

# 呈指数衰变驱动场的作用下二能级 原子系统的瞬态共振荧光光谱\*

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本文研究在呈指数衰变的驱动场的作用下, 二能级原子系统算符间的对易关系, 及所满足的 Bloch 方程的解。通过解 Laplace 变换后的差分方程, 计算出二能级原子系统的瞬态共振荧光光谱。并对计算结果进行分析与讨论。

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## 一、引 言

共振荧光的理论与实验一直是量子光学中令人感兴趣的研究课题。Mollow 的三峰结构理论在实验上已得到验证<sup>[1]</sup>。Mollow 理论模型是假定驱动场为恒定的情形。Huang 等人<sup>[2,3]</sup>研究了矩形脉冲激光作用下二能级原子产生的共振荧光, 且假定脉冲宽度  $\tau_p$  很大, 因此与振幅为恒定的连续场驱动是相近的。后来又有人从二能级原子 Schrödinger 方程出发<sup>[4]</sup>, 计算双曲正割脉冲激光作用下的二能级共振荧光光谱, 但完全略去原子在能级间的弛豫, 以及由此而引起的反转粒子的衰变等。另一方面从近年关于含原子腔的 Q.E.D. (Quantum Electro-dynamics) 研究来看<sup>[5,6]</sup>, 也需要知道在给定腔内初始光子数情况下, 二能级原子共振荧光及腔内辐射场随时间的衰变行为。本文不涉及含原子腔的 Q.E.D 问题, 有兴趣的是在驱动场呈指数衰变情况下二能级原子的瞬态荧光光谱。显然这对前者的研究是有益的。

在 Mollow 的定态共振荧光理论中<sup>[1]</sup>, 主要求解二能级原子系统满足的包括驱动场与原子相互作用在内的 Bloch 方程。体现热库无规力作用项未包括进去。而无规力的引进, 是保证原子算符对易规则所必需的<sup>[7]</sup>。实际上无规力作用也隐含在共振荧光计算中所用到的量子回归理论中<sup>[8]</sup>。对于瞬态共振荧光计算, 本文在文献 [7] 基础上导出原子算符满足的 Langevin 方程。求其形式解, 定义扩散矩阵, 最后导出瞬态共振荧光光谱的计算公式。并具体应用于呈指数衰变驱动场作用下, 二能级原子瞬态共振荧光光谱的计算。而无规力一开始就包含进去。

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## 二、二能级原子系统满足的 Langevin 方程及其解

本文主要研究单模辐射场与原子的相互作用。参照文献 [7], 可导出二能级原子系统与单模驱动场、与热库相互作用的 Langevin 方程如下:

$$\begin{aligned} \frac{d}{dt} \sigma_x &= -\gamma_1 \sigma_x - i\hat{Q}/2\sigma^- + i\hat{Q}/2\sigma^+ + \gamma_1 \sigma_x + \Gamma_x, \\ \frac{d}{dt} \sigma^- &= -i\hat{Q}\sigma_x - (\gamma_2 + i\omega_0)\sigma^- + \Gamma^-, \\ \frac{d}{dt} \sigma^+ &= i\hat{Q}\sigma_x - (\gamma_2 - i\omega_0)\sigma^+ + \Gamma^+, \end{aligned} \quad (1)$$

式中  $\sigma_x, \sigma^\pm$  为二能级原子系统的自旋算符, 是通过二能级原子处于基态与激发态的产生与湮没算符  $a_i^\dagger, a_i$  来定义的。

$$\begin{aligned} \sigma_x &= \frac{1}{2} (a_2^\dagger a_2 - a_1^\dagger a_1), \\ \sigma^- &= a_1^\dagger a_2, \quad \sigma^+ = a_2^\dagger a_1. \end{aligned} \quad (2)$$

(1) 式中  $\bar{\sigma}_x$  为外场为零时, 原子系统半反转粒子数的稳态值。  $\Gamma_x, \Gamma^\pm$  为无规力。文献 [7] 中证明了无规力的引进是保证原子算符间在统计平均的意义上满足对易关系所必需的。设驱动场的振幅为  $b, b^*$ ;  $\hat{Q}$  的定义为

$$\hat{Q} = 2g(b + b^*) = \Omega(e^{i\omega t} + e^{-i\omega t}), \quad (3)$$

$\Omega$  为 Rabi 频率,  $\omega_0$  为原子的跃迁频率,

$$\omega_0 = (E_2 - E_1)/\hbar. \quad (4)$$

设外场频率  $\omega$  与  $\omega_0$  为共振, 对 (1) 式取旋波近似

$$\begin{aligned} \sigma^\pm &\rightarrow \sigma^\pm e^{\pm i\omega_0 t}, \quad \sigma_x \rightarrow \sigma_x, \\ \Gamma^\pm &\rightarrow \Gamma^\pm e^{\pm i\omega_0 t}, \quad \Gamma_x \rightarrow \Gamma_x, \end{aligned}$$

得

$$\frac{d\sigma}{dt} = -M\sigma + \gamma\bar{\sigma} + \Gamma, \quad (5)$$

式中

$$\begin{aligned} \sigma &= \begin{pmatrix} \sigma_x \\ \sigma^- \\ \sigma^+ \end{pmatrix}, \quad \gamma\bar{\sigma} = \begin{pmatrix} \gamma_1 \bar{\sigma}_x \\ \\ \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_x \\ \Gamma^- \\ \Gamma^+ \end{pmatrix}, \\ -M &= \begin{pmatrix} -\gamma_1 & -i\Omega/2 & i\Omega/2 \\ -i\Omega & -\gamma_2 & \\ i\Omega & & -\gamma_2 \end{pmatrix}. \end{aligned} \quad (6)$$

(5) 式的解  $\sigma$  可写为

$$\sigma = \sigma_s + L, \quad \frac{d\sigma_s}{dt} = -M\sigma_s + \gamma\bar{\sigma}, \quad (7)$$

$$\frac{dL}{dt} = -ML + \Gamma. \quad (8)$$

(7),(8) 式的形式解为

$$\begin{aligned} \sigma_s(t+\tau) &= U(t,\tau)\sigma_s(t) + V(t,\tau)\gamma\bar{\sigma}, \\ L(t+\tau) &= U(t,\tau)L(t) + \int_t^{t+\tau} U(t',\tau)\Gamma(t')dt', \\ U &= e^{-\int_t^{t+\tau} Mdt'}, \quad V = \int_t^{t+\tau} U(t',\tau)dt'. \end{aligned} \quad (9)$$

用右上角“r”表示行与列对换的转置矩阵,并定义扩散矩阵  $D(t,\tau)$  为

$$D(t,\tau) = \langle L(t)L^r(t+\tau) \rangle. \quad (10)$$

用(9)式  $L(t+\tau)$  的解代入得

$$D(t,\tau) = \langle L(t)L^r(t) \rangle U^r(t,\tau) + \int_t^{t+\tau} \langle L(t)\Gamma^r(t') \rangle U^r(t',\tau) dt'.$$

由于  $t \neq t'$ ,  $\langle L(t)\Gamma^r(t') \rangle = 0$ , 故上式可写为

$$D(t,\tau) = D(t,0)U^r(t,\tau). \quad (11)$$

而

$$\begin{aligned} \langle \sigma(t)\sigma^r(t) \rangle &= \langle (\sigma_s(t) + L(t))(\sigma_s^r(t) + L^r(t)) \rangle \\ &= \langle \sigma_s(t)\sigma_s^r(t) \rangle + D(t,0), \end{aligned} \quad (12)$$

$$\begin{aligned} \langle \sigma(t)\sigma^r(t+\tau) \rangle &= \langle \sigma_s(t)\sigma_s^r(t+\tau) \rangle + D(t,\tau) \\ &= \langle \sigma_s(t)\sigma_s^r(t) \rangle U^r(t,\tau) + \sigma_s(t)\gamma\sigma^r V^r(t,\tau) \\ &\quad + (\langle \sigma(t)\sigma^r(t) \rangle - \langle \sigma_s(t)\sigma_s^r(t) \rangle) U^r(t,\tau) \\ &= \langle \sigma(t)\sigma^r(t) \rangle U^r(t,\tau) + \sigma_s(t)\gamma\sigma^r V^r(t,\tau). \end{aligned} \quad (13)$$

由(13)式得知,原子算符相关函数  $\langle \sigma(t)\sigma^r(t+\tau) \rangle$  的计算最终归结为  $\langle \sigma(t)\sigma^r(t) \rangle$  的计算. 已知由(2)式定义的自旋算符  $\sigma^\pm, \sigma_z$  满足如下的对易关系:

$$\begin{aligned} \langle \sigma^+\sigma^- - \sigma^-\sigma^+ \rangle &= 2\langle \sigma_z \rangle, \quad \langle \sigma^+\sigma^- + \sigma^-\sigma^+ \rangle = 1, \\ \langle \sigma^+\sigma_z - \sigma_z\sigma^+ \rangle &= \mp \langle \sigma^\pm \rangle, \quad \langle \sigma^\pm\sigma_z + \sigma_z\sigma^\pm \rangle = 0, \\ \langle \sigma^{-2} \rangle &= \langle \sigma^{+2} \rangle = 0, \quad \langle \sigma_z^2 \rangle = 1/4, \end{aligned} \quad (14)$$

得

$$\begin{aligned} \langle \sigma\sigma^r \rangle &= \begin{bmatrix} \langle \sigma_z\sigma_z \rangle & \langle \sigma_z\sigma^- \rangle & \langle \sigma_z\sigma^+ \rangle \\ \langle \sigma^-\sigma_z \rangle & \langle \sigma^{-2} \rangle & \langle \sigma^-\sigma^+ \rangle \\ \langle \sigma^+\sigma_z \rangle & \langle \sigma^+\sigma^- \rangle & \langle \sigma^{+2} \rangle \end{bmatrix} \\ &= \begin{bmatrix} 1/4 & -\langle \sigma^- \rangle/2 & \langle \sigma^+ \rangle/2 \\ \langle \sigma^+ \rangle/2 & 0 & \frac{1}{2} - \langle \sigma_z \rangle \\ -\langle \sigma^+ \rangle/2 & \frac{1}{2} + \langle \sigma_z \rangle & 0 \end{bmatrix}. \end{aligned} \quad (15)$$

又由于  $\langle \sigma \rangle = \sigma_s + \langle L \rangle = \sigma_s$ ,

故有

$$\langle \sigma \sigma^* \rangle = \begin{bmatrix} 1/4 & -\sigma_i^-/2 & \sigma_i^+/2 \\ \sigma_i^-/2 & 0 & \frac{1}{2} + \sigma_{ii} \\ -\sigma_i^+/2 & \frac{1}{2} + \sigma_{ii} & 0 \end{bmatrix}. \quad (16)$$

由(13),(16)式,  $\langle \sigma(t)\sigma^*(t+\tau) \rangle$  的计算归结为  $\sigma_i$ ,  $U(t, \tau)$ ,  $V(t, \tau)$  的计算.

在求得  $\langle \sigma(t)\sigma^*(t+\tau) \rangle$  的基础上可进一步计算二能级原子的瞬态共振荧光光谱.

参照通常求变量  $x(t)$  的谱的方法<sup>[8]</sup>, 先定义谱

$$y(\omega) = \int_0^t e^{-i\omega t'} x(t') dt',$$

再定义谱密度

$$\begin{aligned} S(\omega) &= \lim_{t \rightarrow \infty} \frac{1}{2\pi t} |y(\omega)|^2 \\ &= \lim_{t \rightarrow \infty} \frac{1}{\pi t} \operatorname{Re} \int_0^t e^{-i\omega \tau} d\tau \int_0^{t-\tau} \langle x(t')x(t'+\tau) \rangle dt' \Big|_{t=i\omega}. \end{aligned}$$

定义谱密度矩阵如下:

$$\begin{aligned} G(\omega) &= \frac{1}{\pi t} \operatorname{Re} \int_0^t e^{-i\omega \tau} d\tau \int_0^{t-\tau} \langle \sigma(t')\sigma^*(t'+\tau) \rangle dt' \Big|_{t=i\omega} \\ &= \frac{1}{\pi t} \operatorname{Re} \left\{ \int_0^t e^{-i\omega \tau} d\tau \int_0^{t-\tau} \langle \sigma(t')\sigma^*(t') \rangle u^*(t', \tau) dt' \right. \\ &\quad \left. + \int_0^t e^{-i\omega \tau} d\tau \int_0^{t-\tau} \langle \sigma(t')\gamma\bar{\sigma} \rangle V^*(t', \tau) dt' \right\} \Big|_{t=i\omega}. \end{aligned} \quad (17)$$

当  $t$  很大时, 上式中  $\int_0^{t-\tau} \approx \int_0^t$ , 故有

$$\begin{aligned} G(\omega) &\approx \frac{1}{\pi t} \operatorname{Re} \left\{ \int_0^t \langle \sigma(t')\sigma^*(t') \rangle dt' \int_0^t e^{-i\omega \tau} u^*(t', \tau) d\tau \right. \\ &\quad \left. + \int_0^t \langle \sigma(t')\gamma\bar{\sigma} \rangle dt' \int_0^t e^{-i\omega \tau} V^*(t', \tau) d\tau \right\} \Big|_{t=i\omega}. \end{aligned} \quad (18)$$

瞬态共振荧光光谱所涉及的  $\langle \sigma^+(t')\sigma^-(t'+\tau) \rangle$  即(17),(18)式的  $G_{32}(\omega)$  矩阵元.

### 三、驱动场呈指数衰变情形的瞬态共振荧光

上节虽给出线性方程(5)的通解, 但应用很困难. 当驱动场呈指数衰变时, 发现可以解经 Laplace 变换后的差分方程, 整个计算得以简化. 作为预备, 先讨论一些数学问题.

#### 1. 一维问题

$$\frac{d\sigma}{dt} = -M e^{-\eta t} \sigma, \quad (19)$$

因子  $e^{-\eta t}$  体现驱动场的衰变. 这方程有两种解法. 其一为直接积分

$$\sigma = e^{-\frac{M}{\eta}(1-e^{-\eta t})} \sigma_0 = \sum_n \frac{1}{n!} \left(\frac{-M}{\eta}\right)^n (1-e^{-\eta t})^n. \quad (20)$$

另一方法为对 (19) 式作 Laplace 变换

$$\begin{aligned} s\bar{\sigma}(s) - \sigma_0 &= -M\bar{\sigma}(s + \eta), \quad \bar{\sigma}(s) = \int_0^{\infty} e^{-st} \sigma dt, \\ \bar{\sigma}(s) &= \frac{\sigma_0}{s} - \frac{M}{s} \bar{\sigma}(s + \eta) \\ &= \frac{\sigma_0}{s} + \frac{\sigma_0}{s} \frac{-M}{s + \eta} + \dots + \frac{\sigma_0}{s} \frac{(-M)^n}{s(s + \eta) \dots (s + n\eta)} + \dots \end{aligned} \quad (21)$$

注意到

$$\frac{1}{s(s + \eta) \dots (s + n\eta)} = \frac{1}{\eta^n} \sum_k \frac{(-1)^k}{k!(n-k)!} \frac{1}{s + k\eta},$$

于是有

$$\bar{\sigma}(s) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-M}{\eta}\right)^n \sum_{k=0}^n \frac{(-1)^k n!}{k!(n-k)!} \frac{1}{s + k\eta} \sigma_0. \quad (22)$$

对 (22) 式求反变换得

$$\sigma(t) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-M}{\eta}\right)^n (1 - e^{-\eta t})^n \sigma_0, \quad (23)$$

与 (20) 式一致。

## 2. 三维衰变运动

将 (19) 式推广到三维

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma^- \\ \sigma^+ \end{pmatrix}, \quad M = \begin{pmatrix} -i/2 & i/2 \\ -i & \\ i & \end{pmatrix} Q, \quad (24)$$

解的形式仍为 (23) 式。式中的  $M$  按 (24) 式定义。易证

$$M^2 = \begin{pmatrix} -1 & & \\ & -1/2 & 1/2 \\ & 1/2 & -1/2 \end{pmatrix} = -E, \quad (25)$$

$$M^4 = E, \quad E^2 = E, \quad EM = M. \quad (26)$$

于是有

$$\begin{aligned} \sigma(t) &= \sum_{n=0}^{\infty} \frac{(-Q^2)^n}{(2n)!} \left(\frac{1 - e^{-\eta t}}{\eta}\right)^{2n} E \sigma_0 \\ &\quad + \sum_{n=0}^{\infty} \frac{(-1)^n Q^{2n+1}}{(2n+1)!} \left(\frac{1 - e^{-\eta t}}{\eta}\right)^{2n+1} M \sigma_0. \end{aligned} \quad (27)$$

定义

$$C(Q, \eta, t) = \sum_{n=0}^{\infty} \frac{(-Q^2)^n}{(2n)!} \left( \frac{1 - e^{-\eta t}}{\eta} \right)^{2n},$$

$$S(Q, \eta, t) = \sum_{n=0}^{\infty} \frac{(-1)^n Q^{2n+1}}{(2n+1)!} \left( \frac{1 - e^{-\eta t}}{\eta} \right)^{2n+1} \quad (28)$$

很明显, 当  $\eta \rightarrow 0$  时,

$$C(Q, \eta, t) \rightarrow \cos Qt, \quad \sin(Q, \eta, t) \rightarrow \sin Qt. \quad (29)$$

采用记号 (28) 式后, (27) 式可简写为

$$\sigma(t) = (CE + SM)\sigma_0, \quad (30)$$

即

$$\begin{pmatrix} \sigma_x \\ \sigma^- \\ \sigma^+ \end{pmatrix} = C \begin{pmatrix} \sigma_{x0} \\ 1/2(\sigma_0^- - \sigma_0^+) \\ -1/2(\sigma_0^- - \sigma_0^+) \end{pmatrix} + S \begin{pmatrix} -i/2(\sigma_0^- - \sigma_0^+) \\ -i\sigma_{x0} \\ i\sigma_{x0} \end{pmatrix}. \quad (31)$$

注意到将 (30) 式推广到  $\sigma(t + \tau)$  与  $\sigma(t)$  的情形

$$\sigma(t + \tau) = (CE + SM)\sigma(t),$$

$$C = C(Qe^{-\eta\tau}, \eta, \tau), \quad S = S(Qe^{-\eta\tau}, \eta, \tau). \quad (32)$$

这时  $C, S$  一般为  $t, \tau$  的函数. 而不只是一个变量  $\tau$  的函数.

### 3. 含阻尼和稳态项的三维衰变运动

将三维衰变运动推广到含有阻尼和稳态项的情形.

$$\frac{d\sigma}{dt} = -\gamma(\sigma - \bar{\sigma}) + QMe^{-\eta t}\sigma, \quad (33)$$

式中

$$\gamma = \begin{pmatrix} \gamma_1 & & \\ & \gamma_2 & \\ & & \gamma_2 \end{pmatrix}, \quad \bar{\sigma} = \begin{pmatrix} \bar{\sigma}_x \\ 0 \\ 0 \end{pmatrix}, \quad M = \begin{pmatrix} -i/2 & i/2 \\ -i & \\ & i \end{pmatrix}. \quad (34)$$

对 (33) 式作 Laplace 变换

$$(s + \gamma)\bar{\sigma}(s) - \sigma_0 = \frac{1}{s} \gamma \bar{\sigma} + QM\bar{\sigma}(s + \eta),$$

$$\bar{\sigma}(s) = \left( \frac{1}{s + \gamma} + \frac{1}{s + \gamma} M \frac{Q}{s + \eta + \gamma} \right. \\ \left. + \frac{1}{s + \gamma} M \frac{Q}{s + \eta + \gamma} M \frac{Q}{s + 2\eta + \gamma} + \dots \right) (\sigma_0 - \bar{\sigma}) \\ + \left( \frac{1}{s} + \frac{Q}{s + \gamma} M \frac{1}{s + \eta} \right. \\ \left. + \frac{Q}{s + \gamma} M \frac{Q}{s + \eta + \gamma} M \frac{1}{s + 2\eta} + \dots \right) \bar{\sigma}. \quad (35)$$

若记

$$D(s + \gamma) = \begin{pmatrix} \frac{1}{s + \gamma_1} & & \\ & \frac{1}{s + \gamma_2} & \\ & & \frac{1}{s + \gamma_2} \end{pmatrix}$$

$$\tilde{D}(s + \gamma) = \begin{pmatrix} \frac{1}{s + \gamma_2} & & \\ & \frac{1}{s + \gamma_1} & \\ & & \frac{1}{s + \gamma_1} \end{pmatrix}, \quad (36)$$

易证  $D(s + \gamma)M = M\tilde{D}(s + \gamma)$ ,  $\tilde{D}(s + \gamma) = D(s + \gamma)$ . (37)

于是

$$\begin{aligned} & \frac{1}{s + \gamma} M \frac{Q}{s + \eta + \gamma} \cdots M \frac{Q}{s + n\eta + \gamma} \\ & = D_0 M D_1 \cdots M D_n (Q)^n \\ & = \begin{cases} M^n Q^n \tilde{D}_0 D_1 \cdots D_n & (n \text{ 为奇数}); \\ M^n Q^n D_0 \tilde{D}_1 \cdots D_n & (n \text{ 为偶数}). \end{cases} \end{aligned} \quad (38)$$

由 (35), (38) 式得

$$\begin{aligned} \bar{\sigma}(s) &= \sum_{n=0}^{\infty} Q^{2n} M^{2n} D_0 \tilde{D}_1 D_2 \cdots D_{2n} \left( (\sigma_0 - \bar{\sigma}) + D_{2n}^{-1} \frac{1}{s + 2n\eta} \bar{\sigma} \right) \\ &+ \sum_{n=0}^{\infty} Q^{2n+1} M^{2n+1} \tilde{D}_0 D_1 \tilde{D}_2 \cdots D_{2n+1} \\ &\times \left( (\sigma_0 - \bar{\sigma}) + D_{2n+1}^{-1} \frac{1}{s + (2n+1)\eta} \bar{\sigma} \right). \end{aligned} \quad (39)$$

由于  $D_{2n}^{-1} = s + 2n\eta + \gamma$ ,  $D_{2n+1}^{-1} = s + (2n+1)\eta + \gamma$ ,  
(39) 式可化简为

$$\begin{aligned} \bar{\sigma}(s) &= \sum_{n=0}^{\infty} Q^{2n} M^{2n} D_0 \tilde{D}_1 D_2 \cdots D_{2n} \left( \sigma_0 + \gamma \frac{1}{s + 2n\eta} \bar{\sigma} \right) \\ &+ \sum_{n=0}^{\infty} Q^{2n+1} M^{2n+1} \tilde{D}_0 D_1 \tilde{D}_2 \cdots D_{2n+1} \\ &\times \left( \sigma_0 + \gamma \frac{1}{s + (2n+1)\eta} \bar{\sigma} \right). \end{aligned} \quad (40)$$

由 (26) 式

$$\begin{aligned} M^{2n} &= (-1)^n E, \\ M^{2n+1} &= (-1)^n M, \quad n \neq 0. \end{aligned} \quad (41)$$

故 (40) 式可写为

$$\bar{\sigma}(s) = (1 - E) D_0 \left( \sigma_0 + \gamma \frac{1}{s} \bar{\sigma} \right)$$

$$\begin{aligned}
& + E(\mathcal{J}_1\sigma_0 + \mathcal{J}_3\gamma\bar{\sigma}) + \Omega M(\mathcal{J}_1\sigma_0 + \mathcal{J}_4\gamma\bar{\sigma}), \quad (42) \\
\mathcal{J}_1 &= \sum_{n=0}^{\infty} (-1)^n \Omega^{2n} D_0 \tilde{D}_1 D_2 \cdots D_{2n}, \\
\mathcal{J}_2 &= \sum_{n=0}^{\infty} (-1)^n \Omega^{2n} \tilde{D}_0 D_1 \tilde{D}_2 \cdots D_{2n+1}, \\
\mathcal{J}_3 &= \sum_{n=0}^{\infty} (-1)^n \Omega^{2n} D_0 \tilde{D}_1 D_2 \cdots D_{2n} \frac{1}{s + 2n\eta}, \\
\mathcal{J}_4 &= \sum_{n=0}^{\infty} (-1)^n \Omega^{2n} \tilde{D}_0 D_1 \tilde{D}_2 \cdots D_{2n+1} \frac{1}{s + (2n+1)\eta}. \quad (43)
\end{aligned}$$

$\mathcal{J}_1, \mathcal{J}_4$  的计算由附录 A 给出, 均为对角矩阵。且有

$$\mathcal{J}_j = \begin{pmatrix} i_j & & \\ & \bar{i}_j & \\ & & \bar{i}_j \end{pmatrix} \quad j = 1-4. \quad (44)$$

将  $i_j$  中包含的参数  $\gamma_1, \gamma_2$  对换后, 得  $\bar{i}_j$ 。

经过复杂计算得出 (42) 式的解为

$$\begin{aligned}
\begin{pmatrix} \bar{\sigma}_x \\ \bar{\sigma}^- \\ \bar{\sigma}^+ \end{pmatrix} &= \begin{pmatrix} i_1 \sigma_{x0} \\ \frac{1}{2} \bar{i}_1 (\sigma_0^- - \sigma_0^+) \\ -\frac{1}{2} \bar{i}_1 (\sigma_0^- - \sigma_0^+) \end{pmatrix} + \begin{pmatrix} -i\Omega/2 \bar{i}_2 (\sigma_0^- - \sigma_0^+) \\ -i\Omega i_2 \sigma_{x0} \\ i\Omega i_2 \sigma_{x0} \end{pmatrix} + \begin{pmatrix} i_3 \gamma_1 \bar{\sigma}_x \\ 0 \\ 0 \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ -i\Omega i_4 \gamma_1 \bar{\sigma}_x \\ i\Omega i_4 \gamma_1 \bar{\sigma}_x \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{2} \frac{\sigma_0^- + \sigma_0^+}{s + \gamma_2} \\ \frac{1}{2} \frac{\sigma_0^- + \sigma_0^+}{s + \gamma_2} \end{pmatrix}. \quad (45)
\end{aligned}$$

设  $i_j$  的逆变换为  $I_j$ , 则求 (45) 式的逆变换后, 得

$$\sigma^-(\tau) = \frac{1}{2} \bar{I}_1(\tau) (\sigma_0^- - \sigma_0^+) - i I_2(\tau) \Omega \sigma_{x0} + \frac{1}{2} e^{-\tau s} (\sigma_0^- + \sigma_0^+) - i I_4(\tau) \Omega \gamma_1 \bar{\sigma}_x \quad (46)$$

或

$$\begin{aligned}
\sigma^-(t + \tau) &= \frac{1}{2} \bar{I}_1(\tau) (\sigma_0^-(t) - \sigma_0^+(t)) - i I_2(\tau) \Omega(t) \sigma_{x0}(t) \\
&+ \frac{1}{2} e^{-\tau s} (\sigma_0^-(t) + \sigma_0^+(t)) - i I_4(\tau) \Omega(t) \gamma_1 \bar{\sigma}_x, \quad \Omega(t) = \Omega e^{-\Psi}. \quad (46)'
\end{aligned}$$

故有

$$\begin{aligned}
\langle \sigma^+(t) \sigma^-(t + \tau) \rangle &= \frac{1}{2} (\bar{I}_1(\tau) + e^{-\tau s}) \langle \sigma^+(t) \sigma^-(t) \rangle \\
&- i I_2(\tau) \Omega(t) \langle \sigma^+(t) \sigma_x(t) \rangle - i I_4(\tau) \Omega(t) \gamma_1 \bar{\sigma}_x \langle \sigma^+(t) \rangle. \quad (47)
\end{aligned}$$

参照 (17), (18) 式, 得出瞬态共振荧光光谱为



$$\begin{aligned}
G_{32}(\omega) &= \frac{1}{\pi i} \operatorname{Re} \left\{ \int_0^t e^{-i\tau} d\tau \int_0^{t-\tau} \langle \sigma^+(t') \sigma^-(t'+\tau) \rangle dt' \right\}_{s=i\omega} \\
&\approx \frac{1}{\pi} \operatorname{Re} \left\{ \frac{1}{i} \int_0^t \langle \sigma^+(t') \sigma^-(t') \rangle dt' \int_0^t e^{-i\tau} \left( \frac{1}{2} (\bar{I}_1(\tau) + e^{-\gamma_2 \tau}) \right) d\tau \right. \\
&\quad - i \frac{1}{i} \int_0^t Q(t') \langle \sigma^+(t') \sigma_x(t') \rangle dt' \int_0^t e^{-i\tau} I_2(\tau) d\tau \\
&\quad \left. - i \frac{1}{i} \int_0^t Q(t') \langle \sigma^+(t') \rangle \gamma_1 \bar{\sigma}_z dt' \int_0^t e^{-i\tau} I_4(\tau) d\tau \right\}_{s=i\omega}, \quad (48)
\end{aligned}$$

式中  $\langle \sigma^+(t') \sigma^-(t') \rangle$ ,  $\langle \sigma^+(t') \sigma_x(t') \rangle$ ,  $\langle \sigma^+(t') \rangle$  按 (16) 式均可用  $\sigma_s(t')$  来表示。而  $\sigma_s(t')$  即 (45) 式的逆变换,

$$\begin{aligned}
\begin{pmatrix} \sigma_{\sigma_3}(t) \\ \sigma_{\sigma_1}(t) \\ \sigma_{\sigma_2}(t) \end{pmatrix} &= \begin{pmatrix} I_1(t) \\ \frac{1}{2} \bar{I}_1(t) (\sigma_0^- - \sigma_0^+) \\ -\frac{1}{2} \bar{I}_1(t) (\sigma_0^- - \sigma_0^+) \end{pmatrix} + \begin{pmatrix} -iQ/2 \bar{I}_2(t) (\sigma_0^- - \sigma_0^+) \\ -iQ I_2(t) \sigma_{x0} \\ iQ I_2(t) \sigma_{x0} \end{pmatrix} + \begin{pmatrix} I_3(t) \gamma_1 \bar{\sigma}_z \\ 0 \\ 0 \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ -iQ I_4(t) \gamma_1 \sigma_z \\ iQ I_4(t) \gamma_1 \sigma_z \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{2} \frac{\sigma_0^- + \sigma_0^+}{s + \gamma_2} \\ \frac{1}{2} \frac{\sigma_0^- + \sigma_0^+}{s + \gamma_2} \end{pmatrix}. \quad (49)
\end{aligned}$$

附录 A 给出  $I_1(t) - I_4(t)$  的计算公式。

#### 四、计算结果与分析

1. 图 1 为光抽运脉冲曲线。假定脉冲上升得很快, 以致前沿可略去。当上升到峰值  $Q$  后, 再按  $Q(t) = Q e^{-\eta t}$  衰减。

2. 参照 (48) 式, 定义如下归一化的参量:

$$\begin{aligned}
\text{IMC} &= \frac{1}{i} \int_0^t \langle \sigma^+(t') \sigma^-(t') \rangle dt' / \\
&\quad \langle \sigma^+(\infty) \sigma^-(\infty) \rangle,
\end{aligned}$$

$$\begin{aligned}
\text{INC} &= \frac{1}{i} \int_0^t e^{-\eta t'} \langle \sigma^+(t') \rangle dt' / \\
&\quad \langle \sigma^+(\infty) \rangle,
\end{aligned}$$

$$\langle \sigma^+(\infty) \sigma^-(\infty) \rangle = \frac{Q^2/2}{\gamma_1^2/2 + Q^2}, \quad \langle \sigma^+(\infty) \rangle = \frac{-i(\gamma_1/2)Q}{\gamma_1^2/2 + Q^2}, \quad (50)$$

初值取为  $\langle \sigma_0^+ \rangle = \langle \sigma_0^- \rangle = 0$ ,  $\langle \sigma_{x0} \rangle = \langle \bar{\sigma}_z \rangle = -1/2$ , (51)

IMC, INC 随时间变化的曲线由图 2 给出。

图 3—图 5 分别给出  $\eta = 0.2, 0.5, 1$  的瞬态共振荧光光谱。每一图中又包含  $Q =$

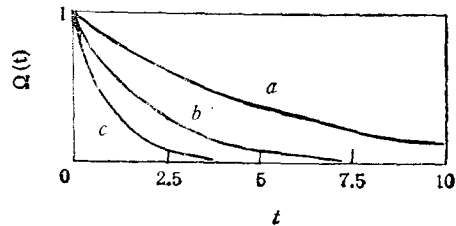


图 1 光抽运脉冲图  $Q(t) = Q e^{-\eta t}$ ;  
a, b, c 分别对应于  $\gamma_1 = 0.2, 0.5, 1$

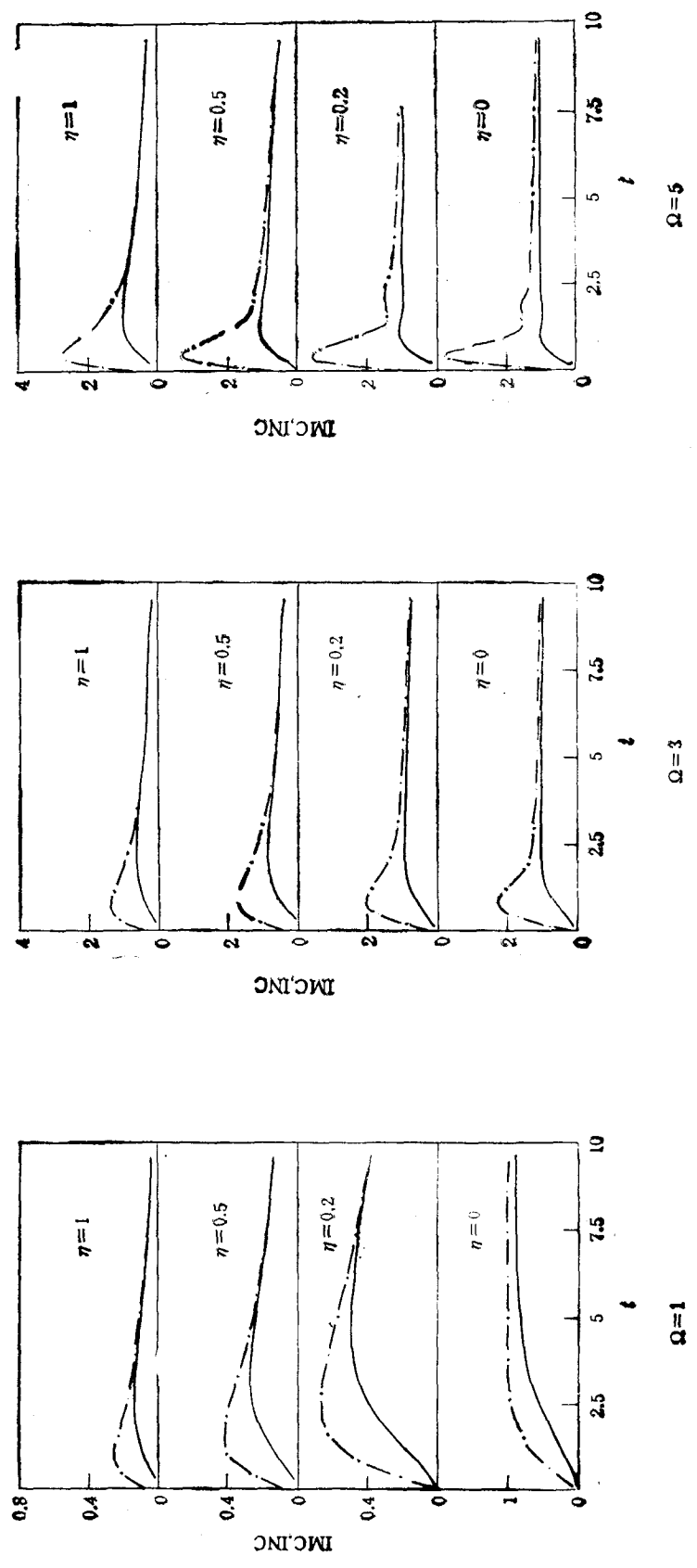


图2 IMC, INC 随时间 t 变化的曲线 —— 为 IMC; - · - 为 INC

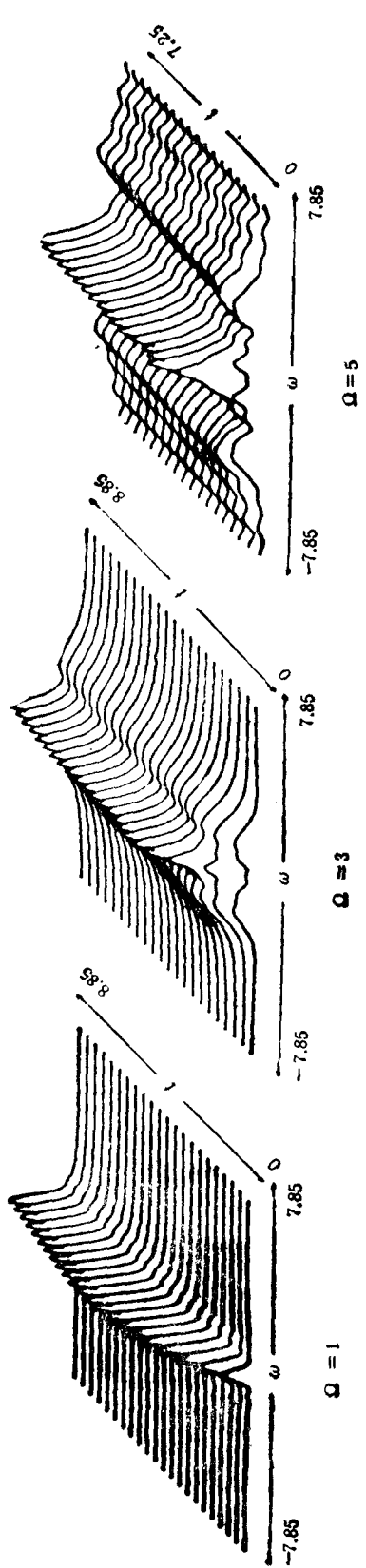


图3 瞬态共振荧光光谱 ( $\eta = 0.2$ )  $\Omega = 1$ ;  $\Omega = 3$ ;  $\Omega = 5$

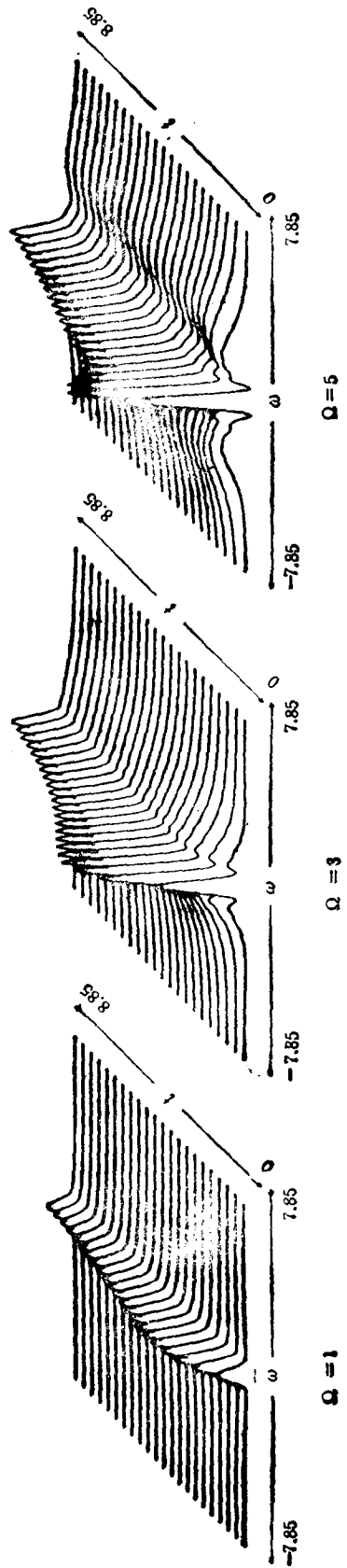


图4 瞬态共振荧光光谱 ( $\eta = 0.5$ )  $\Omega = 1$ ;  $\Omega = 3$ ;  $\Omega = 5$

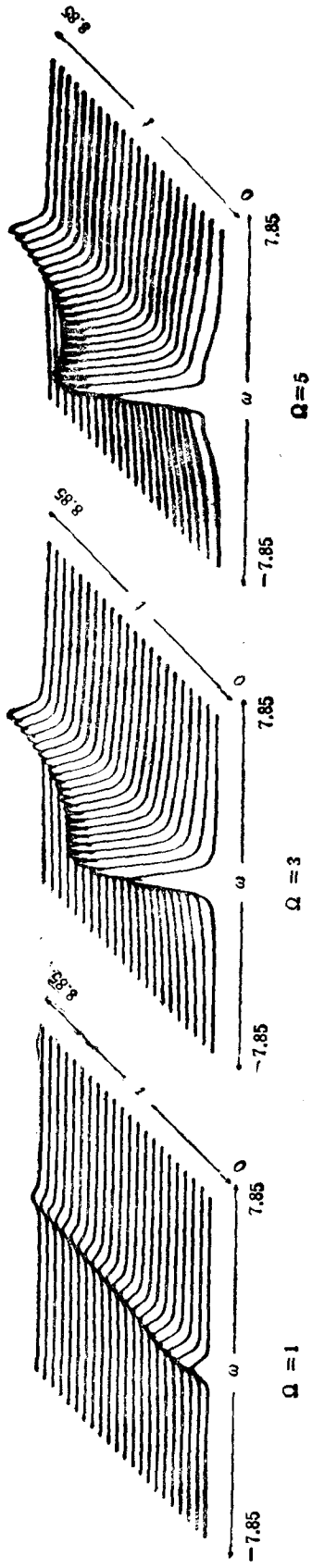


图5 瞬态共振荧光光谱 ( $\eta = 1.0$ )  $Q = 1$ ;  $Q = 3$ ;  $Q = 5$

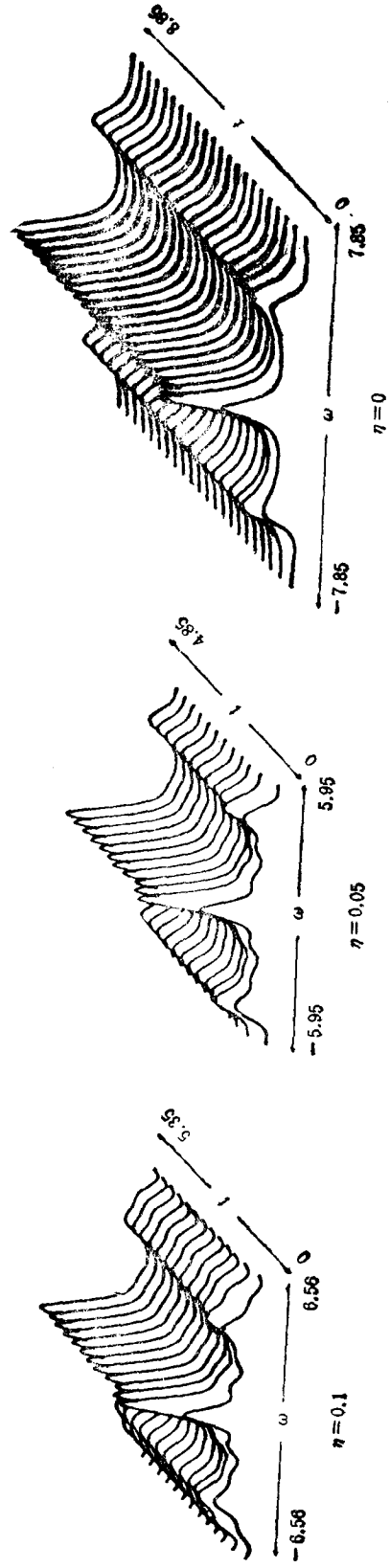


图6  $Q = 5$ ;  $\eta = 0.1, 0.05, 0$  的瞬态共振荧光光谱

1, 3, 5 三种情形,  $\tau$  的取值范围 0—10, 频率  $\omega$  的取值范围 -7.85—7.85.

从这几组图容易看出如下的一些特点:

1) 共振荧光存在的时间, 远大于光抽运衰减时间  $1/\eta$ . 如图 5,  $\eta = 1$ ,  $Q = 1, 3, 5$  的各种共振荧光光谱. 其峰值是随  $\tau$  的增大而指数衰减的. 但比图 1 中相应的  $e^{-\eta t}$  曲线要慢得多. 对于图 3, 图 4,  $\eta = 0.2, 0.5$  的情形也是这样. 由 (48) 式, 影响共振荧光光谱强度的因子  $\frac{1}{\tau} \int_0^t \langle \sigma^+(t') \sigma^-(t') \rangle dt'$ ,  $i/t \int_0^t Q(t') \langle \sigma^+(t') \rangle dt'$  经归一化后便是 IMC, INC, 已在图 2 中给出随  $\tau$  的变化曲线. 对  $\eta = 0$  情形, IMC, INC 随  $\tau$  的增大而趋近于稳态值 1. 但当  $\eta = 0.2, 0.5, 1$  情形, IMC, INC 均随  $\tau$  的增大而衰减. 其中  $\eta = 1$ ,  $Q = 5$  情形明显按指数下降, 但下降速度要比相应的  $e^{-\eta t}$  慢. 对同一  $Q$ , 不同  $\eta$  情形, 则 IMC, INC 的峰值随  $\eta$  加大而下降, 从图 2 能明显地看出这点.

2) 图 3 至图 5 另一重要方面, 即谱时间变化的瞬态行为. 当  $\tau$  增大而又不是很大,  $\eta$  较小的情形, 共振荧光光谱的三峰结构 ( $Q = 3, 5$ ) 基本上与  $\eta = 0$  的稳态情形同. 如图 3 所示的  $\eta = 0.2$  的情形. 但当  $\eta$  增至 0.5, 如图 4, 则边峰减弱; 进一步增至  $\eta = 1$ , 如图 5 则边峰几乎消失, 谱变为单峰. 3) 当  $\tau$  不是很大的瞬态行为. 一般而言, 对  $Q = 1$  情形,  $\tau$  由小到大, 共振荧光光谱均保持单峰结构. 但当  $Q = 3$ ,  $\eta = 0.2, 0.5$  情形瞬态荧光光谱中最先出现的是边峰, 到后来中峰才长出来. 当  $Q = 5$ ,  $\eta = 0.2, 0.5$  的瞬态谱, 中峰的出现, 还伴随着一些细的起伏. 这些细的起伏又随着  $\eta$  的变小而消失. 图 6 给出  $Q = 5$ ,  $\eta = 0.1, 0.05, 0$  的瞬态共振荧光光谱, 显示这一消失过程, 最后过渡到光滑的三峰结构.

## 附 录 A

应用关系式 (20) 求逆变换

$$\begin{aligned} \frac{1}{s(s+\eta)\cdots(s+n\eta)} &= \frac{1}{\eta^n} \sum \frac{(-1)^k}{k!(n-k)!(s+k\eta)} \\ &\Rightarrow \frac{1}{n!\eta^n} (1 - e^{-s\eta})^n. \end{aligned} \quad (\text{A.1})$$

又注意到

$$\begin{aligned} D_0 \tilde{D}_1 D_2 \cdots D_{2n} &= \begin{pmatrix} d_{n1} & & & \\ & d_{n1} & & \\ & & & \\ & & & d_{n1} \end{pmatrix} \\ d_{n1} &= \frac{1}{s+\gamma_1} \cdot \frac{1}{s+\gamma_2+\eta} \cdot \frac{1}{s+\gamma_1+2\eta} \cdots \frac{1}{s+\gamma_1+2n\eta} \\ &= (s+\gamma_2-\eta) \frac{1}{s+\gamma_1} \cdot \frac{1}{s+\gamma_1+2\eta} \cdots \frac{1}{s+\gamma_1+2n\eta} \\ &\quad \cdot \frac{1}{s+\gamma_2-\eta} \cdot \frac{1}{s+\gamma_2+\eta} \cdots \frac{1}{s+\gamma_2+(2n-1)\eta} \\ &\Rightarrow \left( \frac{d}{dt} + \gamma_2 - \eta \right) \frac{1}{(n!)^2} \int_0^t \left( \frac{1 - e^{-2\eta\tau}}{2\eta} \cdot \frac{1 - e^{-2n(\tau-\tau)}}{2\eta} \right)^n \\ &\quad \cdot e^{-\gamma_1\tau - (\gamma_2 - \eta)(t-\tau)} d\tau. \end{aligned} \quad (\text{A.2})$$

$\tilde{d}_{n1}$  的表式与  $d_{n1}$  同, 只是将其中  $\gamma_1, \gamma_2$  对换. 由定义 (42), (43) 和 (A.2) 式得

$$\begin{aligned}
 i_1 &= \sum_{n=0}^{\infty} (-1)^n Q^{2n} d_{n1} \Rightarrow I_{11} \\
 I_1 &= \left(\frac{d}{dt} + \gamma_2 - \eta\right) e^{-(\gamma_2 - \eta)t} \int_0^t e^{-(\gamma_1 - \gamma_2 + \eta)\tau} \\
 &\quad \times \sum_{n=0}^{\infty} \frac{1}{n!^2} \left(\frac{1 - e^{-2\eta\tau}}{2\eta} \times \frac{1 - e^{-2\eta(\tau - \tau')}}{2\eta}\right)^n d\tau \\
 &= \left(\frac{d}{dt} + \gamma_2 - \eta\right) e^{-(\gamma_2 - \eta)t} \int_0^t e^{-(\gamma_1 - \gamma_2 + \eta)\tau} J_0\left(2Q \sqrt{\frac{1 - e^{-2\eta\tau}}{2\eta} \frac{1 - e^{-2\eta(\tau - \tau')}}{2\eta}}\right) d\tau. \quad (\text{A.3})
 \end{aligned}$$

参照积分表<sup>[5]</sup> 736 页, 当  $\eta \rightarrow 0$  时,

$$\begin{aligned}
 I_1 &\rightarrow \left(\frac{d}{dt} + \gamma_2\right) e^{-\gamma_2 t} \int_0^t e^{-(\gamma_1 - \gamma_2)\tau} J_0(2Q \sqrt{\tau(t - \tau)}) d\tau \\
 &= \left(\frac{d}{dt} + \gamma_2\right) e^{-\frac{\gamma_1 + \gamma_2}{2} t} \int_0^t \text{ch}\left(\frac{\gamma_1 - \gamma_2}{2} x\right) J_0(Q \sqrt{t^2 - x^2}) dx \\
 &= \left(\frac{d}{dt} + \gamma_2\right) e^{-\frac{\gamma_1 + \gamma_2}{2} t} \frac{\sin Q' t}{Q'}, \quad Q' = \sqrt{Q^2 - \left(\frac{\gamma_1 - \gamma_2}{2}\right)^2}. \quad (\text{A.4})
 \end{aligned}$$

同样可求  $I_{22}$

$$\begin{aligned}
 \hat{D}_0 D_1 \hat{D}_1 \dots D_{2n+1} &= \begin{pmatrix} d_{n1} & & \\ & d_{n2} & \\ & & d_{n3} \end{pmatrix} \\
 d_{n1} &= \frac{1}{s + \gamma_1} \cdot \frac{1}{s + \gamma_1 + 2\eta} \dots \frac{1}{s + \gamma_1 + 2n\eta} \\
 &\quad \times \frac{1}{s + \gamma_1 + \eta} \cdot \frac{1}{s + \gamma_1 + 3\eta} \dots \frac{1}{s + \gamma_1 + (2n + 1)\eta} \\
 &\Rightarrow \frac{1}{n!^2} \int_0^t \left(\frac{1 - e^{-2\eta\tau}}{2\eta} \frac{1 - e^{-2\eta(\tau - \tau')}}{2\eta}\right)^n e^{-(\gamma_1 + \eta)\tau - \gamma_2(\tau - \tau')} d\tau, \\
 i_2 &= \sum_{n=0}^{\infty} (-1)^n Q^{2n} d_{n2} \Rightarrow I_{22} \\
 I_2 &= e^{-\gamma_2 t} \int_0^t e^{-(\gamma_1 - \gamma_2 + \eta)\tau} J_0\left(2Q \sqrt{\frac{1 - e^{-2\eta\tau}}{2\eta} \frac{1 - e^{-2\eta(\tau - \tau')}}{2\eta}}\right) d\tau. \quad (\text{A.5})
 \end{aligned}$$

当  $\eta \rightarrow 0$  时,

$$\begin{aligned}
 I_2 &\rightarrow e^{-\gamma_2 t} \int_0^t e^{-(\gamma_1 - \gamma_2)\tau} J_0(2Q \sqrt{\tau(t - \tau)}) d\tau \\
 &= e^{-\frac{\gamma_1 + \gamma_2}{2} t} \frac{\sin \sqrt{Q^2 - \left(\frac{\gamma_1 - \gamma_2}{2}\right)^2} t}{\sqrt{Q^2 - \left(\frac{\gamma_1 - \gamma_2}{2}\right)^2}} = e^{-\frac{\gamma_1 + \gamma_2}{2} t} \frac{\sin Q' t}{Q'}. \quad (\text{A.6})
 \end{aligned}$$

由定义式 (43), 容易看出

$$\begin{aligned}
 d_{n3} &= d_{n1} \frac{1}{s + 2n\eta} \Rightarrow \left(\frac{d}{dt} + \gamma_2 - \eta\right) \int_0^t e^{-2n\eta(t - \tau')} d\tau' \\
 &\quad \cdot \frac{1}{(n!)^2} \int_0^{\tau'} \left(\frac{1 - e^{-2\eta\tau}}{2\eta} \frac{1 - e^{-2\eta(\tau' - \tau)}}{2\eta}\right)^n e^{-\gamma_1 \tau - (\gamma_2 - \eta)(\tau' - \tau)} d\tau, \\
 i_3 &= \sum_{n=0}^{\infty} (-1)^n Q^{2n} d_{n3} \Rightarrow I_{33} \\
 I_3 &= \left(\frac{d}{dt} + \gamma_2 - \eta\right) \int_0^t d\tau e^{-(\gamma_2 - \eta)\tau} \int_0^{\tau'} e^{-(\gamma_1 - \tau' + \eta)\tau} \\
 &\quad \cdot J_0\left(2Q e^{-\eta(t - \tau')} \sqrt{\frac{1 - e^{-2\eta\tau}}{2\eta} \frac{1 - e^{-2\eta(\tau' - \tau)}}{2\eta}}\right) d\tau. \quad (\text{A.7})
 \end{aligned}$$

当  $\eta \rightarrow 0$  时,

$$I_3 \rightarrow \left(\frac{d}{dt} + \gamma_2\right) \int_0^t d\tau' e^{-\gamma_2 \tau'} \int_0^{\tau'} e^{-(\gamma_1 - \tau' + \eta)\tau} J_0(2Q \sqrt{\tau(t - \tau)}) d\tau$$

$$\begin{aligned}
 &= \left(\frac{d}{dt} + \gamma_2\right) \int_0^t ds' e^{-\frac{\gamma_1+\gamma_2}{2} t' \frac{\sin Q' t'}{Q'}} \\
 &= e^{-\frac{\gamma_1+\gamma_2}{2} t \frac{\sin Q' t}{Q'}} + \frac{\gamma_2}{2iQ'} \left(\frac{e^{\lambda_1 t} - 1}{\lambda_1} - \frac{e^{\lambda_2 t} - 1}{\lambda_2}\right), \quad (\text{A.8})
 \end{aligned}$$

$$\begin{aligned}
 d_{n+1} &= d_{n2} \frac{1}{s + (2n+1)\eta} \Rightarrow \int_0^t e^{-(2n+1)(t-t')} ds' \\
 &\cdot \left(\frac{1}{n!}\right)^2 \int_0^{t'} \left(\frac{1 - e^{-2\eta\tau}}{2\eta} - \frac{1 - e^{-2\eta(t-\tau)}}{2\eta}\right)^n e^{-(\gamma_1+\eta)\tau - \gamma_2(t-t')} d\tau, \\
 i_4 &= \sum (-1)^n Q^{2n} d_{n+1} \Rightarrow I_4,
 \end{aligned}$$

$$I_4 = \int_0^t e^{-\eta(t-t')} ds' e^{-\gamma_2 t'} \int_0^{t'} e^{-(\gamma_1 - \gamma_2 + \eta)\tau} J_0 \left(2Q e^{-\eta(t-t')} \sqrt{\frac{1 - e^{-2\eta\tau}}{2\eta} - \frac{1 - e^{-2\eta(t-\tau)}}{2\eta}}\right) d\tau. \quad (\text{A.9})$$

当  $\eta \rightarrow 0$  时,

$$\begin{aligned}
 I_4 &\rightarrow \int_0^t ds' e^{-\frac{\gamma_1+\gamma_2}{2} t' \frac{\sin Q' t'}{Q'}} = \frac{1}{2iQ'} \left(\frac{e^{\lambda_1 t} - 1}{\lambda_1} - \frac{e^{\lambda_2 t} - 1}{\lambda_2}\right), \\
 \lambda_1 &= -\frac{\gamma_1 + \gamma_2}{2} + iQ', \quad \lambda_2 = -\frac{\gamma_1 + \gamma_2}{2} - iQ'. \quad (\text{A.10})
 \end{aligned}$$

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## INSTANTANEOUS RESONANCE FLUORESCENCE SPECTRUM OF A TWO LEVEL ATOMIC SYSTEM DRIVEN BY AN EXPONENTIAL BY DCEAYING FIELD

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### ABSTRACT

The commutation relation and quantum mechanical Langevin equations for a two-level system is evaluated at first. Then, the instantaneous fluorescence spectrum of atomic system driven by an exponentially decaying field is calculated.

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