抽运统计对多级联双光子关联自发辐射激光器噪声的影响(I)*

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原子的相干性可导致双光子关联自发辐射激光场的压缩效应，本文利用线性理论证明抽运噪声的抑制可进一步降低光场的噪声.

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1 引　言

自发辐射和抽运噪声是激光场的两个主要噪声来源. 抽运噪声的抑制可使单光子激光器产生光子数压缩效应[1-5],而原子相干性可使两个或多个自发辐射事件关联起来，进而猝灭[6]甚至压缩光场噪声[7].

本文同时考虑抽运统计及原子的相干性对多级联双光子关联自发辐射激光器(CEL)[8,9]噪声的影响. 从线性理论出发，利用密度矩阵方程和 Fokker-Planck 方程证明抽运噪声的抑制可使光场噪声进一步降低.

2 密度矩阵方程

多级联双光子 CEL 的原子能级构型见图1. 在旋转波近似下，$n+1$能级原子与频率为Ω的单模光场相互作用体系的哈密顿量在以激光振荡频率ν旋转的坐标系中为($\hbar=1$)

$$H=(\Omega-\nu)a^{+}a+V, \tag{1a}$$

$$V=ga\left(|n\rangle\langle l|\mathrm{e}^{\mathrm{i}\Delta_{nl1}t}+\sum_{j=1}^{n-1}|l\rangle\langle j|\mathrm{e}^{\mathrm{i}\Delta_{lj1}t}\right)+\mathrm{H.c.}, \tag{1b}$$

其中 $a(a^{+})$ 为光场的湮没(产生)算符，g 为耦合常数；$\Delta_{kk'm}=\omega_k-\omega_{k'}-m\nu$，$\omega_k$ 为原子能级 $|k\rangle$ 的能量. 利用 Scully-Lamb 激光理论[10]解下面密度矩阵方程精确到 g^2，

$$\dot{\rho}=-\mathrm{i}(\Omega-\nu)[a^{+}a,\rho]-\mathrm{i}\mathrm{Tr}_a[V,\rho_{a-f}], \tag{2}$$

得到光场密度矩阵方程[8,9,11]

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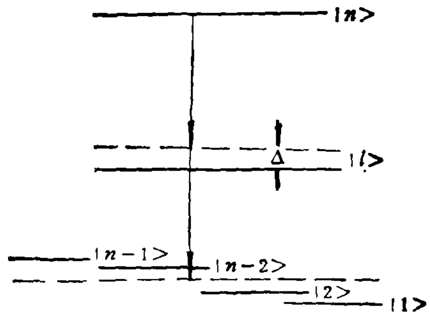

图 1 多级联双光子关联自发辐射激光体系示意图

$$\begin{aligned}\dot{\rho} = & -\mathrm{i}S\Big(\mathscr{L}^*_{nl1}\rho^a_{nl}+\sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{li}\Big)[a^+,\rho] \\ & +\frac{1}{2}\beta\Big[\Big(\rho^a_{nn}\mathscr{L}^*_{nl1}+\rho^a_{ll}\sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\Big)(a^+\rho a-\rho a a^+) \\ & +\Big(\rho^a_{ll}\mathscr{L}^*_{nl1}+\sum_{j\cdot j'=1}^{n-1}\mathscr{L}^*_{jj'0}\mathscr{L}^*_{lj'1}\rho^a_{jj'}\Big)(a\rho a^+-a^+a\rho) \\ & +\Big(-\sum_{j=1}^{n-1}\mathscr{L}^*_{nj2}\mathscr{L}^*_{nl1}\rho^a_{nj}\Big)(\rho a^+a^+-a^+\rho a^+) \\ & +\Big(-\sum_{j=1}^{n-1}\mathscr{L}^*_{nj2}\mathscr{L}^*_{lj}\rho^a_{nj}\Big)(a^+a^+\rho-a^+\rho a^+)\Big]+\mathrm{H.c.} \\ & -\mathrm{i}(\Omega-\nu)[a^+a,\rho]\equiv r_a w\rho-\mathrm{i}(\Omega-\nu)[a^+a,\rho], \end{aligned} \tag{3}$$

其中 $\mathscr{L}_{kk'm}=\Gamma/(\Gamma-\mathrm{i}\Delta_{kk'm})$, $S=r_a g/\Gamma$, $\beta=2r_a g^2/\Gamma^2$, Γ 为原子能级寿命(假定所有原子能级寿命都相同) r_a 为原子的注入速率, ρ^a_{mn} 为初始时刻原子密度矩阵元. 方程(3)加上腔场衰减部分即为泊松注入下体系的密度矩阵方程.

3 抽运统计与 Fokker-Planck 方程

为了计入抽运统计的影响,须对由(3)式确立的密度算符进行抽运统计平均. 由粗粒及 Markov 近似得到的方程(3)可表示为(不考虑最后一项)

$$\dot{\rho}=r_a\delta\rho(t)=r_a[\rho(t)-\rho(0)]=r_a w\rho(t). \tag{4}$$

由(4)式可得到单个原子与光场相互作用对光场密度算符的贡献(在 Markov 近似下)

$$\rho(t)=(1+w)\rho(0). \tag{5}$$

若从 0 到 t 有 l 个原子通过谐振腔,则 t 时刻的光场密度算符为

$$\rho^{(l)}(t)=(1+w)^l\rho(0). \tag{6}$$

若注入原子遵从二项式分布[4a]

$$P(t)=\binom{K}{l}\eta^l(1-\eta)^{K-l}, \tag{7}$$

则(6)式二项式平均后有[4a]

$$\rho(t) = \sum_{l=0}^{K}\binom{K}{l}\eta^l(1-\eta)^{K-l}(1+w)^l\rho(0) = (1+\eta w)^K\rho(0), \tag{8}$$

其中 $K = r_a t$. (8)式对时间微分有[4a]

$$\dot{\rho} = \frac{r_a}{\eta}\ln(1+\eta w)\rho(t). \tag{9}$$

将(9)式展开到 w 的平方项有

$$\dot{\rho} = r_a w\rho(t) - \frac{r_a\eta}{2}w^2\rho(t). \tag{10}$$

将(3)式代入(10)式，只保留到 g 的平方项，且考虑腔场的衰减(衰减常数 γ)有

$$\begin{aligned}\dot{\rho} = f[a^+,\rho] &+ A_1(a^+\rho a - \rho aa^+) + A_2(a\rho a^+ - a^+a\rho) \\ &+ A_3(\rho a^+a^+ - a^+\rho a^+) + A_4(a^+a^+\rho - a^+\rho a^+) \\ &- \frac{\gamma}{2}(\rho a^+a - a\rho a^+) - \mathrm{i}(\Omega-\nu)a^+a\rho + \mathrm{H.c.},\end{aligned} \tag{11}$$

其中

$$f = -\mathrm{i}S\left(\mathscr{L}^*_{nl1}\rho^a_{nl} + \sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{lj}\right), \tag{11a}$$

$$A_1 = \frac{1}{2}\beta\left[\left(\rho^a_{nn}\mathscr{L}^*_{nl1} + \rho^a_{ll}\sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\right) - \frac{\eta}{2}\left|\mathscr{L}^*_{nl1}\rho^a_{nl} + \sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{lj}\right|^2\right], \tag{11b}$$

$$\begin{aligned}A_2 = \frac{1}{2}\beta\Bigg[\Bigg(\rho^a_{ll}\mathscr{L}^*_{nl1} &+ \sum_{j,j'=1}^{n-1}\rho^a_{jj'}\mathscr{L}^*_{jj'0}\mathscr{L}^*_{lj'1}\Bigg) \\ &- \frac{\eta}{2}\left|\mathscr{L}^*_{nl1}\rho^a_{nl} + \sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{lj}\right|^2\Bigg] + \frac{\gamma}{2},\end{aligned} \tag{11c}$$

$$A_3 = \frac{1}{2}\beta\left[-\sum_{j=1}^{n-1}\rho^a_{nj}\mathscr{L}^*_{nj2}\mathscr{L}^*_{nl1} + \frac{\eta}{2}\left(\mathscr{L}^*_{nl1}\rho^a_{nl} + \sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{lj}\right)^2\right], \tag{11d}$$

$$A_4 = \frac{1}{2}\beta\left[-\sum_{j=1}^{n-1}\rho^a_{nj}\mathscr{L}^*_{nj2}\mathscr{L}^*_{lj1} + \frac{\eta}{2}\left(\mathscr{L}^*_{nl1}\rho^a_{nl} + \sum_{j=1}^{n-1}\mathscr{L}^*_{lj1}\rho^a_{lj}\right)^2\right]. \tag{11e}$$

将方程(11)转变为 Glauber-Sudarshan　P 函数所满足的 Fokker-Planck 方程 ($a|\alpha\rangle = \alpha|\alpha\rangle$)

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial\alpha}d_\alpha + \frac{\partial^2}{\partial\alpha^2}D_{\alpha\alpha} + \frac{\partial^2}{\partial\alpha\partial\alpha^*}D_{\alpha\alpha^*} + \mathrm{c.c.}, \tag{12a}$$

有如下的漂移和扩散系数:

$$d_\alpha = (A_1 - A_2)\alpha + (A_4 - A_3)\alpha^* + f - \frac{\gamma}{2}\alpha, \tag{12b}$$

$$D_{\alpha\alpha} = A_4, \tag{12c}$$

$$D_{\alpha\alpha^*} = \mathrm{Re}(A_1). \tag{12d}$$

为了分析光场强度 I 和位相 φ 的起伏，将(12a)式作变数代换: $(\alpha,\alpha^*) \to (I,\varphi)$ 有如下的漂移和扩散系数 ($\alpha = \sqrt{I}\,e^{i\varphi}$)

$$d_I = 2\sqrt{I}\,\mathrm{Re}(d_\alpha e^{-i\varphi}) = 2I\mathrm{Re}[(A_1 - A_2) + (A_4 - A_3)e^{-i2\varphi}]$$
$$+ 2\sqrt{I}\,\mathrm{Re}(fe^{-i\varphi}) - \gamma I, \tag{13a}$$

$$d_\varphi = \frac{1}{\sqrt{I}}\mathrm{Im}(d_\alpha e^{-i\varphi}) = \mathrm{Im}[(A_1 - A_2) + (A_4 - A_3)e^{-i2\varphi}] + \frac{1}{\sqrt{I}}\mathrm{Im}(fe^{-i\varphi}), \tag{13b}$$

$$D_{II} = 2I[D_{\alpha\alpha^*} + \mathrm{Re}(D_{\alpha\alpha}e^{-i2\varphi})] = 2I\mathrm{Re}[A_1 + A_4e^{-i2\varphi}], \tag{13c}$$

$$D_{\varphi\varphi} = \frac{1}{2I}[D_{\alpha\alpha^*} - \mathrm{Re}(D_{\alpha\alpha}e^{-i2\varphi})] = \frac{1}{2I}\mathrm{Re}(A_1 - A_4e^{-i2\varphi}). \tag{13d}$$

由(11)式可以看出，若 $w_{nl} - \nu \neq 0$，$\rho^a_{nl} = \rho^a_{ll} = \rho^a_{li} = 0$（非共振双光子 CEL[9]），则抽运统计对光场噪声无影响；若 $w_{nl} - \nu = 0$，$\rho^a_{nl} \neq 0$，$\rho^a_{li} \neq 0$，$\rho^a_{lj} \neq 0$（共振双光子 CEL[8]），则抽运统计对光场噪声有影响．在此情形下，激光运转有如下两个锁相条件[8]：

$$\varphi_{10} = \theta_{nl} - \pi/2,$$

且

$$\theta_{nl} = \theta_{l1} + \frac{1}{2}(n-2)\theta,\ |\rho^a_{nl}| + |\rho^a_{l1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta > 0,$$

$$\theta_{n1} + \frac{1}{2}(n-2)\theta - 2\varphi_{10} = \pi_0 \tag{14a}$$

$$\varphi_{20} = \theta_{nl} - 3\pi/2,$$

且

$$\theta_{nl} = \theta_{l1} + \frac{1}{2}(n-2)\theta + \pi,\ |\rho^a_{nl}| - |\rho^a_{l1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta < 0,$$

$$\theta_{n1} + \frac{1}{2}(n-2)\theta - 2\varphi_{20} = 2\pi_0 \tag{14b}$$

在(14)式中已取 $\theta_{nj} = \theta_{n1} + (j-1)\theta$，$\theta_{nj} - \theta_{nj'} = \theta_{j'j} = (j-j')\theta$，$|\rho^a_{jj'}| = \rho^a_{l1}(j, j' \neq n, l)$，$|\rho^a_{nj}| = |\rho^a_{n1}|$，$|\rho^a_{mn}|^2 = \rho^a_{nn}\rho^a_{mm}$．对 φ_{10} 有光强

$$\sqrt{I_{10}} = \frac{\dfrac{2S}{\beta}\left[|\rho^a_{nl}| + |\rho^a_{l1}|\sin\dfrac{1}{2}(n-1)\theta/\sin\dfrac{1}{2}\theta\right]}{\gamma/\beta - \rho^a_{nn} - (n-2)\rho^a_{ll} + \rho^a_{l1}\left[\sin\dfrac{1}{2}(n-1)\theta/\sin\dfrac{1}{2}\theta\right]^2} \tag{15a}$$

及光子数起伏 $\langle(\Delta n_1)^2\rangle$ 和位相起伏 $\langle(\Delta\rho_1)^2\rangle$

$$\langle(\Delta n_1)^2\rangle = n_{10} + \frac{D_{II}}{\left|\dfrac{\partial d_I}{\partial I}\right|} = n_{10}$$

$$\times\Bigg\{1 + \Bigg(2\bigg[\rho^a_{nn} + (n-1)\rho^a_{ll} + |\rho^a_{n1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta - \eta\bigg[|\rho^a_{nl}|$$

$$+ |\rho^a_{l1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta\bigg]^2\bigg]\Bigg)\Bigg/\Bigg(\gamma/\beta - \rho^a_{nn} - (n-2)\rho^a_{ll}$$

$$+ \rho^a_{l1}\sin^2\frac{1}{2}(n-1)\theta/\sin^2\frac{1}{2}\theta\Bigg)\Bigg\}, \tag{15b}$$

$$\langle(\Delta\varphi_1)^2\rangle=\frac{1}{4n_{10}}+\frac{D_{\varphi\varphi}}{\left|\dfrac{\partial d_\varphi}{\partial\varphi}\right|}=\frac{1}{4n_{10}}$$

$$\times\left\{1+\frac{2\left[\rho^s_{nn}+(n-1)\rho^s_{ll}-|\rho^s_{n1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta\right]}{\gamma/\beta-\rho^s_{nn}-(n-2)\rho^s_{ll}+\rho^s_{11}\sin^2\frac{1}{2}(n-1)\theta/\sin^2\frac{1}{2}\theta}\right\}. \tag{15c}$$

对 φ_{20} 有光强

$$\sqrt{I_{20}}=\frac{\dfrac{2S}{\beta}\left[|\rho^s_{l1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta-|\rho^s_{nl}|\right]}{\gamma/\beta-\rho^s_{nn}-(n-2)\rho^s_{ll}+\rho^s_{11}\sin^2\frac{1}{2}(n-1)\theta/\sin^2\frac{1}{2}\theta} \tag{16a}$$

及光子数和位相起伏

$$\langle(\Delta n_2)^2\rangle=n_{20}\Big\{1+\Big(2\Big[\rho^s_{nn}+(n-1)\rho^s_{ll}-|\rho^s_{n1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta$$

$$-\eta\Big[|\rho^s_{l1}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta-|\rho^s_{nl}|\Big]^2\Big)\Big/\Big(\gamma/\beta-\rho^s_{nn}-(n-2)\rho^s_{ll}$$

$$+\rho^s_{11}\sin^2\frac{1}{2}(n-1)\theta/\sin^2\frac{1}{2}\theta\Big)\Big\}, \tag{16b}$$

$$\langle(\Delta\varphi_2)^2\rangle=\frac{1}{4n_{20}}\Big\{1+\Big(2\Big[\rho^s_{nn}+(n-1)\rho^s_{ll}+|\rho^s_{nl}|\sin\frac{1}{2}(n-1)\theta/\sin\frac{1}{2}\theta\Big]\Big)\Big/$$

$$\Big(\gamma/\beta-\rho^s_{nn}-(n-2)\rho^s_{ll}+\rho^s_{11}\sin^2\frac{1}{2}(n-1)\theta/\sin^2\frac{1}{2}\theta\Big)\Big\}. \tag{16c}$$

由(15)和(16)式可以看出，由参数 η 表示的抽运统计可降低光子数噪声。对泊松注入统计，$\eta=0$，对亚泊松注入统计 $0<\eta\leqslant 1$，若 $\eta=1$，则注入原子数的起伏为零，这相当于抽运噪声被完全抑制，对此情形由(15)和(16)式可以看出，光子数噪声降低最大。由(15c)式知当(取 $\theta=0$)

$$\frac{n-1}{2n}(1-3\rho^s_{ll})\left(1-\sqrt{1-n\left(\frac{2\rho^s_{ll}}{1-3\rho^s_{ll}}\right)^2}\right)<\rho^s_{nn}<\frac{n-1}{2n}(1-3\rho^s_{ll})$$

$$\times\left(1+\sqrt{1-n\left(\frac{2\rho^s_{ll}}{1-3\rho^s_{ll}}\right)^2}\right) \tag{17}$$

时，光场位相有压缩效应(见文献[8]和[11]的讨论)，而此时光子数噪声在一定条件下[8]将降低

$$\delta_1=\frac{\sqrt{\rho^s_{ll}}}{2\dfrac{\sin\frac{1}{2}(n-1)\theta_1}{\sin\frac{1}{2}\theta_1}\sqrt{\rho^s_{11}}}. \tag{18}$$

若取 $\theta_1=0$，则

$$\delta_1 = \frac{\sqrt{\rho_{ll}^a}}{2(n-1)\sqrt{\rho_{11}^a}}. \tag{19}$$

显见 $n-1$ 越大 δ_1 越小。因 ρ_{ll}^a 越小光场位相压缩效应越大[8]，取 $\rho_{11}^a \approx 10^{-3}$，$\rho_{nn}^a \approx 0.5$，$n-1=1$，则光子数噪声降低≈2.2%。

若 $\eta=0$，由(16b)式可以看出，在条件(17)式下，光子数有压缩效应，若 $\eta=1$，在其它参数相同的条件，光子数压缩效应进一步增加，在一定条件下[8]，压缩效应增加

$$\delta_2 = \frac{\sqrt{\rho_{ll}^a}}{2\dfrac{\sin\frac{1}{2}(n-1)\theta_2}{\sin\frac{1}{2}\theta_2}\sqrt{\rho_{11}^a}}. \tag{20}$$

同 δ_1 可求得 $\delta_2 \approx (2—3)\%$，此时可产生光子数压缩效应的 ρ_{nn}^a 的取值范围也将增加(较 $\eta=0$ 时的(17)式)。若保持抽运噪声抑制前后光子数压缩效应一样，由(16a)和(16b)式可以看出压缩光的强度将增加。

4 结 语

本文利用线性理论证明，在多级联双光子 CEL 中，若原子密度矩阵元 $\rho_{nl}^a = \rho_{lj}^a = 0$，则抽运统计对场噪声没有影响；若 $\rho_{nl}^a \neq 0$，$\rho_{lj}^a \neq 0$，则抽运噪声的抑制可进一步降低光子数噪声，但对光场位相噪声没有影响。

应当指出的是，本文的处理适用于激光振荡的线性区域，若不考虑原子的相干性，本文的线性处理并不能得到光场的稳定性条件，但此处原子的极化 ρ_{nl}^a，ρ_{lj}^a 作为驱动项可得到光场的稳定条件及扩散系数，因而线性理论可给出激光运转的适当描述。

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INFLUENCE OF PUMP STATISTICS ON NOISE OF MULTIPLE CASCADE TWO-PHOTON CORRELATED-SPONTANEOUS-EMISSION LASERS

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ABSTRACT

Atomic coherence can lead to field squeezing of twophoton correlated-spontaneous-emission lasers. In this paper, using linear theory and master equation, we show that quenching of pump fluctuation can reduce the field noise further.

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