第45卷 第2期 1996年2月 物 理 学 报 Vol.45, No.2, Feb., 1996
1000-3290/96/45(02)/0177-08 ACTA PHYSICA SINICA

广义球对称带电蒸发黑洞的量子热效应和非热效应

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(1994年10月18日收到)

研究了广义球对称带电蒸发黑洞周围的时空中 Dirac 粒子的 Hawking 辐射以及 Starobinsky-Unruh 过程.

PACC: 0420; 9760L

1 引 言

广义球对称带电蒸发黑洞的时空线元为[1]

$$ds^2 = e^{2\psi}(1 - 2M/r + Q^2/r^2)dv^2 - 2e^{\psi}dvdr - r^2(d\theta^2 + \sin^2\theta d\varphi^2) . \tag{1}$$

这里采用了超前 Eddington 坐标,上式中 $M = M(r, v)$, $\psi = \psi(r, v)$, $Q = Q(r, v)$. M, Q 分别是黑洞的质量和电荷.度规行列式为

$$g = - r^4\sin^2\theta e^{2\psi}. \tag{2}$$

逆变度规张量为

$$g^{\mu\nu} = \begin{pmatrix} 0 & -e^{-\psi} & 0 & 0 \\ -e^{-\psi} & -\Delta/r^2 & 0 & 0 \\ 0 & 0 & -1/r^2 & 0 \\ 0 & 0 & 0 & -1/r^2\sin^2\theta \end{pmatrix}, \tag{3}$$

式中

$$\Delta = r^2 - 2Mr + Q^2. \tag{4}$$

在这种黑洞的视界附近,不但存在量子热效应,也存在量子非热效应,即 Starobinsky-Unruh 过程和 Misner 超辐射.首先利用 Dirac 方程研究其热效应,然后由 Hamilton-Jacobi 方程研究非热效应.

2 量子热效应

为了研究广义球对称带电蒸发黑洞的量子热效应,构造下面的零标架:

$$l_\mu = (e^{\psi}, 0, 0, 0) ,$$

$$n_\mu = (\frac{1}{2}e^{\psi}\Delta/r^2, -1, 0, 0) ,$$

$$m_\mu = (0,0,\ -r/\sqrt{2},\ -\mathrm{i}r\sin\theta/\sqrt{2})\ ,$$
$$\overline{m}_\mu = (0,0,\ -r/\sqrt{2},\mathrm{i}r\sin\theta/\sqrt{2})\ , \tag{5}$$

其逆变换形式为

$$l^\mu = (0,\ -1,0,0)\ ,$$
$$n^\mu = (\mathrm{e}^{-\psi},\frac{1}{2}\Delta/r^2,0,0)\ ,$$
$$m^\mu = (0,0,1/(\sqrt{2}r),\mathrm{i}/(\sqrt{2}r\sin\theta))\ ,$$
$$\overline{m}^\mu = (0,0,1/(\sqrt{2}r),\ -\mathrm{i}/(\sqrt{2}r\sin\theta))\ . \tag{6}$$

不难验证,(5)和(6)式满足度规条件,零矢条件和伪正交关系[2].

根据公式

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda m}(g_{m\mu,\nu} + g_{m\nu,\mu} - g_{\mu\nu,m})\ , \tag{7}$$

逐一计算出 64 个联络,其中不等于零的有 16 个.然后按照 Newman 和 Penrose 的方法[3]计算 Ricci 旋系数,得

$$\kappa = \sigma = \tau = \pi = \nu = \lambda = 0\ ,$$
$$\rho = 1/r\ ,\ \alpha = \beta = \cot\theta/(2\sqrt{2}r)\ ,$$
$$\gamma = \frac{1}{2}(M\psi'/r - \psi'/2 - Q^2\psi'/2r^2 - M/r^2 + M'/r - QQ'/r^2 + Q^2/r^3)\ ,$$
$$\mu = \frac{1}{2r}(1 - 2M/r + Q^2/r^2)\quad \epsilon = -\psi'/2\ . \tag{8}$$

线元(1)式所表示的时空中的四维电磁势为

$$A_\mu = (-Q/r,0,0,0)\ , \tag{9}$$

方向导数为

$$\mathrm{D} = -\partial/\partial r\ ,$$
$$\Delta = \mathrm{e}^{-\psi}\partial/\partial V + \frac{1}{2}(1 - 2M/r + Q^2/r^2)\partial/\partial r\ ,$$
$$\delta = \frac{1}{\sqrt{2}r}\partial/\partial\theta + \frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\partial/\partial\varphi\ ,$$
$$\overline{\delta} = \frac{1}{\sqrt{2}r}\partial/\partial\theta - \frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\partial/\partial\varphi\ . \tag{10}$$

把(6),(8),(9)和(10)式代入下面的 Dirac 方程的四分量形式[4]:

$$(\mathrm{D} + \epsilon - \rho + \mathrm{i}eA_\mu l^\mu)F_1 + (\overline{\delta} + \pi - \alpha + \mathrm{i}eA_\mu\overline{m}^\mu)F_2 - \frac{\mathrm{i}}{\sqrt{2}}\mu G_1 = 0\ ,$$
$$(\Delta + \mu - \gamma + \mathrm{i}eA_\mu n^\mu)F_2 + (\delta + \beta - \tau + \mathrm{i}eA_\mu m^\mu)F_1 - \frac{\mathrm{i}}{\sqrt{2}}\mu G_2 = 0\ ,$$
$$(\mathrm{D} + \epsilon^* - \rho^* + \mathrm{i}eA_\mu l^\mu)G_2 - (\delta + \pi^* - \alpha^* + \mathrm{i}eA_\mu m^\mu)G_1 - \frac{\mathrm{i}}{\sqrt{2}}\mu F_2 = 0\ ,$$
$$(\Delta + \mu^* - \gamma^* + \mathrm{i}eA_\mu n^\mu)G_1 - (\overline{\delta} + \beta^* - \tau^* + \mathrm{i}eA_\mu\overline{m}^\mu)G_2 - \frac{\mathrm{i}}{\sqrt{2}}\mu F_1 = 0\ . \tag{11}$$

然后分离变量

$$F = e^{im\varphi} f(v, r, \vartheta) ,$$
$$G = e^{im\varphi} g(v, r, \vartheta) , \tag{12}$$

式中 m 为磁量子数,则可得到下面的一阶微分方程组:

$$\sqrt{2} r \mathscr{D}_1 f_1 - (\mathscr{L}_+ - \cot\theta) f_2 + i\mu r g_1 = 0 ,$$
$$\sqrt{2} r \mathscr{D}_2 f_2 + \mathscr{L}_- f_1 - i\mu r g_2 = 0 ,$$
$$\sqrt{2} r \mathscr{D}_1 g_2 + (\mathscr{L}_- - \cot\theta) g_1 + i\mu r f_2 = 0 ,$$
$$\sqrt{2} r \mathscr{D}_2 g_1 - \mathscr{L}_+ g_2 - i\mu r f_1 = 0 , \tag{13}$$

式中

$$\mathscr{D}_1 = \partial/\partial r + 1/r + \psi'/2 ,$$
$$\mathscr{D}_2 = e^{-\psi}\partial/\partial v + \frac{1}{2}(1 - 2M/r + Q^2/r^2)\partial/\partial r + \frac{1}{2r}(1 + \theta^2/r^2)$$
$$- \frac{1}{2}(M\psi'/r - \psi'/2 - Q^2\psi'/2r^2 + M/r^2 + M'/r - QQ'/r^2 + Q^2/r^3)$$
$$- i\frac{eQe^{-\psi}}{r} ,$$
$$\mathscr{L}_\pm = \partial/\partial\theta \pm m/\sin\theta + \frac{1}{2}\cot\theta . \tag{14}$$

(14)式中 $\mathscr{D}_1$ 和 $\mathscr{D}_2$ 可称为广义球对称带电蒸发黑洞时空的径向动量算子,而 $\mathscr{L}_\pm$ 则为角向动量算子,作广义龟变换[5]

$$r_* = r + \frac{1}{2k(v_0, \theta_0)}\ln[r - r_H(v, \theta)] ,$$
$$v_* = v - v_0, \quad \theta_* = \theta - \theta_0 . \tag{15}$$

由上式可得

$$\partial/\partial r = \frac{\rho + 1}{\rho}\partial/\partial r_* , \quad \partial/\partial v = \partial/\partial v_* - \frac{\dot{r}_H}{\rho}\partial/\partial r_* , \quad \partial/\partial\theta = \partial/\partial\theta_* - \frac{r_{H,\theta}}{\rho}\partial/\partial r_* , \tag{16}$$

式中

$$\rho = 2\kappa(r - r_H), \quad \dot{r}_H = \partial r_H/\partial v, \quad r_{H,\theta} = \partial r_H/\partial\theta . \tag{17}$$

经 Tortoise 变换后(14)式中的 $\mathscr{D}_1$, $\mathscr{D}_2$ 和 $\mathscr{L}_\pm$ 可整理为

$$\mathscr{D}_1 = \frac{\rho + 1}{\rho}\frac{\partial}{\partial r_*} + \psi'/2 + 1/r$$
$$\mathscr{D}_2 = e^{-\psi}\partial/\partial v_* + \left[\frac{\Delta(\rho + 1) - 2r^2\dot{r}_H e^{-\psi}}{2\rho r^2}\right]\partial/\partial r_* + \frac{1}{2r}(1 + Q^2/r^2)$$
$$- \frac{1}{2}(M\psi'/r - \psi'/2 - Q^2\psi'/2r^2 + M/r^2 + M'/r - QQ'/r^2 + Q^2/r^3) - ieQe^{-\psi}/r,$$
$$\mathscr{L}\pm\partial/\partial\theta_* \pm m/\sin\theta + \frac{1}{2}\cot\theta - \frac{r_{H,\theta}}{\rho}\partial/\partial r_* . \tag{18}$$

考察(18)式,可以得出下述结论:

1)因 $\mathscr{L}_{\pm}$ 为角向动量算符,$\partial/\partial r_*$ 的系数应为零,

$$r_{\mathrm{H},\theta} = 0 . \tag{19}$$

这表明广义球对称带电蒸发黑洞的视界不随 θ 角变化.

2) $\mathscr{D}_2$ 中 $\partial/\partial r_*$ 的系数在 $r \to r_{\mathrm{H}}$ 时应为有限值,即

$$\lim_{r \to r_{\mathrm{H}}} \frac{\Delta(\rho + 1) - 2r^2 \dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi}}{2\rho r^2} = A_0(\text{有限值}) . \tag{20}$$

欲使(20)式成立,必须使

$$\lim_{r \to r_{\mathrm{H}}} [\Delta(\rho + 1) - 2r^2 \dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi}] = 0 .$$

由此可得视界面方程

$$\Delta - 2r^2 \dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi} = 0 ,$$

即

$$(1 - 2\dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi}) r_{\mathrm{H}}^2 - 2Mr_{\mathrm{H}} + Q^2 = 0 . \tag{21}$$

3)由上式可求得黑洞的内外视界为

$$r_{\mathrm{H}}^{\pm} = \frac{M \pm [M^2 - (1 - 2\dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi}) Q^2]^{1/2}}{1 - 2\dot{r}_{\mathrm{H}} \mathrm{e}^{-\psi}} . \tag{22}$$

4)利用 L'Hospital 法则,对(20)式求极限得

$$\kappa = \frac{\mathrm{e}^{\psi} \psi'(r_{\mathrm{H}}^2 - 2Mr_{\mathrm{H}} + Q^2) + 2\mathrm{e}^{\psi}(r_{\mathrm{H}} - M'r_{\mathrm{H}} - M + QQ') - 4r_{\mathrm{H}}\dot{r}_{\mathrm{H}}}{2A_0 r_{\mathrm{H}}^2 - 2\mathrm{e}^{\psi}(r_{\mathrm{H}}^2 - 2Mr_{\mathrm{H}} + Q^2)} .$$

当 $Q=0$ 时有

$$\kappa = \frac{\mathrm{e}^{\psi} \psi'(r_{\mathrm{H}} - 2M) + \mathrm{e}^{\psi}(1 - 2M') - 2\dot{r}_{\mathrm{H}}}{2A_0 r_{\mathrm{H}} - 2\mathrm{e}^{\psi}(r_{\mathrm{H}} - 2M)} . \tag{23}$$

这里利用了 $2r\dot{r}_{\mathrm{H}} = \mathrm{e}^{\psi}(r_{\mathrm{H}} - 2M)$,它可由(21)式导出.将(23)式与广义球对称蒸发黑洞的 κ 相比较[6]得

$$A_0 = 1 .$$

故广义球对称带电蒸发黑洞的视界温度函数为

$$\kappa = \frac{\mathrm{e}^{\psi} \psi'(r_{\mathrm{H}}^2 - 2Mr_{\mathrm{H}} - Q^2) + 2\mathrm{e}^{\psi}(r_{\mathrm{H}} - M'r_{\mathrm{H}} - M + QQ') - 4r_{\mathrm{H}}\dot{r}_{\mathrm{H}}}{2r_{\mathrm{H}}^2 - 2\mathrm{e}^{\psi}(r_{\mathrm{H}}^2 - 2Mr_{\mathrm{H}} + Q^2)} . \tag{24}$$

下面讨论(22)和(24)式.(22)式表明广义球对称带电蒸发黑洞有两个视界面:外视界 r_{H}^{+} 和内视界 r_{H}^{-},在 $r_{\mathrm{H}}^{-} < r < r_{\mathrm{H}}^{+}$ 区域为单向膜区.该黑洞的无限红移面也有两个

$$r_{\mathrm{s}}^{\pm} = M \pm \sqrt{M^2 - Q^2} . \tag{25}$$

所以此黑洞有两个能层

$$\text{外能层} \quad r_{\mathrm{H}}^{+} < r < r_{\mathrm{s}}^{+};$$

$$\text{内能层} \quad r_{\mathrm{s}}^{-} < r < r_{\mathrm{H}}^{-}. \tag{26}$$

在(22)式中令 $Q=0$，可得广义球对称蒸发黑洞的视界为

$$r_{\mathrm{H}}^{外}=\frac{2M}{1-2\dot{r}_{\mathrm{H}}\mathrm{e}^{-\psi}},\qquad r_{\mathrm{H}}^{内}=0\ . \tag{27}$$

这种黑洞实际上只有一个视界，它可以看作广义球对称带电蒸发黑洞的基态.

若 $Q=Q'=0$，由(24)式可得广义球对称蒸发黑洞的视界温度为

$$\kappa=\frac{\mathrm{e}^{\psi}\psi'(r'_{H}-2Mr_{\mathrm{H}})+2\mathrm{e}^{\psi}(r_{\mathrm{H}}-M'r_{\mathrm{H}}-M)-4r_{\mathrm{H}}\dot{r}_{\mathrm{H}}}{2r_{\mathrm{H}}^{2}-2\mathrm{e}^{\psi}(r_{\mathrm{H}}^{2}-2Mr_{\mathrm{H}})}\ . \tag{28}$$

若 $\psi=\psi'=0$，则可得到球对称带电蒸发黑洞的视界温度为

$$\kappa=\frac{r_{\mathrm{H}}-M-M'r_{\mathrm{H}}+QQ'-2r_{\mathrm{H}}\dot{r}_{\mathrm{H}}}{2Mr_{\mathrm{H}}-Q^{2}}\ . \tag{29}$$

若 $M^2=(1-2\dot{r}_{\mathrm{H}}\mathrm{e}^{-\psi})Q^2$，则由(22)式知两视界面重合为

$$r_{\mathrm{H}}=\frac{M}{1-2\dot{r}_{\mathrm{H}}\mathrm{e}^{-\psi}}\ .$$

将此结果代入(24)式，可以证明

$$\kappa=0\ . \tag{30}$$

这违背黑洞热力学第三定律.故广义球对称带电蒸发黑洞的两视界面永不重合.

将(18)，(19)式代入(13)式，并再次分离变量，令

$$f_1=k_+(v_*,r_*)\Theta_+(\theta_*)=R_+\Theta_+,$$
$$f_2=k_-\Theta_-,\quad g_1=k_-\Theta_+,\quad g_2=k_+\Theta_-, \tag{31}$$

则可得到

$$\frac{1}{k_-}\sqrt{2}r\mathscr{D}_1k_++\mathrm{i}\mu r=\frac{1}{\Theta_+}(\mathscr{L}'_+-\cot\theta)\Theta_-=-\lambda_1,$$
$$\frac{1}{k_+}\sqrt{2}r\mathscr{D}_2k_--\mathrm{i}\mu r=-\frac{1}{\Theta_-}\mathscr{L}'_-\Theta_+=-\lambda_2,$$
$$\frac{1}{k_-}\sqrt{2}r\mathscr{D}_1k_++\mathrm{i}\mu r=-\frac{1}{\Theta_-}(\mathscr{L}'_--\cot\theta)\Theta_+=-\lambda_1,$$
$$\frac{1}{k_+}\sqrt{2}r\mathscr{D}_2k_--\mathrm{i}\mu r=\frac{1}{\Theta_+}\mathscr{L}'_+\Theta_-=-\lambda_2, \tag{32}$$

式中 $\mathscr{L}'_\pm=\partial/\partial\theta_*\pm m/\sin\theta+\frac{1}{2}\cot\theta$，$\mathscr{D}_1$ 和 $\mathscr{D}_2$ 仍如(18)式所示.解径向方程组

$$\sqrt{2}r\mathscr{D}_1k_++(\lambda_1+\mathrm{i}\mu r)k_-=0\ ,$$
$$\sqrt{2}r\mathscr{D}_2k_-+(\lambda_2-\mathrm{i}\mu r)k_+=0\ . \tag{33}$$

在视界附近，它可以化成波动方程的标准形式

$$\partial^2k_-/\partial r_*^2+2\mathrm{e}^{-\psi}\partial^2k_-/\partial v_*\partial r_*+(\alpha_0+\mathrm{i}w_0)\partial k_-/\partial r_*=0\ , \tag{34}$$

其解为

$$k_-(v_*,r_*)_{\mathrm{in}}=\mathrm{e}^{-\mathrm{i}wv_*}$$
$$k_-(v_*,r_*)_{\mathrm{out}}=\mathrm{e}^{-\mathrm{i}wv_*}\mathrm{e}^{-\alpha_0r_*}\mathrm{e}^{\mathrm{i}(w+w_0)r_*}\ . \tag{35}$$

将出射波作解析延拓后，按照 Sannan 等提供的方法[7]可得出射波的黑体谱为

$$N(w) = [\exp(w - w_0)/k_B T + 1]^{-1}, \tag{36}$$

式中 $T = k/2\pi k_B$ 为 Hawking 温度，k_B 为 Boltzmann 常量，$w_0 = e\theta/r$ 为时空(1)式中的 Dirac 粒子的固有电磁能，而

$$\alpha_0 = e^{-\psi}\psi'\dot{r}_H + M/2r^2 - M'/r - \dot{r}_H(\psi'/2 + 1/r) + \frac{1}{2}(1 - 2M/r + Q^2/r^2)$$

$$\times(\psi'/2 + 1/r) + \frac{1}{2r}(1 + Q^2/r^2) - M\psi'/2r + \psi'/4 .$$

3 量子非热效应

广义球对称带电蒸发黑洞不但有量子热效应，也有量子非热效应. Dirac 真空是量子场的基态，在一定条件下它可以被激发而产生正负实粒子对. 黑洞视界的静电势会导致视界附近 Dirac 真空的正负能级交错. 如果 Dirac 真空中的正负虚粒子对在电磁场的作用下分离到 Compton 波长 λ 之前得到了足够的能量，则可以实化[8]能量为 $2\mu c^2$. 因此正负虚粒子对实化的必要条件是

$$ev = eE\lambda \geqslant 2\mu ,$$

或

$$E \geqslant \frac{2\mu}{e\lambda} . \tag{37}$$

这里已令 $c = 1$.

下面推导非热辐射粒子的能量范围. 荷电为 e、质量为 μ 的粒子在弯曲时空中的运动方程是 Hamilton-Jacobi 方程[9]

$$g^{\mu\nu}\left(\frac{\partial S}{\partial \chi^\mu} - eA_\mu\right)\left(\frac{\partial S}{\partial \chi^\nu} - eA_\nu\right) - \mu^2 = 0 , \tag{38}$$

式中 $S = \int L\mathrm{d}\tau$ 是 Hamilton 主函数. 把(3)，(9)两式代入(38)式得

$$2e^{-\psi}(\partial S/\partial v + eQ/r)(\partial S/\partial r) + \frac{\Delta}{r^2}(\partial S/\partial r)^2 + \frac{1}{r^2}(\partial S/\partial \theta)^2$$

$$+ \frac{1}{r^2\sin^2\theta}(\partial S/\partial \varphi)^2 + \mu^2 = 0 . \tag{39}$$

再把(16)式代入上式，得

$$2e^{-\psi}\left[(\partial S/\partial v_* - \frac{\dot{r}_H}{\rho}\partial S/\partial r_*) + eQ/r\right]\frac{\rho + 1}{\rho}\partial S/\partial r_* + \frac{\Delta}{r^2}\left(\frac{\rho + 1}{\rho}\right)^2(\partial S/\partial r_*)^2$$

$$+ \frac{1}{r^2}(\partial S/\partial \theta_* - \frac{r_{H,\theta}}{\rho}\partial S/\partial r_*)^2 + \frac{1}{r^2\sin^2\theta}(\partial S/\partial \varphi)^2 + \mu^2 = 0 . \tag{40}$$

可以证明，线元(1)式描述的时空存在 Killing 矢量$(\partial/\partial\varphi)^a$，故有

$$\partial S/\partial \varphi = m(\text{常数}) . \tag{41}$$

定义

$$w = - \partial S/\partial v_* , \tag{42}$$

$$l = \partial S/\partial \theta_* , \tag{43}$$

显然，w 和 l 分别为 Dirac 粒子的能量和角动量. 于是(40)式变成

$$[\frac{\Delta}{r^2}(\frac{\rho+1}{\rho})^2 - 2e^{-\psi}\frac{\dot{r}_H}{\rho}\frac{(\rho+1)}{\rho} + (r_{H,\theta}/r\rho)^2](\partial S/\partial r_*)^2 + [2e^{-\psi}(eQ/r - w)\frac{\rho+1}{\rho}$$
$$-\frac{2lr_{H,\theta}}{r^2\rho}](\partial S/\partial r_*) + l^2/r^2 + m^2/r^2\sin^2\theta + \mu^2 = 0\ . \tag{44}$$

S 和 $\partial S/\partial r_*$ 都不应为虚数,故必有

$$[\frac{lr_{H,\theta}}{r^2\rho} - e^{-\psi}(\frac{eQ}{r} - w)(\frac{\rho+1}{\rho})]^2 - [(\frac{\Delta}{r^2}(\frac{\rho+1}{\rho})^2 - 2e^{-\psi}\frac{\dot{r}_H(\rho+1)}{\rho^2}$$
$$+ r_{H,\theta}^2/r^2\rho^2](l^2/r^2 + m^2/r^2\sin^2\theta + \mu^2) \geqslant 0\ . \tag{45}$$

这就是动态广义球对称带电蒸发黑洞周围时空中粒子能量 w 的方程,满足此方程的粒子能级才能存在,由此式可得

$$w^{\pm} = eQ/r - lr_{H,\theta}e^{\psi}/(\rho+1)r^2$$
$$\pm\sqrt{(A\rho^2 + r_{H,\theta}/r^2)(l^2/r^2 + m^2/r^2\sin^2\theta + \mu^2)}/(\rho+1)e^{-\psi}, \tag{46}$$

式中 $A = \frac{\Delta}{r^2}(\frac{\rho+1}{\rho})^2 - 2e^{-\psi}\frac{\dot{r}_H(\rho+1)}{\rho^2}$.

Dirac 真空能级分布是

$$w \geqslant w^+, \qquad w \leqslant w^-. \tag{47}$$

粒子能态的禁区是 $w^- < w < w^+$,禁区宽度为

$$\Delta\omega = 2\sqrt{(A\rho^2 + r_{H,\theta}^2/r^2)(l^2/r^2 + m^2/r^2\sin^2\theta + \mu^2)}/(\rho+1)e^{-\psi}. \tag{48}$$

讨论:

1)当 r 很大时,电磁场可以忽略,且时空(1)式应渐近平直,即应有 $e^{\psi}\to 1$,故由(46)式可得

$$w^{\pm} = \pm e^{\psi}\mu = \pm\mu\ . \tag{49}$$

2)当 $r\to r_H$ 时,即在黑洞视界附近,由(19),(21)式

$$\rho^2 A\,|_{r\to r_H} = 0 \qquad r_{H,\theta} = 0\ ,$$

所以

$$w^{\pm} = eQ/r = w_0, \tag{50}$$

$$\Delta w\,|_{r\to r_H} = 0\ . \tag{51}$$

可见在黑洞视界附近,禁区宽度为零,正负能级发生交错现象.在交错区域,处于负能态而能量高于处于正能态的最低能量的粒子,将由于量子隧道效应而穿过禁区,成为正能粒子出射到远方.这是一种与温度无关的自发辐射过程,称作 Starobinsky-Unruh 过程.粒子能量 w 只要满足

$$w_0 \geqslant w > \mu\ , \tag{52}$$

便可离开视界到达远方.

对于玻色子,伴随自发辐射,还会有类似原子受激发射的受激辐射发生,通常称为 Misner 超辐射.自发辐射和受激辐射都是非热辐射.非热辐射使广义球对称带电蒸发黑洞最终变成广义 Vaidya 黑洞.

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THE QUANTUM THERMAL AND NON - THERMAL EFFECTS IN THE SPACE-TIME DETERMINED BY GSSEC BLACK HOLE

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(Received 18 October 1994)

ABSTRACT

The Hawking radiation and the Starobinsky - Unruh process of Dirac particles in space-time determined by the generalized spherical symmetric evaporating charged (GSSEC) black hole are studied.

PACC: 0420; 9760L