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$n+1$ 维双 sine-Gordon 模型的 Coleman 相变

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本文计算了 $n+1$ 维双 sine-Gordon 模型的高斯有效势，在高斯有效势近似下，证明：$n \geqslant 3$ 的模型是平凡的，$n<3$ 的模型存在一个 Coleman 相变点，临界耦合参数为

$$\beta_{cr}=\frac{2^{n+2}\pi^{(n+1)/2}(\alpha_{1R}+\alpha_{2R}/4)^{(3-n)/2}}{\Gamma\left(\frac{1}{2}(3-n)\right)(\alpha_{1R}+\alpha_{2R}/16)}.$$

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一、引　言

由于对线性系统的研究及微扰论等近似理论的日趋完善，正把人们的注意力逐步引向非线性、非微扰理论的研究.双 sine-Gordon 模型在场论、非线性光学和凝聚态物理中具有广泛应用，如：超流液氦 ^{3}HeB 相中的自旋波[1,2]；通过简并介质的共振超短光波的传播[1,3]；象 TTF-TCNQ 这样的准一维有机导体中的电荷密度波[4]；铁磁链[5]和反铁磁链[6]；铁电体[7]等等. 对该模型的量子力学性质[8]和热力学性质[9]都已做了一定的研究，在一定初条件下孤子的演化[1,3,10,11]，孤子反孤子的相互作用[1,8]等也得到了许多有意义的结果.文献[12]指出，对于 1＋1 维的双 sine-Gordon 模型，当耦合常数超过某个临界值时，体系的基态成为不稳定的，模型成为非物理的，即该模型将发生 Coleman 相变. 本文将给出一般维度($n+1$维)下的高斯有效势，并由此确定 $n \geqslant 3$ 的量子双 sine-Gordon 模型是平凡的，还讨论了非平凡的 2＋1 维模型的 Coleman 相变.

二、高斯有效势

$n+1$ 维双 sine-Gordon 模型的哈密顿量为

$$H=\int d^n x\left[\frac{1}{2}\pi^2+V(\phi)\right], \tag{1}$$

式中

$$\pi\equiv\frac{\partial\phi}{\partial t}, \tag{2}$$

$$V(\phi)\equiv\sum_{i=1}^{n}\left(\frac{\partial\phi}{\partial x_i}\right)^2-\frac{\alpha_1}{\beta^2}[\cos(\beta\phi)-1]-\frac{\alpha_2}{\beta^2}\left[\cos\frac{\beta\phi}{2}-1\right]. \tag{3}$$

象 ϕ^4 模型一样，双 sine-Gordon 模型也是一个不完全可积的非线性场论模型. 对于不可积模型的量子化问题还没有完全解决，通常的微扰论方法对处理象 ϕ^4 理论的平凡性问题[13]遇到了一定的困难. 近年来，非微扰的高斯有效势方法有了更进一步的发展[14-16]. 本文将采用这种近似量子化方案将双 sine-Gordon 模型量子化. 在量子场论的薛定谔图景下，通过引入泛函导数

$$\pi(x)=-i\frac{\delta}{\delta\phi(x)}, \tag{4}$$

即可得到量子化所需要的通常的对易关系[15,16]

$$[\phi(x),\ \pi(y)]=i\delta(x-y). \tag{5}$$

采用 Ingermanson 的方法[15]，取一般的高斯波泛函为

$$\begin{aligned}&\psi[\phi,\Phi,f,P]\\&\equiv N_f\exp\left[i\int_x P(x)\phi(x)-\frac{1}{2}\int_{xy}(\phi(x)-\Phi(x))f_{xy}(\phi(y)\right.\\&\qquad\left.-\Phi(y))\right],\end{aligned} \tag{6}$$

式中 $\int_x\equiv\int d^nx$, $\int_{xy}\equiv\int d^nxd^ny$ 及 N_f 由归一化条件

$$\langle\psi|\psi\rangle\equiv\int\mathscr{D}_\phi\psi^*\psi=1 \tag{7}$$

确定。为了计算哈密顿量(1)式在高斯波泛函(6)式下的泛函平均，引入

$$\begin{aligned}|\psi\rangle_J=N_f\exp\left\{i\int_x P(x)\phi(x)-\frac{1}{2}\int_{xy}(\phi(x)-\Phi(x))f_{xy}(\phi(y)\right.\\\left.-\Phi(y))+\frac{1}{2}\int_x J(x)\phi(x)\right\}.\end{aligned} \tag{8}$$

归一化条件 ${}_J\langle\psi|\psi\rangle_J|_{J=0}=1$ 导致

$${}_J\langle\psi|\psi\rangle_J=\exp\left\{\int_x J(x)\Phi(x)+\frac{1}{4}\int_{xy}J(x)f^{-1}_{xy}J(y)\right\}, \tag{9}$$

式中 f^{-1}_{xy} 为 f_{xy} 的逆，由下式决定：

$$\int_y f_{xy}f^{-1}_{yz}=\delta(x-z). \tag{10}$$

利用(9)式易得

$$\begin{aligned}\langle\psi|\phi(x)|\psi\rangle&\equiv\int\mathscr{D}\phi\psi^*\phi(x)\psi\\&=\left[\frac{\delta}{\delta J(x)}{}_J\langle\psi|\psi\rangle_J\right]_{J=0}=\Phi(x),\end{aligned} \tag{11}$$

$$\begin{aligned}\langle\psi|\phi(x)\nabla^2\phi(x)|\psi\rangle&=\left\{\frac{\delta}{\delta J(x)}\nabla^2\frac{\delta}{\delta J(x)}{}_J\langle\psi|\psi\rangle_J\right\}_{J=0}\\&=\Phi(x)\nabla^2\Phi(x)+\frac{1}{2}\int_y\delta(x-y)\nabla^2_x f^{-1}_{xy},\end{aligned} \tag{12}$$

$$\langle\psi|\pi(x)|\psi\rangle = {}_J\left\langle\psi\right|-i\frac{\delta}{\delta\phi(x)}\left|\psi\right\rangle_J\Big|_{J=0} = P(x), \tag{13}$$

$$\langle\psi|(\pi(x))^2|\psi\rangle = {}_J\left\langle\psi\right|\left(-i\frac{\delta}{\delta\phi(x)}\right)^2\left|\psi\right\rangle_J\Big|_{J=0} = (P(x))^2 + \frac{1}{2}f_{xx}, \tag{14}$$

$$\langle\psi|e^{\pm i\beta\phi(x)}|\psi\rangle = {}_J\langle\psi|\psi\rangle_J\big|_{J(y)=\pm i\beta\delta(y-x)}$$

$$= \exp\left(-\frac{1}{4}\beta^2 f_{xx}^{-1} \pm i\beta\Phi(x)\right). \tag{15}$$

由(11)—(15)式得变分态的能量为(哈密顿量(1)式在(6)式下的泛函平均)

$$E(\Phi,P,f) \equiv \langle\psi|H|\psi\rangle$$

$$= \int_x \left\{\frac{1}{2}(P(x))^2 + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial}{\partial x_i}\Phi\right)^2 - \frac{\alpha_1}{\beta^2}\left[Z_{1m}\cos\beta\Phi(x) - 1\right]\right.$$

$$\left. - \frac{\alpha_2}{\beta^2}\left[Z_{2m}\cos\frac{\beta}{2}\Phi(x) - 1\right] + \frac{1}{4}\left[f_{xx} - \int_y \delta_{xy}\nabla_x^2 f_{xy}^{-1}\right]\right\}, \tag{16}$$

式中

$$Z_{1m} = \exp\left[-\frac{1}{4}\beta^2 f_{xx}^{-1}\right], \tag{17}$$

$$Z_{2m} = \exp\left[-\frac{1}{16}\beta^2 f_{xx}^{-1}\right]. \tag{18}$$

由于感兴趣的是有效势问题,由此可取 $\frac{\partial}{\partial x_i}\Phi(x) = 0$, 要求能量对于 $P(x)$ 取极值得 $P(x) = 0$, 最后,从变分条件

$$\frac{\delta E}{\delta f_{xy}} = 0, \tag{19}$$

得 f_{xy} 的形式解

$$f_{xy} = \int \frac{d^n p}{(2\pi)^n}\cos[\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})]\sqrt{p^2+m^2}, \tag{20}$$

$$m^2 = \alpha_1 Z_{1m}\cos\beta\Phi + \frac{\alpha_2}{4}Z_{2m}\cos\frac{1}{2}\beta\Phi \tag{21}$$

及其逆 f_{xy}^{-1} 为

$$f_{xy}^{-1} = \int \frac{d^n p}{(2\pi)^n}\frac{\cos[\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})]}{\sqrt{p^2+m^2}}. \tag{22}$$

引入记号

$$I_0(m^2) = f_{xx} = \int \frac{d^n p}{(2\pi)^n}\sqrt{p^2+m^2}, \tag{23}$$

$$I_1(m^2) = f_{xx}^{-1} = \int \frac{d^n p}{(2\pi)^n}\frac{1}{\sqrt{p^2+m^2}}, \tag{24}$$

则能量密度 $\mathscr{E}$ 可写为

$$\mathscr{E}(\Phi, m^2) = \frac{1}{2}\ I_0(m^2) - \frac{1}{4}\ m^2 I_1(m^2) - \frac{\alpha_1}{\beta^2}\ [\, \mathrm{Z}_{1m}\cos\beta\Phi - 1\,]$$

$$- \frac{\alpha_2}{\beta^2}\left[\mathrm{Z}_{2m}\cos\frac{\beta\Phi}{2} - 1\right]. \tag{25}$$

定义 $\mu(\Phi)$ 为使 $\mathscr{E}$ 取极值的 m 值，即

$$\left.\frac{\partial\mathscr{E}}{\partial(m^2)}\right|_{m^2=\mu^2(\Phi)} = 0 \tag{26}$$

及有效势 $\mathscr{V}(\Phi)$

$$\mathscr{V}(\Phi) \equiv \mathscr{E}(\Phi,\ \mu^2(\Phi)), \tag{27}$$

得

$$\mu^2(\Phi) = \alpha_1 \mathrm{Z}_{1\mu}\cos\beta\Phi + \frac{1}{4}\ \alpha_2 \mathrm{Z}_{2\mu}\cos\frac{1}{2}\ \beta\Phi \tag{28}$$

及

$$\mathscr{V}(\Phi) = \frac{1}{2}\ I_0(\mu^2) - \frac{1}{4}\ \mu^2 I_1(\mu^2) - \frac{\alpha_1}{\beta^2}\ [\, \mathrm{Z}_{1\mu}\cos\beta\Phi - 1\,]$$

$$- \frac{\alpha_2}{\beta^2}\left[\mathrm{Z}_{2\mu}\cos\frac{1}{2}\ \beta\Phi - 1\right], \tag{29}$$

式中

$$\mathrm{Z}_{1\mu} = \exp\left[-\frac{1}{4}\ \beta^2 I_1(\mu^2)\right], \tag{30}$$

$$\mathrm{Z}_{2\mu} = \exp\left[-\frac{1}{16}\ \beta^2 I_1(\mu^2)\right]. \tag{31}$$

三、重整化与 Coleman 相变条件

设 Φ_0 使 $\mathscr{V}(\Phi)$ 相对于 Φ 取极小，

$$\left.\frac{\partial\mathscr{V}}{\partial\Phi}\right|_{\Phi=\Phi_0} = \frac{\alpha_1}{\beta}\ \mathrm{Z}_{1\mu}\sin\beta\Phi_0 + \frac{1}{2}\ \frac{\alpha_2}{\beta}\ \mathrm{Z}_{2\mu}\sin\frac{1}{2}\ \beta\Phi_0 = 0, \tag{32}$$

$$\left.\frac{\partial^2\mathscr{V}}{\partial\Phi^2}\right|_{\Phi=\Phi_0} > 0, \tag{33}$$

则有

$$\Phi_0 = 0, \tag{34}$$

$$\alpha_2 \mathrm{Z}_{2\mu}|_{\Phi=0} > -4\alpha_1 \mathrm{Z}_{1\mu}|_{\Phi=0}. \tag{35}$$

由此可以定义重整化参数

$$m_R^2 \equiv \left.\frac{d^2\mathscr{V}}{d\Phi^2}\right|_{\Phi=\Phi_0} = \alpha_{1R} + \frac{1}{4}\ \alpha_{2R}, \tag{36}$$

$$-\left(\alpha_{1R} + \frac{1}{16}\ \alpha_{2R}\right)\beta_R^2 \equiv \left.\frac{d^4\mathscr{V}}{d\Phi^4}\right|_{\Phi=\Phi_0}. \tag{37}$$

使重整化量 m_R^2，β_R^2 与有效势 $\mathscr{V}(\Phi)$ 的关系和经典量 $m^2 \equiv \alpha_1 + \frac{1}{4}\alpha_2$，$\beta^2$ 与经典势 $\mathscr{V}(\phi)$ 的关系相同．由(28)和(29)式，有

$$m_R^2 = \alpha_1 Z_{1\mu}|_{\Phi=0} + \frac{1}{4}\alpha_2 Z_{2\mu}\Big|_{\Phi=0} \equiv \alpha_{1R} + \frac{1}{4}\alpha_{2R}$$

$$= \mu^2|_{\Phi=0}, \tag{38}$$

$$\beta_R^2 = \frac{(2\theta+1)(\alpha_{1R}+\alpha_{2R}/16)}{(1-\theta)(\alpha_{1R}+\alpha_{2R}/4)}\beta^2 \equiv Z_\beta\beta^2, \tag{39}$$

$$\theta = \frac{\beta^2}{8}(\alpha_{1R}+\alpha_{2R}/16)I_2(m_R^2) \tag{40}$$

及

$$\frac{\partial\mu^2}{\partial\Phi} = \frac{\beta\alpha_1 Z_{1\mu}\sin\beta\Phi + \frac{1}{8}\alpha_2 Z_{2\mu}\sin\frac{1}{2}\beta\Phi}{1-\frac{1}{8}I_2(\mu^2)\left(\beta^2\alpha_1 Z_{1\mu}\cos\beta\Phi + \frac{1}{16}\beta^2\alpha_2 Z_{2\mu}\cos\frac{1}{2}\beta\Phi\right)}, \tag{41}$$

$$I_2(\mu^2) = \int_{-\Lambda}^{+\Lambda}\frac{d^n p}{(2\pi)^n}\frac{1}{(p^2+\mu^2)^{3/2}}. \tag{42}$$

由于 $I_2(\mu^2)$ 当 $n \geqslant 3$ 时是发散的，所以从(41)式有

$$\frac{\partial\mu^2}{\partial\Phi}\Big|_{\Lambda\to\infty} = 0\,(n \geqslant 3), \tag{43}$$

也就是说 $\mu^2(\Phi)$ 为一个常质量参数，

$$\mu^2(\Phi) = m_R^2 = \mu^2(\Phi=0) \quad (n \geqslant 3). \tag{44}$$

比较(28)与(44)式可知，要使(28)和(44)式没有矛盾，仅有 $\beta_R \approx \beta = 0$，即当 $n \geqslant 3$ 时，量子的双 sine-Gordon 模型是平凡的自由场论．对于 $n = 2$ 的情况，由于

$$I_0(\mu^2) = \frac{1}{6\pi}\left[\Lambda^3\left(1+\frac{\mu^2}{\Lambda^2}\right)^{3/2} - |\mu|^3\right]$$

$$= \frac{1}{6\pi}\left[\Lambda^3 + \frac{3}{2}\Lambda\mu^2 - |\mu|^3 + o\left(\frac{1}{\Lambda}\right)\right], \tag{45}$$

$$I_1(\mu^2) = \frac{1}{2\pi}\left[\Lambda^2\left(1+\frac{\mu^2}{\Lambda^2}\right)^{\frac{1}{2}} - |\mu|\right]$$

$$= \frac{1}{2\pi}\left(\Lambda - |\mu| + o\left(\frac{1}{\Lambda}\right)\right), \tag{46}$$

$$I_2(\mu^2) = \frac{1}{2\pi}\left(\frac{1}{\mu} - \frac{1}{\mu(1+\mu^2/\Lambda^2)^{1/2}}\right) = \frac{1}{2\pi\mu}\left(1+o\left(\frac{1}{\Lambda}\right)\right), \tag{47}$$

得 2 + 1 维双 sine-Gordon 场重整化后的有效势为

$$\mathscr{V}(\Phi) = \mathscr{V}_{\text{vac}} + \frac{1}{24\pi}\mu^3 - \frac{\alpha_{1R}}{\beta^2}\left[\exp\left(-\frac{\beta^2}{8\pi}(m_R-\mu)\right)\cdot\cos\beta\Phi\right]$$

$$-\frac{\alpha_{2R}}{\beta^2}\left[\exp\left(-\frac{\beta^2}{32\pi}(m_R-\mu)\right)\cdot\cos\frac{1}{2}\beta\Phi\right], \tag{48}$$

$$\mu^2=\alpha_{1R}\exp\left[-\frac{\beta^2}{8\pi}(m_R-\mu)\right]\cdot\cos\beta\Phi+\frac{1}{4}\alpha_{2R}\exp\left[-\frac{\beta^2}{32\pi}(m_R-\mu)\right]\cdot\cos\frac{1}{2}\beta\Phi. \tag{49}$$

(48)式中 $\mathscr{V}_{\rm vac}$ 为真空能量 ($\sim\Lambda^3$).

最后,若有效势(48)式有意义,则由(26)式确定的 m^2 要求有效势取极小而不是极大,因此必须有

$$\left.\frac{\partial^2\mathscr{E}}{\partial(m^2)^2}\right|_{m^2=\mu^2(\Phi)}=\frac{I_2(\mu^2)}{8}\left[1-\frac{\beta^2}{8}I_2(\mu^2)\left(\alpha_1 Z_{1\mu}\cos\beta\Phi+\frac{\alpha_2}{16}Z_{2\mu}\cos\frac{\beta\Phi}{2}\right)\right]>0. \tag{50}$$

将 $\Phi=0$ 代入上式即得 Coleman 相变条件

$$\beta^2<\frac{16\pi(\alpha_{1R}+\alpha_{2R}/4)^{1/2}}{\alpha_{1R}+\alpha_{2R}/16}. \tag{51}$$

与文献[12] 1 + 1 维的结果比较,可以统一地表示为

$$\beta^2<\frac{2^{n+2}\pi^{\frac{n+1}{2}}\left(\alpha_{1R}+\frac{1}{4}\alpha_{2R}\right)^{\frac{3-n}{2}}}{\left(\alpha_{1R}+\frac{1}{16}\alpha_{2R}\right)\Gamma\left(\frac{1}{2}(3-n)\right)}=\beta_{\rm cr}^2, \tag{52}$$

式中 $\Gamma(x)$ 为伽马函数. 当 $\alpha_{2R}\to 0$ (或 $\alpha_{1R}\to 0$) 时, (52)式回到文献 [15] 的单 sine-Gordon 模型的结果.

四、结果与讨论

本文利用高斯有效势方法讨论了一般维度下双 sine-Gordon 模型的一些有趣性质. 分析表明: 空间维度 $n\geqslant 3$ 的量子双 sine-Gordon 模型当 $\Lambda\to\infty$ 时成为一个平凡的自由场论. 实际上对于 $n=3$ 的情况,类似于 ϕ^4 模型,很容易讨论其"Precarious"理论[14]和"Autonomous"理论[17,18]存在的可能性,然而对单 sine-Gordon 模型的研究[18]表明作这种讨论并不能得到很有意义的结果,因此,在此不进行这种讨论.

当 $n<3$ 时,对于体系的耦合参数 β 存在着一个临界值 $\beta_{\rm cr}$,当 $\beta>\beta_{\rm cr}$ 时体系的基态不再稳定(能量无下限), 即双 sine-Gordon 体系在 $\beta_{\rm cr}$ 处发生了相变. Coleman 相变条件(52)式当 $\alpha_{2R}\to 0$ (或 $\alpha_{1R}\to 0$) 时, 即回到 Ingermanson[15] 的单 sine-Gordon 模型的 Coleman 相变条件.

双 sine-Gordon 模型根据参数 α_1, α_2 的不同选择存在四种不同的特征结构[11]. 本文讨论只适用于讨论条件(35)式满足的特征结构. 至于对其它特征结构, Coleman 相变点是否存在,有限温度对 Coleman 相变点有何影响, 以及在有限温度下不同特征结构间是否会发生另一种形式的相变——结构相变等有关课题的研究正在进行中.

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THE COLEMAN PHASE TRANSITION FOR THE DOUBLE sine-GORDON MODEL IN $n+1$ DIMENSIONS

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ABSTRACT

The Gaussian effective potential of the double sine-Gordon model in $n+1$ dimensions is computed. Within the Gaussian approximation, we prove that the model is trivial for $n \geqslant 3$ and there exists a Coleman phase transition point for $n < 3$, the critical coupling parameter reads:

$$A_{cr} = \frac{2^{n+2}\pi^{(n+1)/2}(\alpha_{1R} + \alpha_{2R}/4)^{(3-n)/2}}{\Gamma\left(\frac{1}{2}(3-n)\right)(\alpha_{1R} + \alpha_{2R}/16)}.$$

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