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二维费密液体理论（III）

输运性质和声传播特性

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本文研究了二维费密液体的输运性质和声传播特性，得到了扩散系数、粘滞系数和热导率的微观表达式以及零声和第一声的有关结果．通过对碰撞积分的讨论，认为须用一系列弛豫时间参数来描述二维液体，这与在三维情形可用一个唯象参数来描述不同．

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一、引　言

实验表明超流 ^{4}He 表面和碳膜上的 ^{3}He 原子层都具有二维简并费密液体特性[1]，其输运性质与三维体系有明显不同．文献[2]曾用熵极值方法估算其输运性质，但必须仔细选择试探函数．Resibois 已引入一套完美的办法[3]来推求输运系数．我们用来研究二维费密液体的扩散系数，粘滞系数和热导率．首先将朗道费密液体的动力学方程线性化，得到其简正模式，然后用微扰法求微观流体力学的频率，从而得到输运系数．

本文同时研究了与输运性质相关的声传播特性．文献[4]第一次研究了二维费密液体的声传播，因玻耳兹曼方程中碰撞积分的复杂性，引用了唯象参数 τ．在研究了碰撞积分的结构后，我们认为须用一系列弛豫参数 τ_n 来描述声传播特性．

二、动力学方程的线性化

玻耳兹曼方程为

$$\frac{\partial n_{\boldsymbol{p}_1\sigma_1}(\boldsymbol{r},t)}{\partial t}+\frac{\partial \varepsilon_{\boldsymbol{p}_1\sigma_1}}{\partial \boldsymbol{p}_1}\cdot\frac{\partial n_{\boldsymbol{p}_1\sigma_1}(\boldsymbol{r},t)}{\partial \boldsymbol{r}}-\frac{\partial \varepsilon_{\boldsymbol{p}_1\sigma_1}}{\partial \boldsymbol{r}}\cdot\frac{\partial n_{\boldsymbol{p}_1\sigma_1}(\boldsymbol{r}t)}{\partial \boldsymbol{p}_1}=I(\boldsymbol{p}_1\sigma_1,\boldsymbol{r}). \tag{1}$$

式中碰撞积分为

$$I(\boldsymbol{p}_1\sigma_1,\boldsymbol{r})=\sum_{\boldsymbol{p}_2\sigma_2\boldsymbol{p}_3\sigma_3\boldsymbol{p}_4\sigma_4}\frac{2\pi}{\hbar}|\langle \boldsymbol{p}_1\sigma_1\boldsymbol{p}_2\sigma_2|A|\boldsymbol{p}_3\sigma_3\boldsymbol{p}_4\sigma_4\rangle|^2$$
$$\cdot\delta(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_3-\boldsymbol{p}_4)\delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4}\cdot\delta(\varepsilon_{\boldsymbol{p}_1\sigma_1}(\boldsymbol{r},t)+\varepsilon_{\boldsymbol{p}_2\sigma_2}(\boldsymbol{r},t)$$

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$$- \varepsilon_{p_3\sigma_3}(\boldsymbol{r},t) - \varepsilon_{p_4\sigma_4}(\boldsymbol{r},t)) \cdot [n_{p_3\sigma_3}(\boldsymbol{r}t) n_{p_4\sigma_4}(\boldsymbol{r},t) \cdot (1 - n_{p_1\sigma_1}(\boldsymbol{r}t))(1 - n_{p_2\sigma_2}(\boldsymbol{r},t)) - n_{p_1\sigma_1}(\boldsymbol{r}t) n_{p_2\sigma_2}(\boldsymbol{r}t) \cdot (1 - n_{p_3\sigma_3}(\boldsymbol{r}t))(1 - n_{p_4\sigma_4}(\boldsymbol{r}t))] \tag{2}$$

式中 $\frac{2\pi}{\hbar}|\langle|A|\rangle|^2$ 为准粒子散射幅．当偏离平衡不远时，有

$$\begin{aligned} n_{p\sigma}(\boldsymbol{r}t) &= n^0_{p\sigma} + \delta n_{p\sigma}(\boldsymbol{r}t) \\ &= n^0_{p\sigma}(\boldsymbol{r}t) + \delta n_{p\sigma}(\boldsymbol{r}t) - \frac{\partial n^0_{p\sigma}}{\partial \varepsilon^0_{p\sigma}} \sum_{p'\sigma'} f_{p\sigma p'\sigma'} \delta n_{p'\sigma'}(\boldsymbol{r},t). \end{aligned} \tag{3}$$

式中 $n^0_{p\sigma}(\boldsymbol{r}t)$ 为

$$n^0_{p\sigma}(\boldsymbol{r}t) = (1 + \exp[\beta(\varepsilon^0_{p\sigma}(\boldsymbol{r}t) - \mu)])^{-1}, \tag{4}$$

定义

$$\phi_{p\sigma}(\boldsymbol{r}t) = \frac{\partial \varepsilon^0_{p\sigma}(\boldsymbol{r}t)}{\partial n^0_{p\sigma}(\boldsymbol{r}t)} \left(\delta n_{p\sigma}(\boldsymbol{r}t) - \frac{\partial n^0_{p\sigma}}{\partial \varepsilon^0_{p\sigma}} \sum_{p'\sigma'} f_{p\sigma,p'\sigma'} \delta n_{p'\sigma'}(\boldsymbol{r}t) \right), \tag{5}$$

为了将碰撞积分线性化，引入线性碰撞算符 $\hat{I}_{p\sigma}$ 和相互作用算符 $\hat{F}_{p\sigma}$

$$\begin{aligned} \hat{I}_{p_1\sigma_1}\phi_{p_1\sigma_1} = \beta \left(\frac{\partial n^0_{p_1\sigma_1}}{\partial \varepsilon^0_{p_1\sigma_1}} \right)^{-1} \sum_{p_2\sigma_2 p_3\sigma_3 p_4\sigma_4} \frac{2\pi}{\hbar} |\langle A \rangle|^2 \cdot \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{p}_4) \delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4} \\ \cdot \delta(\varepsilon^0_{p_1\sigma_1} + \varepsilon^0_{p_2\sigma_2} - \varepsilon^0_{p_3\sigma_3} - \varepsilon^0_{p_4\sigma_4}) n^0_{p_1\sigma_1} n^0_{p_2\sigma_2} (1 - n^0_{p_3\sigma_3})(1 - n^0_{p_4\sigma_4})(\phi_{p_1\sigma_1} \\ + \phi_{p_2\sigma_2} - \phi_{p_3\sigma_3} - \phi_{p_4\sigma_4}), \end{aligned} \tag{6}$$

$$\hat{F}_{p_1\sigma_1}\phi_{p_1\sigma_1} = \sum_{p_1'\sigma_1'} f_{p_1\sigma_1 p_1'\sigma_1'} \frac{\partial n^0{}_{p_1'\sigma_1'}}{\partial \varepsilon^0{}_{p_1'\sigma_1'}} \phi_{p_1'\sigma_1'}. \tag{7}$$

定义 $\nu_{p\sigma}(\boldsymbol{r}t)$，并进行空间傅氏变换

$$\delta n_{p\sigma}(\boldsymbol{r}t) = \frac{\partial n^0_{p\sigma}}{\partial \varepsilon^0_{p\sigma}} \nu_{p\sigma}(\boldsymbol{r}t), \tag{8}$$

$$\nu_{p\sigma}(\boldsymbol{r}t) = \nu_{p\sigma}(\boldsymbol{k}t) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{9}$$

得到线性化方程

$$\begin{aligned} \frac{\partial \nu_{p\cdot\sigma}(\boldsymbol{k}t)}{\partial t} + i\boldsymbol{k} \cdot \frac{\partial \varepsilon^0_{p\sigma}}{\partial \boldsymbol{p}} (1 - \hat{F}_{p\sigma}) \nu_{p\sigma}(\boldsymbol{k}t) \\ = \hat{I}_{p\sigma}(1 - \hat{F}_{p\sigma}) \nu_{p\sigma}(\boldsymbol{k}t). \end{aligned} \tag{10}$$

相应地定义分布函数 $\nu^+_p(\boldsymbol{k}t)$ 和 $\nu^-_p(\boldsymbol{k}t)$，得到线性化流体动力学方程

$$\nu^+_p(\boldsymbol{k}t) = \nu_{p\uparrow}(\boldsymbol{k}t) + \nu_{p\downarrow}(\boldsymbol{k}t), \tag{11}$$

$$\nu^-_p(\boldsymbol{k}t) = \nu_{p\uparrow}(\boldsymbol{k}t) - \nu_{p\downarrow}(\boldsymbol{k}t), \tag{12}$$

$$\frac{\partial \nu^+_p(\boldsymbol{k}t)}{\partial t} + i\boldsymbol{k} \cdot \frac{\partial \varepsilon^0_p}{\partial \boldsymbol{p}} (1 - F^+_p) \nu^+_p(\boldsymbol{k}t) = \hat{I}^+_p (1 - \hat{F}^+_p) \nu^+_p(\boldsymbol{k}t), \tag{13}$$

$$\frac{\partial \nu^-_p(\boldsymbol{k}t)}{\partial t} + i\boldsymbol{k} \cdot \frac{\partial \varepsilon^0_p}{\partial \boldsymbol{p}} - (1 - F^-_p) \nu^-_p(\boldsymbol{k}t) = \hat{I}^-_p (1 - \hat{F}^-_p) \nu^-_p(\boldsymbol{k}t), \tag{14}$$

$\hat{F}^+_p, \hat{F}^-_p$ 和 $\hat{I}^+_p, \hat{I}^-_p$ 相应地为

$$\hat{F}^+_p \phi_p = 2 \sum_{p'} \frac{\partial n^0_{p'}}{\partial \varepsilon^0_{p'}} f^{(s)}_{pp'} \phi_{p'}, \tag{15}$$

$$\hat{F}_p^- \phi_p = 2 \sum_{p'} \frac{\partial n_{p'}^0}{\partial \varepsilon_{p'}^0} f_{pp'}^{(a)} \phi_p, \tag{16}$$

$$\begin{aligned}\hat{I}_{p_1}^+ \phi_{p_1} = \beta \left(\frac{\partial n_{p_1}^0}{\partial \varepsilon_{p_1}^0}\right)^{-1} \sum_{p_2 p_3 p_4} n_{p_1}^0 n_{p_2}^0 (1 - n_{p_3}^0)(1 - n_{p_4}^0) \\ \cdot \frac{2\pi}{\hbar}\left(\frac{1}{2}|\langle p_1\uparrow p_2\uparrow|A|p_3\uparrow p_4\uparrow\rangle|^2 + \frac{1}{2}|\langle p_1\uparrow p_2\downarrow|A|p_3\uparrow p_4\downarrow\rangle|^2 \right. \\ \left. + \frac{1}{2}|\langle p_1\uparrow p_2\downarrow|A|p_3\downarrow p_4\uparrow\rangle|^2\right)\delta(p_1 + p_2 - p_3 - p_4)\delta(\varepsilon_{p_1}^0 \\ + \varepsilon_{p_2}^0 - \varepsilon_{p_3}^0 - \varepsilon_{p_4}^0) \cdot (\phi_{p_1} + \phi_{p_2} - \phi_{p_3} - \phi_{p_4}),\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{I}_{p_1}^- \phi_{p_1} = \beta \left(\frac{\partial n_{p_1}^0}{\partial \varepsilon_{p_1}^0}\right)^{-1} \sum_{p_2 p_3 p_4} n_{p_1}^0 n_{p_2}^0 (1 - n_{p_3}^0)(1 - n_{p_4}^0)\delta(p_1 + p_2 - p_3 \\ - p_4) \cdot \delta(\varepsilon_{p_1}^0 + \varepsilon_{p_2}^0 - \varepsilon_{p_3}^0 - \varepsilon_{p_4}^0) \cdot \frac{2\pi}{\hbar} \cdot \left[\frac{1}{2}|\langle\uparrow\uparrow|A|\uparrow\uparrow\rangle|^2(\phi_{p_1} \right. \\ + \phi_{p_2} - \phi_{p_3} - \phi_{p_4}) + \frac{1}{2}|\langle\uparrow\downarrow|A|\uparrow\downarrow\rangle|^2(\phi_{p_1} - \phi_{p_2} - \phi_{p_3} + \phi_{p_4}) \\ \left. + \frac{1}{2}|\langle\uparrow\downarrow|A|\downarrow\uparrow\rangle|^2(\phi_{p_1} - \phi_{p_2} + \phi_{p_3} - \phi_{p_4})\right].\end{aligned} \tag{18}$$

由(17)和(18)式知 $\hat{I}_p^+$ 的零本征矢为 1，p，ε_p^0，$\hat{I}_p^-$ 的唯一零本征矢为 1.（11)至(18)式是研究输运系数和声传播特性的出发点.

三、输运系数的计算

1. 流体动力学方程的简正模式

定义平均粒子数密度 $\langle n_\sigma\rangle$

$$\langle n_\sigma\rangle = \sum_p n_{p\sigma}(r), \tag{19}$$

其他物理量的平均值均按类似下式的方法定义:

$$\langle v_\sigma\rangle = \frac{1}{\langle n_\sigma\rangle}\int dp\, n_{p\sigma}(r)\frac{\partial \varepsilon_{p\sigma}(r)}{\partial p}, \tag{20}$$

设

$$n = n_0 + \Delta n, \tag{21}$$

$$\Delta m = n_\uparrow - n_\downarrow, \tag{22}$$

由(10)式得到

$$\frac{\partial}{\partial t}\langle \Delta n\rangle + n_0 \nabla_r \langle v\rangle = 0, \tag{23a}$$

$$m n_0 \frac{\partial}{\partial t}\langle v\rangle + \nabla_r \mathrm{P} = 0, \tag{23b}$$

$$mn_0T_0\frac{\partial}{\partial t}\langle\Delta s\rangle+\nabla_r J^Q=0, \tag{23c}$$

$$\frac{\partial}{\partial t}\langle\Delta m\rangle+\nabla_r\cdot J^D=0, \tag{23d}$$

式中 $\langle v\rangle$, $\mathbf{P}$, J^Q 和 J^D 分别为平均粒子流、压强张量、能流和扩散流．引入扩散系数 D、粘滞系数 η, ζ 和热导率 K

$$P_{ij}(rt)=p(rt)\delta_{ij}-\zeta\delta_{ij}\nabla_r\langle v\rangle-\eta\left(\frac{\partial v_i}{\partial r_j}+\frac{\partial v_j}{\partial r_i}-\delta_{ij}\nabla_r\langle v\rangle\right), \tag{24a}$$

$$J^Q=K\nabla_r T, \tag{24b}$$

$$J^D=-D\nabla_r\langle\Delta m\rangle, \tag{24c}$$

(23)和(24)式给出简正模频率

$$\omega_v=-ik^2\eta/(mn_0), \tag{25a}$$

$$\omega_D=ik^2D, \tag{25b}$$

$$\omega_s=-ik^2K/(mn_0C_p), \tag{25c}$$

$$\omega_\pm=\pm C_0k-ik^2\left(\frac{\eta}{2}+\zeta+K\left(\frac{1}{C_v}-\frac{1}{C_p}\right)\right)/(mn_0). \tag{25d}$$

式中 $C_0=\left[\frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_s\right]^{\frac{1}{2}}$.

2. 输运系数的微观表述

引入两矢量之标量积．

$$\langle\phi|\psi\rangle=-\frac{2\pi}{m^*}\sum_p\frac{\partial n_p^0}{\partial\varepsilon_p^0}\phi_p\psi_p, \tag{26}$$

令

$$|\chi_n(p)\rangle_+=(1-\hat{F}_p^+)|\nu_p^+e^{i\omega_n^+t}, \tag{27}$$

$$|\chi_n(p)\rangle_-=(1-\hat{F}_p^-)|\nu_p^-e^{i\omega_n^-t}, \tag{28}$$

由(13)和(14)式得到

$$\omega_n^+(1-\hat{F}_p^+)^{-1}|\chi_n(p)\rangle_+=\left(i\hat{I}_p^++k\cos\theta\frac{\partial\varepsilon_p^0}{\partial p}\right)|\chi_n(p)\rangle_+ \tag{29}$$

$$\omega_n^-(1-\hat{F}_p^-)^{-1}|\chi_n(p)\rangle_-=\left(i\hat{I}_p^-+k\cos\theta\frac{\partial\varepsilon_p^0}{\partial p}\right)|\chi_n(p)\rangle_-. \tag{30}$$

利用瑞利微扰法，(29)式四个本征值中有两个与 k 成正比，是动力学区域的声模，另两个本征值分别对应于粘滞系数和热导率，(30)式唯一本征值对应于扩散系数．与(25)式对比得到

$$D=\frac{\left\langle\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\frac{1}{\hat{I}_p^-}\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right\rangle}{\langle(1-\hat{F}_p^-)^{-1}\rangle}, \tag{31a}$$

$$\eta=-mn_0\frac{\left\langle p\sin\theta\left|\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\frac{1}{\hat{I}_p^+}\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|p\sin\theta\right\rangle}{\langle p\sin\theta|(1-\hat{F}_p^+)^{-1}|p\sin\theta\rangle},\tag{31b}$$

$$K=-mn_0C_p\frac{\left\langle\chi_K\left|\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\frac{1}{\hat{I}_p^+}\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|\chi_K\right\rangle}{\langle\chi_K|(1-\hat{F}_p^+)^{-1}|\chi_K\rangle},\tag{31c}$$

式中 $|\chi\rangle$ 为

$$|\chi_K\rangle=\left(1-\frac{\left\langle 1\left|\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|p\cos\theta\right\rangle}{\left\langle\left|\left(\frac{p^2}{k_F^2}-1\right)\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|p\cos\theta\right\rangle}\right)\times\left(1+\frac{\pi^2}{3\beta^2\varepsilon_F^2}\cdot\frac{\left\langle 1\left|\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|p\cos\theta\right\rangle^2}{\left\langle\left(\frac{p^2}{k_F^2}-1\right)\left|\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\right|p\cos\theta\right\rangle^2}\right)^{\frac{1}{2}}\Big|\Big\rangle\tag{32}$$

3. 输运系数的计算

从(31)式直接的简化得到

$$D=-\frac{(1+F_0^a)}{2\pi m^*}\int d\boldsymbol{p}\,\frac{\partial n_p^0}{\partial\varepsilon_p^0}\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\phi_D(\boldsymbol{p}),\tag{33a}$$

$$\hat{I}_p^-\phi_D(p)=\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta,\tag{33b}$$

$$\eta=\frac{\left(1+\frac{F_1^s}{2}\right)mn_0}{\pi m^*k_F^2}\int d\boldsymbol{p}\,\frac{\partial n_p^0}{\partial\varepsilon_p^0}\frac{\partial\varepsilon_p^0}{\partial p}p\cos\theta\sin\theta\phi_\eta(\boldsymbol{p}),\tag{34a}$$

$$\hat{I}_p^+\phi_\eta(p)=\frac{\partial\varepsilon_p^0}{\partial p}p\cos\theta\sin\theta,\tag{34b}$$

$$K=\frac{mn_0C_p}{2\pi m^*}\int d\boldsymbol{p}\,\frac{\partial n_p^0}{\partial\varepsilon_p^0}\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta\left[\frac{p^2}{k_F^2}-1-(1+F_0^s)\frac{\pi^2}{3\beta^2\varepsilon_F^2}\right]\phi_K(\boldsymbol{p}),\tag{35a}$$

$$\hat{I}_p^+\phi_K(p)=\left(\frac{3\beta^2\varepsilon_F^2}{\pi^2}\left(\frac{p^2}{k_F^2}-1\right)-1\right)\frac{\partial\varepsilon_p^0}{\partial p}\cos\theta.\tag{35b}$$

利用文献[5]对碰撞积分的简化,可进一步化简上式.

$$\Sigma(\delta\cdots)=\frac{m^{*3}}{(2\pi)^3\beta^2k_F^2}\left[\int_\Delta^{\pi-\Delta}+\int_{\pi+\Delta}^{2\pi-\Delta}\right]\frac{1}{|\sin\theta|}\cdot\frac{d\theta}{2\pi}\iiint d\chi_2d\chi_3d\chi_4(\delta\cdots).\tag{36}$$

式中各符号意义同文献[5].

设

$$\phi_D(\boldsymbol{p})=Q_D(p)\cos\theta\frac{\partial\varepsilon_p^0}{\partial p},\tag{37a}$$

$$\phi_\eta(\boldsymbol{p}) = Q_\eta(p)\frac{\partial \varepsilon_p^0}{\partial p} p \sin\theta \cos\theta, \tag{37b}$$

$$\phi_K(\boldsymbol{p}) = Q_K(p)\frac{\partial \varepsilon_p^0}{\partial p} \cos\theta, \tag{37c}$$

利用(34)至(36)和(17),(18)式得到下述方程:

$$1 = \frac{1}{\tau}\int (Q_D(t) - \lambda_D Q_D(x)) K(t,x) dx, \tag{38a}$$

$$-1 = \frac{1}{\tau}\int (Q_\eta(t) - \lambda_\eta Q_\eta(x)) K(t,x) dx, \tag{38b}$$

$$-\frac{3\beta\varepsilon_F}{\pi^2} t = \frac{1}{\tau}\int (Q_K(t) - \lambda_K Q_K(x)) K(t,x) dx. \tag{38c}$$

式中

$$K(t,\ x) = \frac{e^{-t}+1}{e^{-x}+1} \cdot \frac{x-t}{e^{x-t}-1}, \tag{39}$$

$$\tau^{-1} = \frac{\langle W\rangle m^{*3}}{(2\pi)^3 \beta^2 k_F^2} \tag{40}$$

$$\lambda_\eta = \frac{1}{\langle W\rangle}\int_\Delta^{\pi-\Delta} \frac{d\theta}{2\pi} \cdot \frac{2W(\theta)}{\sin\theta} (1 + 2\cos 2\theta), \tag{41a}$$

$$\lambda_K = \frac{1}{\langle W\rangle}\int_\Delta^{\pi-\Delta} \frac{d\theta}{2\pi} \cdot \frac{2W(\theta)}{\sin\theta} (1 + 2\cos\theta), \tag{41b}$$

$$\lambda_D = \frac{1}{\langle W\rangle}\int_\Delta^{\pi-\Delta} \frac{d\theta}{2\pi} \cdot \frac{2W_1(\theta)(1+2\cos\theta) + 2W_2(\theta)(1-2\cos\theta) - 2W_3(\theta)}{\sin\theta}, \tag{41c}$$

$$\langle W\rangle = \int_\Delta^{\pi-\Delta} \frac{d\theta}{2\pi} \cdot \frac{2W(\theta)}{\sin\theta}, \tag{42}$$

(38)式可按文献[6]的方法来解出. 值得注意的是二维情形使得可以不必对散射幅作任何近似而求出 η, K, D

$$\eta = C_\eta(-T^{-2}\ln^{-1}\Delta), \tag{43a}$$

$$K = C_K(-T^{-1}\ln^{-1}\Delta), \tag{43b}$$

$$D = C_D(-T^{-2}\ln^{-1}\Delta). \tag{43c}$$

式中系数项为

$$C_\eta = \frac{k_F^6}{\pi m^{*2}} \cdot \frac{1}{|A(0)|^2 + |A(\pi)|^2} \cdot \frac{C(\lambda_\eta)}{1-\lambda_\eta},$$

$$C_K = \frac{4\pi k_F^4}{m^{*2}} \frac{1}{|A(0)|^2 + |A(\pi)|^2} \frac{H(\lambda_K)}{1-\lambda_K},$$

$$C_D = \frac{8(1+F_0^a)k_F^4}{m^{*3}} \frac{1}{|A(0)|^2 + |A(\pi)|^2} \cdot \frac{C(\lambda_D)}{1-\lambda_D},$$

$$\lambda_\eta = 3,$$

$$\lambda_K = \frac{3W(0) - W(\pi)}{W(0) + W(\pi)}$$

$$\lambda_D = \frac{4W_1(0)+4W_2(\pi)}{W(0) + W(\pi)} - 1,$$

$$C(\lambda) = \frac{\lambda - 1}{2\lambda}\Big(\gamma + \ln 2 + \frac{1}{2}\psi\Big(\frac{3}{4} + \sqrt{8\lambda + 1}\Big) + \frac{1}{2}\psi\Big(\frac{3}{4} - \sqrt{8\lambda + 1}\Big)\Big),$$

$$H(\lambda) = \frac{\lambda - 3}{2\lambda}\Big(\gamma + \ln 2 - 1 - \frac{1}{2\lambda} + \frac{1}{2}\psi\Big(\frac{1}{4} + \sqrt{8\lambda + 1}\Big) + \frac{1}{2}\psi\Big(\frac{1}{4} - \sqrt{8\lambda + 1}\Big)\Big),$$

式中 γ 为欧拉数，ψ 为普西函数.

文献 [2] 由于利用了三维的试探函数 $\phi_\eta = p_i p_j/m_i^*$ 和消除了相空间奇异性因子 $\ln\Delta$，导致粘滞系数与温度关系为 $\eta \propto T^{-2}$，与三维情形相同，不同于(43)式. 这表明试探函数选择的困难带来的熵极值法的局限性. 另外，在二维情形不必作散射幅近似就能得到输运系数，而文献[2]由于类比三维情形的做法采用了近似.

四、声传播特性

定义

$$\phi_p^+ = (1 - \hat{P}_p^+)\nu_p^+, \tag{44}$$

$$\phi_p^- = (1 - \hat{P}_p^-)\nu_p^-, \tag{45}$$

从(13)和(14)式得到

$$\frac{\partial \nu_p^+}{\partial t} + ik\frac{\partial \varepsilon_p^0}{\partial p}\phi_p^+ = \hat{I}_p^+\phi_p^+, \tag{46}$$

$$\frac{\partial \nu_p^-}{\partial t} + ik\frac{\partial \varepsilon_p^0}{\partial p}\phi_p^- = \hat{I}_p^-\phi_p^-. \tag{47}$$

(46)式描述声传播特性，而(47)式描述自旋波传播特性. 为书写方便，下面我们略去右上标“+”“−”，然后分节讨论.

设 $\nu_p \sim e^{i\omega t}$，(46)，(47)式成为

$$-i(\omega\nu_p - kv_F\cos\theta\phi_p) = I_p\phi_p, \tag{48}$$

作傅氏变换

$$\nu_p = \nu(p,\theta) = \sum_{n=0}^{\infty}(\nu_n^c \cos n\theta + \nu_n^s \sin n\theta), \tag{49}$$

类似地，对 ϕ_p，$J_p = -I_p\phi_p$ 作傅氏变换，给出系数 ϕ_n^c，ϕ_n^s 和 J_n^c，J_n^s. 同时按下式定义 ν_p，ϕ_p，J_p 在费密环上的量:

$$\nu^c_{Fn}=\int_{-\infty}^{\infty}dt\frac{dn_0}{dt}\nu^c_n(t),\tag{50}$$

波传播时，动量流为

$$\delta\Pi_{ik}=\int d\tau p_i\frac{\partial}{\partial p_k}\frac{\partial n_0}{\partial\varepsilon}\phi(p),\tag{51}$$

具体地为 $\delta\Pi_{11}=N\left(\phi^c_{F0}+\frac{1}{2}\phi^c_{F2}\right),\delta\Pi_{12}=N\frac{1}{2}\phi^s_{F2},\ \delta\Pi_{22}=N\left(\phi^c_{F0}-\frac{1}{2}\phi^c_{F2}\right)$，表明 ν^c_F 或 ϕ^c_F 波有纵波特性，ν^s_F 或 ϕ^s_F 波有横波特性.

定义一系列驰豫时间参数 τ^c_n，τ^s_n，每一分量对应一个参数

$$J^c_{Fn}=\nu^c_{Fn}/\tau^c_n,\tag{52}$$

$$J^s_{Fn}=\nu^s_{Fn}/\tau^s_n,\tag{53}$$

1. 声传播特性

由 $\hat{I}^+_F$ 的四个零本征值得到 $J_{F0}=J_{F1}=0$，因而要求(52)，(53)式中 $n\geqslant 2$. 利用(36)式得到 $J^c_{Fn}=J^s_{Fn}=0$，表明

$$\tau^{c-1}_n=\tau^{s-1}_n=0.\tag{54}$$

我们认为，在二维情形的费密液体中，只要有波模存在，总可以激发起来. 由(48)式得到

$$\nu^c_{Fm}(1+\delta_{m,0})-\sum_n F^s_n\nu^c_{Fn}(1+\delta_{n,0})\Omega^c_{mn}\left(\frac{\omega}{kv_F}\right)=0\qquad(m\geqslant 0),\tag{55a}$$

$$\nu^s_{Fm}-\sum_n F^s_n\nu^s_{Fn}\Omega^s_{mn}\left(\frac{\omega}{kv_F}\right)=0\qquad(m\geqslant 1),\tag{55b}$$

式中 Ω^c_{mn}，Ω^s_{mn} 为

$$\Omega^c_{mn}(s)=\int_0^{2\pi}\frac{d\theta}{2\pi}\cdot\frac{\cos m\theta\cos n\theta}{s-\cos\theta}\cos\theta,$$

$$\Omega^s_{mn}(s)=\int_0^{2\pi}\frac{d\theta}{2\pi}\cdot\frac{\sin m\theta\sin n\theta}{s-\cos\theta}\cos\theta.$$

纵波情形下，若略去 $F^s_l\ (l\geqslant 1)$ 的贡献，色散关系为

$$\frac{\omega}{kv_F}=\frac{1+F^s_0}{\sqrt{1+2F^s_0}},\tag{56}$$

这就是零声解，对二维 ^{3}He 因 F^s_0 较大，与理想情形相差较大. 横波情形下，必须考虑 F^s_1，这时横波模存在条件为 $F^s_1>2$，但当考虑到更高阶有效场，如 F^s_2，色散关系为

$$\Omega^s_{00}\left(\frac{\omega}{kv_F}\right)=\frac{F^s_1-2+F^s_2\left(4\left(\frac{\omega}{kv_F}\right)^2-1+\frac{F^s_1}{2}\right)}{\left[\left(\frac{\omega}{kv_F}\right)^2-1\right]\left(2F^s_1+F^s_2\left(F^s_1+8\left(\frac{\omega}{kv_F}\right)^2\right)\right)}.\tag{57}$$

表明若 F^s_2 为正存在横波模. 二维 ^{3}He 中满足这一条件.

2. 自旋波传播特性

由 $\hat{I}^-_F$ 仅有一个零本征值得到 $J_{F0}=0$，因而要求(52)，(53)式中 $n\geqslant 1$. 由(48)式

得到

$$\nu_{Fm}^c(1+\delta_{m,0})-\sum_{n=0}F_n^a\nu_{Fn}^c(1+\delta_{n,0})\Omega_{mn}^c(\xi^c)$$

$$+\frac{2}{\sigma^c}\left(\nu_{F0}^c\Gamma_{m0}^c(\xi^c)-\sum_{n=2}\beta_n^c\nu_{Fn}^c\Gamma_{mn}^c(\xi^c)\right)=0, \tag{58a}$$

$$\nu_{Fm}^s-\sum_{n=0}F_n^a\nu_{Fn}^s\Omega_{mn}^s(\xi^s)-\frac{2}{\sigma^s}\sum_{n=2}\beta_n^s\nu_{Fn}^s\Gamma_{mn}^s(\xi^s)=0, \tag{58b}$$

式中 $\beta_n^{c(s)}$, $\zeta^{c(s)}$, $\sigma^{c(s)}$, 和 Γ_{mn}^c Γ_{mn}^s 为

$$\beta_n=\tau_1/\tau_n-1, \tag{59}$$

$$\zeta=(i\omega\tau_1-1)/(ikv_F\tau_1), \tag{60}$$

$$\sigma=ikv_F\tau_1, \tag{61}$$

$$\Gamma_{mn}^c(s)=\int\frac{d\theta}{2\pi}\cdot\frac{\cos m\theta-\cos n\theta}{s-\cos\theta},$$

$$\Gamma_{mn}^s(s)=\int\frac{d\theta}{2\pi}\cdot\frac{\sin m\theta\sin n\theta}{s-\cos\theta}.$$

在二维情形下,无法对 β_n 作出一般性近似(下面将作说明),因而讨论 $\omega\tau\ll1$ 和 $\omega\tau\gg1$ 两极限情形.

(1) $\omega\tau\ll1$ 情形

对小参量 $\omega\tau$ 进行展开,在一级近似下得到

$$\frac{kv_F}{\omega}=\frac{v_F}{c}+i\gamma\frac{v_F}{\omega}, \tag{62}$$

式中 c 和 γ 纵波情形为

$$c=\frac{1+F_0^a}{2}v_F \tag{63a}$$

$$\gamma=\frac{4\omega^2\tau}{\pi^2(1+F_0^a)^2v_F}\left(2+\frac{1+F_2^a/2}{1+F_0^a}\right)\cdot\frac{C(\lambda_1)}{1-\lambda_1}. \tag{63b}$$

横波情形为

$$c=\left(\frac{2\omega\tau_3}{\pi^2F_2^a}\cdot\frac{c(\lambda_1)}{1-\lambda_1}\right)^{\frac{1}{2}}\left(1+\frac{F_2^a}{2}\right), \tag{64a}$$

$$\gamma=\left(\frac{\pi^2F_2^a\omega}{2\tau_3}\cdot\frac{1-\lambda_1}{C(\lambda_1)}\right)^{\frac{1}{2}}\frac{1}{\left(1+\frac{F_2^a}{2}\right)v_F}, \tag{64b}$$

式中 λ_1 为

$$\lambda_1=1+\frac{1}{\langle W\rangle}\int_{\Delta}^{\pi-\Delta}\frac{d\theta}{2\pi}\frac{4W_3(\pi)(\cos\theta-1)}{\sin\theta}. \tag{65}$$

在展开过程中,注意到偶 n 次的 $\nu_{Fn}^{c(s)}$ 比奇 n 次的 $\nu_{Fn}^{c(s)}$ 高一个 $(\omega\tau)^{1(1/2)}$ 量级,似可不计及 ν_{F2m+1}^c,但实际上碰撞积分中这时偶 n 次 $\nu_{Fn}^{c(s)}$ 不存在,奇 n 次的 $\nu_{Fn}^{c(s)}$ 全在一个数量级上,无法近似已找到一个与唯象参数 τ 对应的 $\tau_n^{c(s)}$.

(2) $\omega\tau \gg 1$ 情形

由(48)式有

$$\nu^c_{Fm}(1+\delta_{m0}) - \sum_{n=0}\Omega^c_{mn}\left(\frac{\omega}{kv_F}\right)F^a_n\nu^c_{Fn}(1+\delta_{n,0}) - \sum_n \frac{\nu^c_{Fn}}{i\omega\tau^c_n}\cdot\frac{\omega}{kv_F}\Gamma^c_{mn}\left(\frac{\omega}{kv_F}\right) = 0, \tag{66a}$$

$$\nu^s_{Fm} - \sum_{n=1}\Omega^s_{mn}\left(\frac{\omega}{kv_F}\right)F^a_n\nu^s_{Fn} - \sum_n \frac{\nu^s_{Fn}}{i\omega\tau^s_n}\cdot\frac{\omega}{kv_F}\cdot\Gamma^s_{mn}\left(\frac{\omega}{kv_F}\right) = 0. \tag{66b}$$

对小参量 $(\omega\tau)^{-1}$ 进行展开,纵模情形下当仅计及 F^a_0 时，F^a_0 为正有 ν^c 模，计及 F^a_1 后表明 F^a_1 为正时有 ν^c 模. 对二维 ^{3}He 液体,因 F^a_2 较大还需计及其贡献，结果表明有自旋零声纵模存在. 横模情形下,考虑到 F^a_2 色散关系同(57)式，其衰减为

$$\gamma = \frac{1}{s_0v_F\tau^s_1\tau^s_2}\cdot\frac{(1-F^a_2\Omega^s_{22}(s_0))\tau^s_2\Gamma^s_{11}(s_0) + (F^a_1\tau^s_1 + F^a_2\tau^s_2)\Gamma^s_{12}(s_0)\Omega^s_{12}(s_0) + (1-F^a_1\Omega^s_{11}(s_0))\tau^s_1\Gamma^s_{22}(s_0)}{(1-F^a_2\Omega^s_{22}(s_0))\Delta^s_{11}(s_0)F^a_1 + 2F^a_1F^a_2\Omega^s_{12}(s_0)\Delta^s_{12}(s_0) + (1-F^a_1\Omega^s_{11}(s_0))\Delta^s_{22}(s_0)F^a_2} \tag{67}$$

式中 Δ 为

$$\Delta^s_{mn}(s) = \int\frac{d\theta}{2\pi}\cdot\frac{\sin m\theta\sin n\theta}{(s-\cos\theta)^2}\cos\theta.$$

我们引入了一系列弛豫时间参数,同时注意到它们与唯象参数 τ 关系不明显,简单地引入 τ 来描述二维费密液体的传播特性是不完全的.

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THEORY OF TWO-DIMENSIONAL FERMI LIQUIDS (III) TRANSPORT PROPERTIES AND SOUND PROPAGATION

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ABSTRACT

We discuss transport properties and sound propagation of 2-D Fermi liquids. Microscopic expressions for the coefficients of diffusion, viscosity and thermal conductivity are derived using Resibois method. Velocities of the zeroth and first sounds are calculated. Based on an analysis of collision integral, it is shown that a series of relaxtion time parameters is necessary to define precisely the sound propagation properties in 2-D Fermi liquids, in contrast to the 3-D case.

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