电磁引起透明:强信号、反对称失谐情形

罗振飞　徐至展　陈荣清

中国科学院上海光学精密机械研究所,上海　201800

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在强信号场与产生相干的外场分别与Λ原子两个偶极允许跃迁具有反对称失谐情况下严格求解了原子密度矩阵方程. 在等拉比频率(G)条件下发现失谐很小时存在近似的相干粒子俘陷现象,这导致介质对强信号光的$O(G^{-2})$级透明. 当失谐量为$G/2$时,原子近似等概率地处于三个能级,透明仅具有$O(G^{-1})$量级.

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一、引　　言

已揭示的电磁引起透明[1-6]适用于小信号入射光. 对强信号光,文献[7]通过数值计算发现当外加电磁场与信号场分别与Λ原子两个偶极跃迁具有反对称失谐(见图1),且当其拉比频率相等并与失谐量保持一定关系时,原子将近似等几率地处于三个能级. 在这种情况下,容易看出吸收正比于联系信号场的上、下能级布居数差,因而Λ原子介质将对强信号场近似透明.

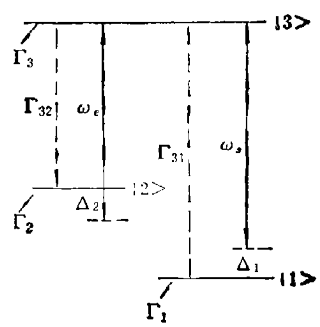

图1　三能级Λ系统　$|2\rangle \longleftrightarrow |1\rangle$跃迁是偶极禁戒的

本文将在信号场与外场分别与Λ原子两偶极跃迁具有反对称失谐情况下严格求解原子密度矩阵方程,并据此研究原子布居特点及强信号的透明. 在等拉比频率条件下得到了下述结论. 1)若失谐很小,则Λ原子上能级布居概率几乎为零(而非1/3),原子近似等概率地处于两个下能级($\rho_{11}=\rho_{22}\doteq 1/2$). 此时介质对强信号场的透明主要起源于强场导致的近似相干粒子俘陷机制. 2)若失谐为信号场拉比频率的一半,原子几乎等概率地处于三个能级,介质对强信号场的透明比小失谐时低$O(G^2)$量级,透明的机制是跃迁饱和(第二点解析地证明了文献[7]的数值计算结果).

二、模型及其解

对图1所示的Λ系统旋转坐标系中的原子演化方程写为

$$\dot{\rho}_{21}=-\mathrm{i}\frac{G_s}{2}\rho_{23}+\mathrm{i}\frac{G_e}{2}\rho_{31}-\left[\frac{\Gamma_1+\Gamma_2}{2}+\mathrm{i}(\Delta_1-\Delta_2)\right]\rho_{21},\tag{1}$$

$$\dot{\rho}_{31}=-\mathrm{i}\frac{G_s}{2}(\rho_{33}-\rho_{11})+\mathrm{i}\frac{G_e}{2}\rho_{21}-\left[\frac{\Gamma_1+\Gamma_3}{2}+\mathrm{i}\Delta_1\right]\rho_{31},\tag{2}$$

$$\dot{\rho}_{32}=-\mathrm{i}\frac{G_e}{2}(\rho_{33}-\rho_{22})+\mathrm{i}\frac{G_s}{2}\rho_{12}-\left[\frac{\Gamma_2+\Gamma_3}{2}+\mathrm{i}\Delta_2\right]\rho_{32},\tag{3}$$

$$\dot{\rho}_{11}=\mathrm{i}\frac{G_s}{2}(\rho_{31}-\rho_{13})+\Gamma_{31}\rho_{33},\tag{4}$$

$$\dot{\rho}_{22}=\mathrm{i}\frac{G_e}{2}(\rho_{32}-\rho_{23})+\Gamma_{32}\rho_{33},\tag{5}$$

式中 $G_s=\mu_{31}E_s$，$G_e=\mu_{32}E_e$ 分别是信号场（s）和产生相干的外场（e）的拉比频率，并取为正实数。$\Delta_s=\omega_{31}-\omega_s$ 和 $\Delta_e=\omega_{32}-\omega_e$ 是场一原子频率失谐。Γ_i 是处于能级 $|i\rangle$ 的原子的总自发衰变几率，Γ_{31} 和 Γ_{32} 是偶极跃迁几率，$\Gamma_{21}\doteq 0$。

首先从方程(1)—(3)写出非对角元的稳态严格解如下:

$$\rho_{21}=\frac{G_sG_e}{4(\delta-\mathrm{i}\Gamma)}\cdot\left[\frac{\rho_{11}-\rho_{33}}{\Delta_1-\mathrm{i}\dfrac{\Gamma_1+\Gamma_3}{2}}-\frac{\rho_{22}-\rho_{33}}{\Delta_2+\mathrm{i}\dfrac{\Gamma_2+\Gamma_3}{2}}\right],\tag{6}$$

$$\rho_{31}=\frac{\Delta_1+\mathrm{i}\dfrac{\Gamma_1+\Gamma_3}{2}}{\Delta_1^2+\left(\dfrac{\Gamma_1+\Gamma_3}{2}\right)^2}\cdot\left[\frac{G_s}{2}(\rho_{11}-\rho_{33})+\frac{G_e}{2}\rho_{21}\right],\tag{7}$$

$$\rho_{32}=\frac{\Delta_2+\mathrm{i}\dfrac{\Gamma_2+\Gamma_3}{2}}{\Delta_2^2+\left(\dfrac{\Gamma_2+\Gamma_3}{2}\right)^2}\cdot\left[\frac{G_e}{2}(\rho_{22}-\rho_{33})+\frac{G_s}{2}\rho_{12}\right],\tag{8}$$

式中

$$\delta=(\Delta_1-\Delta_2)-\frac{\Delta_1\cdot G_e^2/4}{\Delta_1^2+\left(\dfrac{\Gamma_1+\Gamma_3}{2}\right)^2}+\frac{\Delta_2\cdot G_s^2/4}{\Delta_2^2+\left(\dfrac{\Gamma_2+\Gamma_3}{2}\right)^2},\tag{9}$$

$$\Gamma=\frac{\Gamma_1+\Gamma_2}{2}+\frac{(\Gamma_1+\Gamma_3)\cdot G_e^2/8}{\Delta_1^2+\left(\dfrac{\Gamma_1+\Gamma_3}{2}\right)^2}+\frac{(\Gamma_2+\Gamma_3)\cdot G_s^2/8}{\Delta_2^2+\left(\dfrac{\Gamma_2+\Gamma_3}{2}\right)^2}.\tag{10}$$

在等衰变概率 $\Gamma_{31}=\Gamma_{32}\equiv\Gamma_{3d}$，$\Gamma_1=\Gamma_2\equiv\Gamma_t$ 和反对称失谐 $\Delta_1=-\Delta_2=\Delta$ 条件下，(6)—(10)式简化为

$$\rho_{21}=\frac{G_sG_e}{4D(\delta^2+\Gamma^2)}\cdot\left[\left(\delta\Delta-\Gamma\frac{\Gamma_t+\Gamma_3}{2}\right)+\mathrm{i}\left(\Gamma\Delta+\delta\frac{\Gamma_t+\Gamma_3}{2}\right)\right]\cdot(\rho_{11}+\rho_{22}-2\rho_{33}),\tag{6'}$$

$$\rho_{31} = \frac{\Delta + \mathrm{i}\dfrac{\Gamma_e + \Gamma_s}{2}}{D} \cdot \left[\frac{G_s}{2}(\rho_{11} - \rho_{33}) + \frac{G_e}{2}\rho_{21}\right], \tag{7$'$}$$

$$\rho_{32} = \frac{-\Delta + \mathrm{i}\dfrac{\Gamma_e + \Gamma_s}{2}}{D} \cdot \left[\frac{G_e}{2}(\rho_{22} - \rho_{33}) + \frac{G_s}{2}\rho_{12}\right], \tag{8$'$}$$

$$\delta = 2\Delta - \frac{G_e^2 + G_s^2}{4D} \cdot \Delta, \tag{9$'$}$$

$$\Gamma = \Gamma_e + \frac{G_e^2 + G_s^2}{8D}(\Gamma_0 + \Gamma_s), \tag{10$'$}$$

式中 $D = \Delta^2 + \left(\dfrac{\Gamma_e + \Gamma_s}{2}\right)^2$.

从(6)′—(8)′式有

$$\rho_{21} + \rho_{12} = \frac{G_s G_e}{4D(\delta^2 + \Gamma^2)} \cdot 2\left(\delta\Delta - \Gamma\frac{\Gamma_e + \Gamma_s}{2}\right) \cdot (\rho_{11} + \rho_{22} - 2\rho_{33}), \tag{11}$$

$$\rho_{21} - \rho_{12} = \frac{\mathrm{i}G_s G_e}{4D(\delta^2 + \Gamma^2)} \cdot 2\Delta(2\Gamma_e + \Gamma_s) \cdot (\rho_{11} + \rho_{22} - 2\rho_{33}), \tag{12}$$

$$\rho_{31} - \rho_{13} = \frac{\mathrm{i}G_s(\Gamma_e + \Gamma_s)}{2D}(\rho_{11} - \rho_{33}) + \frac{G_e\Delta}{2D}(\rho_{21} - \rho_{12}) + \frac{\mathrm{i}G_e(\Gamma_e + \Gamma_s)}{4D}(\rho_{21} + \rho_{12}), \tag{13}$$

$$\rho_{32} - \rho_{23} = \frac{\mathrm{i}G_e(\Gamma_e + \Gamma_s)}{2D}(\rho_{22} - \rho_{33}) + \frac{G_s\Delta}{2D}(\rho_{21} - \rho_{12}) + \frac{\mathrm{i}G(\Gamma_e + \Gamma_s)}{4D}(\rho_{21} + \rho_{12}). \tag{14}$$

从(4),(5)式知

$$G_s(\rho_{31} - \rho_{13}) = G_e(\rho_{32} - \rho_{23}). \tag{15}$$

利用(13),(14)式,有

$$G_s^2(\rho_{11} - \rho_{33}) = G_e^2(\rho_{22} - \rho_{33}). \tag{16}$$

将(6)′式代入(7)′式并利用(16)式,我们得到决定介质对信号场吸收的极化虚部为

$$\mathrm{Im}\rho_{31} = \frac{G_s \cdot A}{4D} \cdot (\rho_{11} - \rho_{33}), \tag{17}$$

式中

$$A = (\Gamma_e + \Gamma_s) + \frac{G_s^2 + G_e^2}{2D(\delta^2 + \Gamma^2)} \cdot \left[\Delta^2(2\Gamma_e + \Gamma_s) + \frac{\Gamma_e + \Gamma_s}{2}\left(\delta\Delta - \Gamma\frac{\Gamma_e + \Gamma_s}{2}\right)\right]. \tag{18}$$

另一方面,将(11)—(13)式代入(4)式得

$$\frac{G_i^2A}{4D}\cdot(\rho_{33}-\rho_{11})+\Gamma_{3d}\rho_{33}=0. \tag{19}$$

联立(16),(19)式以及归一化条件,可求出Λ原子三个能级的布居概率为

$$\rho_{11}=\left(1+\frac{4D\Gamma_{3d}}{G_i^2A}\right)\rho_{33}, \tag{20}$$

$$\rho_{22}=\left(1+\frac{4D\Gamma_{3d}}{G_e^2A}\right)\rho_{33}, \tag{21}$$

$$\rho_{33}=\left(3+\frac{4D\Gamma_{3d}}{G_i^2A}+\frac{4D\Gamma_{3d}}{G_e^2A}\right)^{-1}. \tag{22}$$

至此，我们在等衰变概率和反对称失谐情况下求得了原子密度矩阵方程的严格解析解。(17)和(20)—(22)式是我们下面研究强信号透明及机制的基本方程。

三、强信号透明及机制

为讨论简洁起见,我们选取外场强度,使得其拉比频率与信号场拉比频率相等($G_e=G_i\equiv G$)。

1. 小失谐($|\Delta|\ll G$)情形

因

$$\frac{1}{\delta^2+\Gamma^2}=\frac{4D}{G^4}\left\{1+\frac{2}{G^2}[4\Delta^2-\Gamma_0(\Gamma_0+\Gamma_3)]\right\}+O(G^{-8}), \tag{23}$$

代入(18)式并经化简有

$$A=(\Gamma_0+\Gamma_3)-\frac{4}{G^2}\left\{\frac{G^2}{4}(\Gamma_0+\Gamma_3)-\Gamma_0D\right\}+O(G^{-4})$$

$$=\frac{4\Gamma_0D}{G^2}+O(G^{-4}). \tag{24}$$

由(24)式可见,A的$O(G^0)$次项被消除,其主要项为$O(G^{-2})$。因此,我们不能从(22)式得到$\rho_{33}=\frac{1}{3}+O(G^{-2})$的结论。

将(24)式代入(17)式有

$$\mathrm{Im}\rho_{31}=\frac{\Gamma_0}{G}(\rho_{11}-\rho_{33})+O(G^{-3}). \tag{25}$$

最常见的Λ型系统是两个下能级为(亚)稳态($\Gamma_0=0$)。此时

$$\mathrm{Im}\rho_{31}=O+O(G^{-3}). \tag{26}$$

即在等拉比频率,反对称失谐(且很小)时,介质对强信号场有$O(G^{-3})$级透明。

将(24)式代入(20)—(22)式得

$$\rho_{11}=\rho_{22}=\frac{\Gamma_{3d}+\Gamma_0}{2\Gamma_{3d}+3\Gamma_0}+O(G^{-2}), \tag{27}$$

$$\rho_{33} = \frac{\Gamma_0}{2\Gamma_{3d} + 3\Gamma_0} + O(G^{-2}). \tag{28}$$

在 $\Gamma_0 = 0$ 时 $\rho_{33} \doteq 0$, $\rho_{11} = \rho_{22} \doteq 1/2$. 此时,原子几乎等概率地被"锁"在两个下能级,而不被强场激发.

2. 大失谐（$|\Delta| = G/2$）情形

因 $|\Delta| = G/2$, 失谐量不能再当作小量处理. 通过展开化简得

$$A = \frac{G^2}{2\Gamma_0 + \Gamma_3} + (\Gamma_0 + \Gamma_3)\left[1 - \frac{\Gamma_0(\Gamma_0 + \Gamma_3)(3\Gamma_0 + \Gamma_3)}{(2\Gamma_0 + \Gamma_3)^3}\right] + O(G^{-2}). \tag{29}$$

利用上式,从(20)—(22)式有

$$\rho_{11} = \rho_{22} = \frac{1}{3} + O(G^{-2}), \quad \rho_{33} = \frac{1}{3} + O(G^{-2}). \tag{30}$$

从(17),(20)—(22),(29)式可得

$$\mathrm{Im}\rho_{31} = \frac{\Gamma_{3d}}{3G} + O(G^{-3}). \tag{31}$$

随 G 增大, $\mathrm{Im}\rho_{31} \to 0$. 由(30)式知,此时透明的机制是强场引起的跃迁饱和效应.

四、讨论与结论

将(20)式直接代人(17)式可得

$$\mathrm{Im}\rho_{31} = \frac{\Gamma_{3d}}{G}\rho_{33}. \tag{32}$$

因此,无需对外场强度和(反对称失谐的)失谐量大小作特别的假定,即可发现介质对强信号场的吸收与其拉比频率成反比,随信号场增强,介质趋向透明(两能级原子的跃迁饱和).

然而,从上节对等拉比频率和失谐量很小(大)时所作的具体分析,我们对原子布居概率、强信号场透明的量级及机制获得了更深入的理解. 具体体现在:

1. 在(反对称)失谐量很小时,证明了近似的相干粒子俘陷现象(准确的相干粒子俘陷出现在对称失谐 $\Delta_1 = \Delta_2$ 处(任意场强))的存在. 当 Λ 原子两下能级为理想的(亚)稳态时($\Gamma_0 = 0$), $\rho_{33} = O(G^{-2})$, 原子对强信号场展现出 $O(G^{-3})$ 级的透明.

2. 在失谐等于 $G/2$ 时, 得到了原子将近似等概率地处于三个能级的结论. 这解析地证明了文献[7]的数值分析结果. 这时介质对强信号场仅具有 $O(G^{-1})$ 级的透明.

简言之,本文在反对称失谐条件下得到了 Λ 系统原子密度矩阵方程的严格解析解.当外场与强信号场具有等拉比频率时,发现若失谐量为场拉比频率的一半, Λ 原子三能级的布居概率几乎相等,强信号透明起源于强场引起的跃迁饱和效应. 而在失谐很小时,存在近似的相干粒子俘陷现象,原子将被"锁"在两个下能级而不被强场激发,这时强信号透明机制主要源于这种粒子俘陷机制. 与跃迁饱和导致的透明($O(G^{-1})$级)相比,此时透明具有 $O(G^{-3})$ 的量级.

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ELECTROMAGNETICALLY-INDUCED TRANSPARENCY: STRONG SIGNAL AND ANTI-SYMMETRICAL DETUNING CASE

Luo Zhen-fei, Xu Zhi-zhan Chen Rong-qing
Shanghai Institute of Optics and Fine Mechanics, Academia Sinica, Shanghai 201800
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Abstract

The equations of atomic density matrix are solved without any approximation when the strong signal field and external coherence-driving field are detuned to respective dipole-allowed transition of the Λ atom in such a way that $\Delta_1 = -\Delta_2$. Under the equal Rabi frequency (G) condition, it is found that approximate coherent population trapping appears for small detuning, which leads to the transparency of the medium to the strong signal having an order of $O(G^{-3})$. If the detuning is $G/2$, the atom nearly equally populates three levels and the transparency is of only order $O(G^{-1})$.

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