a \equiv (W/c, P')$, 而在局部静止坐标系(等离子体波架)中的四维动量为 $P^a \equiv (W/c, P)$, 那么电子在两个坐标系中四维动量的变换由下面关系式联系:

$$\begin{pmatrix} \frac{W}{c} \\ P \end{pmatrix} = \begin{pmatrix} \frac{\partial t}{\partial t'} & \frac{c \partial t}{\partial x'} \\ \frac{\partial x}{c \partial t'} & \frac{\partial x}{\partial x'} \end{pmatrix} \begin{pmatrix} \frac{W'}{c} \\ P' \end{pmatrix}$$

$$\equiv \begin{pmatrix} \frac{\pm [\bar{g}_{00}^2 + 2 \frac{v_x}{c} \bar{g}_{00} \bar{g}_{01} + \frac{v_x^2}{c^2} \bar{g}_{01}^2]^{1/2}}{[-\bar{g}_{00} - \frac{\partial v_x}{c} \bar{g}_{01} - \frac{v_x^2}{c^2} \bar{g}_{11}]^{1/2}} & \frac{\mp [\bar{g}_{01}^2 + 2 \frac{v_x}{c} \bar{g}_{01} \bar{g}_{11} + \frac{v_x^2}{c^2} \bar{g}_{11}^2]^{1/2}}{[-\bar{g}_{00} - 2 \frac{v_x}{c} \bar{g}_{01} - \frac{v_x^2}{c^2} \bar{g}_{11}]^{1/2}} \\ \frac{\mp v_x (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2}}{c (-\bar{g}_{00} - 2 \frac{v_x}{c} \bar{g}_{01} - \frac{v_x^2}{c^2} \bar{g}_{11})^{1/2}} & \frac{\pm (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2}}{(-\bar{g}_{00} - 2 \frac{v_x}{c} \bar{g}_{01} - \frac{v_x^2}{c^2} \bar{g}_{11})^{1/2}} \end{pmatrix} \begin{pmatrix} \frac{W'}{c} \\ P' \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\pm [\bar{g}_{00}^2 + 2 \beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2]^{1/2}}{[-\bar{g}_{00} - 2 \beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11}]^{1/2}} & \frac{\mp [\bar{g}_{01}^2 + 2 \beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2]^{1/2}}{[-\bar{g}_{00} - 2 \beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11}]^{1/2}} \\ \frac{\mp \beta_\phi (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2}}{(-\bar{g}_{00} - 2 \beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2}} & \frac{\pm (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2}}{(-\bar{g}_{00} - 2 \beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2}} \end{pmatrix} \begin{pmatrix} \frac{W'}{c} \\ P' \end{pmatrix}, \quad (20)$$

式中

$$P = P_x = \frac{mc\beta}{(1-\beta^2)^{1/2}}, \quad W = \frac{mc^2}{(1-\beta^2)^{1/2}}$$

$$P' = P_{x'} = \frac{mc\beta'}{[-\bar{g}_{00} - 2\beta' \bar{g}_{01} - \beta'^2 \bar{g}_{11}]^{1/2}}, \quad W' = \frac{mc^2}{[-\bar{g}_{00} - 2\beta' \bar{g}_{01} - \beta'^2 \bar{g}_{11}]^{1/2}},$$

$$\beta = \frac{v}{c}, \quad \beta' = \frac{v'}{c}, \quad \beta_\phi = \frac{v_\phi}{c} = \frac{v_x}{c},$$

v , 和 v' 分别为粒子在 (ct, x, y, z) 和 (ct', x', y', z') 坐标系中的 x 方向速度。

三、结果讨论

(20) 式比文献[1]给出的复杂得多。(20) 式给出了在存在外引力场、激光产生的引力场和运动的不均匀介质最普遍情况下的四维动量的变换式, (20) 式给出

$$\begin{aligned} \frac{W'}{c} &= \left\{ (-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} \left[\frac{W}{c} (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} \right. \right. \\ &\quad \left. \left. + P(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2} \right] \right\} / [(-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} \{ (\bar{g}_{00}^2 \\ &\quad + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi (\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2} \}], \quad (21) \\ P' &= [(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} \left\{ \frac{W}{c} \beta_\phi (-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} \right. \end{aligned}$$

$$+ P(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} \Big] \Big/ [(-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} \{(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}\}], \quad (22)$$

在实验室坐标系中初始能量为 $W'_0 = mc^2/[-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11}]^{1/2}$ 的电子经等离子体波加速后达到 (21) 式给出的能量 W' , 能量增益 $\Delta W'$ 为

$$\begin{aligned} \Delta W' &= W' - W'_0 \\ &= \frac{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} [W(-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} + cP(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2}]}{(-\bar{g}_{00} \bar{g}_{11} + \bar{g}_{01}^2)^{1/2} [(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}]} \\ &\quad - \frac{mc^2}{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2}}. \end{aligned} \quad (23)$$

当粒子在局域静止坐标系中的动量 $P = 0$ 时, 有

$$\frac{W'}{c} = \frac{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} \frac{W}{c}}{\{(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}\}}, \quad (24)$$

$$P' = \frac{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} \beta_\phi \frac{W}{c}}{\{(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}\}}, \quad (25)$$

$$\begin{aligned} \Delta W' &= \frac{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} W}{\{(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}\}} \\ &\quad - \frac{mc^2}{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2}}. \end{aligned} \quad (26)$$

从 (26) 式, 得到当静止于局域静止架中的电子加速到 v 时 ($\beta = v/c$), 在实验室坐标系中观察到的电子能量增益为

$$\begin{aligned} \Delta W' &= \frac{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2} mc^2}{(1 - \beta^2)^{1/2} \{(\bar{g}_{00}^2 + 2\beta_\phi \bar{g}_{00} \bar{g}_{01} + \beta_\phi^2 \bar{g}_{01}^2)^{1/2} - \beta_\phi(\bar{g}_{01}^2 + 2\beta_\phi \bar{g}_{01} \bar{g}_{11} + \beta_\phi^2 \bar{g}_{11}^2)^{1/2}\}} \\ &\quad - \frac{mc^2}{(-\bar{g}_{00} - 2\beta_\phi \bar{g}_{01} - \beta_\phi^2 \bar{g}_{11})^{1/2}}. \end{aligned} \quad (27)$$

下面根据上节得到的结果, 讨论几种特殊情况:

1) 时空是平直的, 其光学度规

$$\bar{g}_{00} = -1, \bar{g}_{01} = 0, \bar{g}_{11} = 1.$$

在这种情况下, 不仅光束的引力场不考虑, 而且讨论的是真空; 介质背景的影响亦不存在. 这时 (21), (22) 式可简化成

$$\frac{W'}{c} = \frac{(1 - \beta_\phi^2)^{1/2} \left[\frac{W}{c} + \beta_\phi P \right]}{(1 - \beta_\phi^2)} = \frac{\frac{W}{c} + \beta_\phi P}{(1 - \beta_\phi^2)^{1/2}}, \quad (28)$$

$$P' = \frac{(1 - \beta_\phi^2)^{1/2} \left[\frac{W}{c} \beta_\phi + P \right]}{(1 - \beta_\phi^2)} = \frac{\frac{W}{c} \beta_\phi + P}{(1 - \beta_\phi^2)^{1/2}}. \quad (29)$$

当 $P = 0$ 时, 有

$$\frac{W'}{c} = \frac{W}{c(1 - \beta_\phi^2)^{1/2}}, \quad (30)$$

$$P' = \frac{W\beta_\phi}{c(1 - \beta_\phi^2)^{1/2}}. \quad (31)$$

由 (27) 式给出能量增益

$$\Delta W' = \frac{(1 + \beta\beta_\phi)mc^2}{(1 - \beta^2)^{1/2}(1 - \beta_\phi^2)^{1/2}}. \quad (32)$$

(30)–(32) 式就是 Chen¹⁾ 导出的能量-动量变换式和能量增益表达式。由此可见, 洛伦兹变换对应的仅是广义协变坐标变换在真空情况下的一种特殊情况。本节导出的广义协变变换式既包含了真实的外加引力场和激光产生的引力场的影响, 也包括了介质对电子加速的影响。

$$2) \quad \varepsilon = \varepsilon, \quad \mu = 1, \quad g_{00} = -\frac{1}{\varepsilon}, \quad g_{01} = -\left(1 - \frac{1}{\varepsilon}\right)\beta_\phi,$$

$$g_{11} = \left[1 + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2\right] \simeq 1, \quad g_{22} = g_{33} = 0.$$

在这种情况下, 不考虑激光场本身的引力效应, 只考虑介电常数为 ε 的介质对其的影响。于是 (21)–(23) 式给出

$$\begin{aligned} \frac{W'}{c} = & \left\langle \left[\frac{1}{\varepsilon} + 2\beta_\phi^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_\phi^2 \right]^{1/2} \left\{ \frac{W}{c} \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \right. \right. \\ & \left. \left. + P \left[\left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 - 2\beta_\phi^2 \left(1 - \frac{1}{\varepsilon}\right) + \beta_\phi^2 \right]^{1/2} \right\} \right\rangle / \\ & \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \left\{ \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_\phi^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_\phi^2 \right]^{1/2} \right. \right. \\ & \left. \left. - \beta_\phi \left[\left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 - 2 \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 + \beta_\phi^2 \right]^{1/2} \right\} \right\rangle, \end{aligned} \quad (33)$$

$$\begin{aligned} P' = & \left\langle \left[\frac{1}{\varepsilon} + 2\beta_\phi^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_\phi^2 \right]^{1/2} \left\{ \frac{W}{c} \beta_\phi \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \right. \right. \\ & \left. \left. + P \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_\phi^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_\phi^2 \right]^{1/2} \right\} \right\rangle / \\ & \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \left\{ \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_\phi^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_\phi^2 \right]^{1/2} \right. \right. \\ & \left. \left. - \beta_\phi \left[\left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 - 2 \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 + \beta_\phi^2 \right]^{1/2} \right\} \right\rangle, \end{aligned} \quad (34)$$

$$\begin{aligned} \Delta W' = & \left\langle \left[\frac{1}{\varepsilon} + 2\beta_\phi^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_\phi^2 \right]^{1/2} \left\{ \frac{mc^2}{(1 - \beta^2)^{1/2}} \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \right. \right. \\ & \left. \left. + \frac{mc^2\beta}{(1 - \beta^2)^{1/2}} \left[\left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 - 2\beta_\phi^2 \left(1 - \frac{1}{\varepsilon}\right) + \beta_\phi^2 \right]^{1/2} \right\} \right\rangle / \\ & \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)\beta_\phi^2 \right]^{1/2} \left\{ \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_\phi^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_\phi^2 \right]^{1/2} \right. \right. \end{aligned}$$

1) F. F. Chen, Laser-driven particle accelerators, UCLA, pp.-833, (1984), (no be published).

$$- \beta_{\phi} \left[\left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 - 2 \left(1 - \frac{1}{\varepsilon}\right) \beta_{\phi}^2 + \beta_{\phi}^2 \right]^{1/2} \Bigg\rangle \\ - \frac{mc^2}{\left[\frac{1}{\varepsilon} + 2\beta_{\phi}^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_{\phi}^2 \right]^{1/2}}. \quad (35)$$

当粒子在局域洛伦兹坐标系中的动量为零 ($P = 0$) 时, 有

$$\frac{W'}{c} = \left\langle \left[\frac{1}{\varepsilon} + 2\beta_{\phi}^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_{\phi}^2 \right]^{1/2} \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} \frac{W}{c} \right\rangle / \\ \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} \left\{ \left[\frac{1}{\varepsilon^2} + \frac{2\beta_{\phi}^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^4 \right]^{1/2} \right. \right. \right. \\ \left. \left. - \beta_{\phi} \left[\left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 - 2 \left(1 - \frac{1}{\varepsilon}\right) \beta_{\phi}^2 + \beta_{\phi}^2 \right]^{1/2} \right\} \right\rangle, \quad (36)$$

$$P' = \left\langle \left[\frac{1}{\varepsilon} + 2\beta_{\phi}^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_{\phi}^2 \right]^{1/2} \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} \beta_{\phi} \frac{W}{c} \right\rangle \\ \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} \left\{ \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_{\phi}^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^4 \right]^{1/2} \right. \right. \right. \\ \left. \left. - \beta_{\phi} \left[\left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 - 2 \left(1 - \frac{1}{\varepsilon}\right) \beta_{\phi}^2 + \beta_{\phi}^2 \right]^{1/2} \right\} \right\rangle, \quad (37)$$

$$\Delta W' = \left\langle \left[\frac{1}{\varepsilon} + 2\beta_{\phi}^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_{\phi}^2 \right]^{1/2} \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} mc^2 \right\rangle / \\ \left\langle \left[\frac{1}{\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 \right]^{1/2} \left\{ \left[\left(\frac{1}{\varepsilon}\right)^2 + \frac{2\beta_{\phi}^2}{\varepsilon} \left(1 - \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^4 \right]^{1/2} \right. \right. \right. \\ \left. \left. - \beta_{\phi} \left[\left(1 - \frac{1}{\varepsilon}\right)^2 \beta_{\phi}^2 - 2 \left(1 - \frac{1}{\varepsilon}\right) \beta_{\phi}^2 + \beta_{\phi}^2 \right]^{1/2} \right\} \right\rangle \\ - \frac{mc^2}{\left[\frac{1}{\varepsilon} + 2\beta_{\phi}^2 \left(1 - \frac{1}{\varepsilon}\right) - \beta_{\phi}^2 \right]^{1/2}}. \quad (38)$$

由文献[5]知道, 色散关系是不受背景时空的曲率和介质的不均匀性的影响的。即仍有

$$\omega^2 = (c^2 k^2 + \omega_p^2), \quad (39)$$

相速度

$$v_{\phi} = \frac{c}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}} \approx c \left(1 + \frac{\omega_p^2}{2\omega^2}\right) = c \left(1 + \frac{n}{2n_c}\right), \quad (40)$$

群速度

$$v_g = \frac{d\omega}{dk} = c \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} = c \left(1 - \frac{n}{n_c}\right)^{1/2} \approx c \left(1 - \frac{n}{2n_c}\right). \quad (41)$$

当 $n \ll n_c$ 时, 有 $v_{\phi} = v_g \approx c$ 。

而 $\varepsilon = 1 - n/n_c$, n 为电子密度, n_c 为电子的临界密度。

(36)–(38) 式表明, 即使不存在外引力场, 由于运动不均匀介质的存在, 电子被激光

拍波加速器加速的能量-动量变换关系和能量增益将不同于文献[1,8]导出的结果。

最大的能量增益可以用(38)式对 ε 求极值得到。即由

$$\frac{d\Delta W'}{dn} = \frac{mc^2}{(1-\beta^2)^{1/2}} \frac{d}{d\varepsilon} \left\{ \frac{\left(\frac{4}{\varepsilon^2} - \frac{6}{\varepsilon^3} + \frac{6}{\varepsilon^4} - \frac{2}{\varepsilon^5}\right)^{1/2} + \beta \left(\frac{2}{\varepsilon^4} - \frac{2}{\varepsilon^5}\right)^{1/2}}{\left(\frac{8}{\varepsilon^3} - \frac{16}{\varepsilon^4} + \frac{14}{\varepsilon^5} - \frac{6}{\varepsilon^6} + \frac{1}{\varepsilon^7}\right)^{1/2} - \left(\frac{2}{\varepsilon^5} - \frac{2}{\varepsilon^6} + \frac{1}{\varepsilon^7}\right)^{1/2}} \right\} - mc^2 \frac{d}{d\varepsilon} \left[\frac{1}{\left(\frac{2}{\varepsilon} - \frac{2}{\varepsilon^2}\right)^{1/2}} \right] = 0 \quad (42)$$

求得。因而,可以通过设计等离子体背景密度 n 和激光频率 ω ,使注入电子获得最大能量增益。

当激光场很强时,必须计及激光场的引力效应,其度规系数可以按照Chen的算出。因而本文给出的结果既包含了背景介质的影响,又包括了引力场的作用,具有相当的普遍性。

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A GENERAL COVARIANT DERIVATION OF ELECTRON ENERGY GAIN IN A LASER ACCELERATOR

ZHU SHI-TONG

(Shanghai Institute of Optics and Fine Mechanics, Academia Sinica)

SHEN WEN-DA

(Department of Physics, Shanghai University of Science and Technology)

DENG XI-MING WANG ZHI-JIANG

(Shanghai Institute of Optics and Fine Mechanics, Academia Sinica)

ABSTRACT

In this paper, we derive an expression of energy gain in a laser electron accelerator by using a general covariant coordinate transformation. It is pointed out that even when the external gravitational field, is absent, the electrons will still be additionally accelerated by the influence of background medium. In the vacuum case, the obtained results agree with those obtained by the Lorentz transformation.