

黑洞的温度和熵变化是时间尺度变换的 补偿效应^{*}

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以史瓦西黑洞和 Vaidya 黑洞为例, 研究了时间尺度变换的补偿效应. 规范势就是仿射联络的缩并, 黑洞的熵变化和 Hawking 温度都可看作补偿场的纯规范势.

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1 引 言

最近, 我们以稳态黑洞为例, 论证了黑洞的量子热效应可以看作坐标尺度变换的补偿效应^[1], Hawking 温度以补偿场纯规范势的形式出现. 本文把这一工作推广到动态黑洞, 指出不仅 Hawking 温度, 而且黑洞的熵变化, 都可看作补偿场的纯规范势. 还论证了坐标尺度变换实质上是时间尺度变换, 补偿场的规范势即时空联络的缩并. 规范场强恒为零, 因而时间尺度虽然是时空点的函数, 但与平移路径无关.

2 史瓦西时空

史瓦西时空线元

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (1)$$

在乌龟坐标

$$r_* = r + \frac{1}{2\kappa} \ln \left(\frac{r}{2M} - 1 \right) \quad (2)$$

下, 可以写成

$$ds^2 = \left(1 - \frac{2M}{r} \right) (-dt^2 + dr_*^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (3)$$

或

$$ds^2 = \frac{1}{2\kappa r} e^{2\kappa(r_* - r)} (-dt^2 + dr_*^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (4)$$

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变换到 Kruskal 坐标,

$$T = \frac{1}{\kappa} e^{\kappa r_*} \sinh \kappa t, \quad R = \frac{1}{\kappa} e^{\kappa r_*} \cosh \kappa t, \quad (5)$$

线元又可改写成

$$ds^2 = \frac{1}{2\kappa r} e^{-2\kappa r} (-dT^2 + dR^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (6)$$

显然, 坐标变换(5)式在二维 \$(t, r_*)\$ 时空中诱导出一个共形等度规映射

$$-dt^2 + dr_*^2 = e^{-2\kappa r_*} (-dT^2 + dR^2), \quad (7)$$

共形因子为

$$\Omega^2 = e^{2\kappa r_*}. \quad (8)$$

从(4)和(6)式可知, 二维子时空 \$(t, r_*)\$ 或 \$(T, R)\$ 的线元可写为

$$d\tilde{s}^2 = \Omega_1^2 d\tilde{s}_1^2, \quad d\tilde{s}^2 = \Omega_2^2 d\tilde{s}_2^2, \quad (9)$$

其中

$$\Omega_1^2 = \frac{1}{2\kappa r} e^{-2\kappa r}, \quad \Omega_2^2 = \frac{1}{2\kappa r} e^{2\kappa(r_* - r)}, \quad (10)$$

$$d\tilde{s}_1^2 = -dT^2 + dR^2, \quad d\tilde{s}_2^2 = -dt^2 + dr_*^2. \quad (11)$$

显然, (8)式所示的 \$\Omega\$ 满足

$$\Omega = \Omega_2 / \Omega_1. \quad (12)$$

(9)式所示的两个线元, 都显式共形于二维闵氏时空. 我们定义 \$d\tilde{s}^2\$ 为二维时空的坐标长度. 线元(9)式表现为坐标长度与共形因子的乘积. 其中共形因子 \$\Omega_1\$ 与 \$\Omega_2\$ 表征坐标长度的尺度伸缩. 所以坐标变换(5)式可以看作坐标长度的尺度变换. 另一方面, 由于 \$\Omega_1\$ 与 \$\Omega_2\$ 均是时空点的函数, 坐标长度的尺度标准还随时空点而异, 甚至可能与平移路径有关.

假定有一个二维无穷小矢量 \$B_\mu\$, 它在 \$P\$ 点的固有长度为 \$L = (B_\mu B^\mu)^{1/2} = d\tilde{s}\$, 而坐标长度定义为 \$l \equiv d\tilde{s}_1\$, \$l' \equiv d\tilde{s}_2\$. 在广义相对论中, 联络被选作克氏符, 因而 \$L\$ 在平移下不变. 于是从(9)式可得

$$\delta L = 0, \quad \Omega_1 \delta l + l \delta \Omega_1 = 0, \quad \Omega_2 \delta l' + l' \delta \Omega_2 = 0. \quad (13)$$

当取 \$x^\mu = (T, R)\$, \$x'^\mu = (t, r_*)\$ 时, 二维时空的联络的缩并可写为

$$\Gamma_{\alpha\mu}^\alpha = \frac{\partial}{\partial x^\mu} \ln \sqrt{-g} = 2 \frac{\partial \ln \Omega_1}{\partial x^\mu}, \quad \Gamma_{\alpha\mu}^{\prime\alpha} = \frac{\partial}{\partial x'^\mu} \ln \sqrt{-g'} = 2 \frac{\partial \ln \Omega_2}{\partial x'^\mu}. \quad (14)$$

上式又可写作

$$\begin{aligned} \frac{1}{2} \Gamma_{\alpha\mu}^\alpha dx^\mu &= \frac{\partial \ln \Omega_1}{\partial x^\mu} dx^\mu = \delta \ln \Omega_1; \\ \frac{1}{2} \Gamma_{\alpha\mu}^{\prime\mu} dx'^\mu &= \frac{\partial \ln \Omega_2}{\partial x'^\mu} dx'^\mu = \delta \ln \Omega_2. \end{aligned} \quad (15)$$

从(13)式可得

$$\delta \ln l = -A_\mu dx^\mu, \quad \delta \ln l' = -A'_\mu dx'^\mu, \quad (16)$$

其中

$$A_\mu = \frac{1}{2} \Gamma_{\alpha\mu}^\alpha, \quad A'_\mu = \frac{1}{2} \Gamma_{\alpha\mu}^{\prime\alpha}. \quad (17)$$

(16)式表示坐标长度在平移下发生了变化, \$A_\mu\$ 就是变化率. 这可归因于坐标长度的尺度基准在平移下发生了变化. \$A_\mu\$ 与 \$A'_\mu\$ 不同, 表明尺度基准在不同坐标系下的变化规律也不同. 众所周知, 联络的缩并在坐标变换下的变换式为

$$\Gamma_{\alpha\mu}^{\prime\alpha} = \Gamma_{\alpha\nu}^\alpha \frac{\partial x^\nu}{\partial x'^\mu} + \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^\alpha}{\partial x'^\mu} \right) \quad (18)$$

$$\text{或} \quad A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^\alpha}{\partial x'^\mu} \right). \quad (19)$$

(19)式表示尺度基准在坐标变换下如何变化. 显然, A_μ 不是矢量, 它在坐标变换下不按矢量的规律变化. 依据 Weyl^[2] 的思想, 我们认为坐标长度尺度基准的变化, 产生了一个补偿场, A_μ 就是补偿场的势. 补偿场强可用二维曲率张量

$$R_{\lambda\mu\nu}^\sigma = \Gamma_{\lambda\nu, \mu}^\sigma - \Gamma_{\lambda\mu, \nu}^\sigma + \Gamma_{\sigma\mu}^\rho \Gamma_{\lambda\nu}^\sigma - \Gamma_{\sigma\nu}^\rho \Gamma_{\lambda\mu}^\sigma \quad (20)$$

的缩并来定义

$$F_{\mu\nu} \equiv \frac{1}{2} R_{\lambda\mu\nu}^\lambda = \frac{1}{2} \Gamma_{\lambda\nu, \mu}^\lambda - \frac{1}{2} \Gamma_{\lambda\mu, \nu}^\lambda = A_{\nu, \mu} - A_{\mu, \nu}. \quad (21)$$

对换指标 σ 与 λ , 可知上式中联络的乘积项相互抵消. 从(14)式可知, $A_{\nu, \mu} = A_{\mu, \nu}$, 所以 $F_{\mu\nu} = 0$. 从曲率张量的对称性也可知 $F_{\mu\nu} = 0$. 总之, 补偿场强是张量, 但恒为零. 这表明, 尺度基准(因而坐标长度)虽然随时空点变化, 但与平移路径无关.

从(7)式到(21)式的讨论均是在二维子时空 (t, r_*) 或 (T, R) 中进行的. 所涉及的矢量、联络、曲率张量等, 均是二维子时空中的量. 由于我们采用自然单位制, 从(12)式可知, 时间坐标 t (或 T) 与空间坐标 r_* (或 R) 有相同的坐标尺度. 所以, 上面讨论的坐标尺度变换, 实际上就是时间尺度变换. 由于二维子时空与四维时空的坐标时间相同, 上述尺度变换不仅可看作二维子时空中的时间尺度变换, 也可看作四维时空中的时间尺度变换. 因此, A_μ 就是时间尺度变换产生的补偿场的规范势.

由于联络选作克氏符, (21)式中 $F_{\mu\nu}$ 为零, 使得坐标时间的尺度与平移路径无关. 即钟速同步具有传递性, 热力学第零定律成立^[3].

从(9)式可知, $L^2 = \Omega_1^2 t^2 = \Omega_2^2 l'^2$, 即

$$l' = \Omega^{-1} l. \quad (22)$$

$$\text{利用(16)和(17)式可得} \quad A'_\mu dx'^\mu = \left(A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{\partial \ln \Omega}{\partial x'^\mu} \right) dx'^\mu. \quad (23)$$

由于 dx'^μ 的任意性, 有

$$A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{\partial \ln \Omega}{\partial x'^\mu}, \quad (24)$$

其中 Ω 如(8)式所示. 与(19)式比较, 可知

$$\frac{\partial \ln \Omega}{\partial x'^\mu} = \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^\alpha}{\partial x'^\mu} \right). \quad (25)$$

它是补偿场的线规范势. 用坐标变换(5)式, 也可证明(25)式. 把(8)式代入, 可得

$$\frac{\partial \ln \Omega}{\partial t} = 0, \quad \frac{\partial \ln \Omega}{\partial r_*} = \kappa. \quad (26)$$

可见, 事件视界的表面重力 κ 作为时间尺度变换的纯规范势出现. 也即, 黑洞的 Hawking 温度

$$T = \kappa / 2\pi K_B \quad (27)$$

作为时间尺度变换的纯规范势出现. 所以, 可以把 Hawking 效应看作时间尺度变换下的补偿效应.

3 爱丁顿坐标下的史瓦西时空

用超前爱丁顿坐标表示的史瓦西时空

$$ds^2 = -(1 - 2M/r)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (28)$$

式中超前与迟后爱丁顿坐标定义为

$$v = t + r_*, \quad u = t - r_*, \quad (29)$$

r_* 为(2)式所示的乌龟坐标. 把 r 换成 r_* , (28)式改写为

$$\begin{aligned} ds^2 &= (1 - 2M/r)(-dv^2 + 2dvdr_*) + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\ &= \frac{e^{2\kappa(r_* - r)}}{2\kappa r}(-dv^2 + 2dvdr_*) + r^2(d\theta^2 + \sin^2\theta d\phi^2), \end{aligned} \quad (30)$$

式中 $\kappa = 1/4M$. 引入类光 Kruskal 坐标

$$U = -\frac{1}{\kappa}e^{-\kappa u}, \quad V = \frac{1}{\kappa}e^{\kappa v}. \quad (31)$$

它与(5)式所示的 Kruskal 坐标有以下关系:

$$T = (V + U)/2, \quad R = (V - U)/2. \quad (32)$$

于是(30)式可化成

$$ds^2 = \frac{e^{-2\kappa r}}{2\kappa r}(-dV^2 + 2dVdR) + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (33)$$

仿照前面的讨论, 可得

$$d\tilde{s}^2 = \Omega_1^2 d\tilde{s}_1^2, \quad d\tilde{s}^2 = \Omega_2^2 d\tilde{s}_2^2, \quad (34)$$

$$d\tilde{s}_1^2 = -dV^2 + 2dVdR, \quad d\tilde{s}_2^2 = -dv^2 + 2dvdr_*, \quad (35)$$

$$\Omega_1^2 = \frac{1}{2\kappa r}e^{-2\kappa r}, \quad \Omega_2^2 = \frac{1}{2\kappa r}e^{2\kappa(r_* - r)}, \quad \Omega^2 = \Omega_2^2/\Omega_1^2 = e^{2\kappa r_*}. \quad (36)$$

同样存在时间尺度变换的补偿场

$$A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^\alpha}{\partial x'^\mu} \right) \quad (37)$$

或

$$A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{\partial \ln \Omega}{\partial x'^\mu}. \quad (38)$$

同样有(26)式成立, Hawking 温度以补偿场纯规范势的形式出现.

4 Vaidya 时空

下面考察 Vaidya 时空^[4].

$$ds^2 = -[1 - 2M(v)/r]dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (39)$$

作广义乌龟坐标变换

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa} \ln \left[\frac{r - r_H(v)}{r_0} \right]; \\ v_* &= v - v_0, \end{aligned} \quad (40)$$

$$dr = \frac{2\kappa(r-r_H)}{2\kappa(r-r_H)+1}dr_* + \frac{r_H}{2\kappa(r-r_H)+1}dv_*; \quad (41)$$

$$dv = dv_*,$$

式中 $r_H = r_H(v)$; $r_H = dr_H/dv$; $r_0 = r_H(v_0)$; v_0 是所研究的热辐射粒子脱离黑洞表面的时刻^[4]; κ 为可调节参数. κ , v_0 和 r_0 在乌龟坐标变换下均是常数. 于是, 线元(39)式可改写为

$$ds^2 = \frac{(r-2M)[2\kappa(r-r_H)+1]-2r\dot{r}_H}{r[2\kappa(r-r_H)+1]} - dv_*^2 + 2 \frac{2\kappa r(r-r_H)}{(r-2M)[2\kappa(r-r_H)+1]-2r\dot{r}_H} dv_* dr_* + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (42)$$

$$\text{引入} \quad u_* = v_* - 2r_* \quad (43)$$

及广义 Kruskal 坐标

$$U = -\frac{1}{\kappa}e^{-\kappa u_*}, \quad V = \frac{1}{\kappa}e^{\kappa v_*}, \quad (44)$$

$$T = (V+U)/2, \quad R = (V-U)/2,$$

于是有

$$dv_*^2 = e^{-2\kappa v_*} dV^2,$$

$$dv_* dr_* = e^{-2\kappa r_*} dV dR = \frac{1}{2}(e^{-2\kappa r_*} - e^{-2\kappa v_*}) dV^2.$$

线元(42)式可重写为

$$ds^2 = \frac{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2\dot{r}_H) - 2M[2\kappa(r-r_H)+1]\}e^{-2\kappa v_*}}{r[2\kappa(r-r_H)+1]} - dV^2 + 2 \frac{2\kappa r(r-r_H)e^{-2\kappa r_*}}{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2\dot{r}_H) - 2M[2\kappa(r-r_H)+1]\}e^{-2\kappa v_*}} dV dR + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (46)$$

利用 $\kappa = 1/2 r_H(v_0) = [1-2\dot{r}_H(v_0)]/4M$ 及 $r_H = 2M/(1-2\dot{r}_H)$ 不难证明

$$\lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{2\kappa r(r-r_H)}{(r-2M)[2\kappa(r-r_H)+1]-2r\dot{r}_H} = 1, \quad (47)$$

$$\lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{2\kappa r(r-r_H)e^{-2\kappa r_*}}{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2\dot{r}_H) - 2M[2\kappa(r-r_H)+1]\}e^{-2\kappa v_*}} = 1. \quad (48)$$

所以, (42)和(46)式的二维子时空线元为

$$d\tilde{s}^2 = \Omega_1^2 d\tilde{s}_1^2, \quad d\tilde{s}^2 = \Omega_2^2 d\tilde{s}_2^2, \quad (49)$$

$$\Omega_1^2 = \frac{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2\dot{r}_H) - 2M[2\kappa(r-r_H)+1]\}e^{-2\kappa v_*}}{r[2\kappa(r-r_H)+1]} \quad (50)$$

$$\Omega_2^2 = \frac{(r-2M)[2\kappa(r-r_H)+1]-2r\dot{r}_H}{r[2\kappa(r-r_H)+1]},$$

$$\begin{aligned} d\tilde{s}_1^2 &= -dV^2 + 2 \frac{2\kappa r(r-r_H)e^{-2\kappa r_*}}{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2r_H) - 2M[2\kappa(r-r_H) + 1]\}e^{-2\kappa r_*}} dV dR, \\ d\tilde{s}_2^2 &= -dv_*^2 + 2 \frac{2\kappa r(r-r_H)}{(r-2M)[2\kappa(r-r_H) + 1] - 2r\dot{r}_H} dv_* dr_*. \end{aligned} \quad (51)$$

在事件视界附近有

$$d\tilde{s}_1^2 \rightarrow -dV^2 + 2dV dR, \quad d\tilde{s}_2^2 \rightarrow -dv_*^2 + 2dv_* dr_*. \quad (52)$$

因此,在事件视界附近,从(42)式到(46)式的坐标变换(44)式,仍可看作诱导出一个共形等度规映射

$$d\tilde{s}_2^2 = \Omega^{-2} d\tilde{s}_1^2 \quad (53)$$

$$\text{或} \quad -dv_*^2 + 2dv_* dr_* \simeq \Omega^{-2}(-dV^2 + 2dV dR), \quad (54)$$

其中

$$\Omega^2 \equiv \frac{\Omega_2^2}{\Omega_1^2} = \frac{(r-2M)[2\kappa(r-r_H) + 1] - 2r\dot{r}_H}{2\kappa r(r-r_H)e^{-2\kappa r_*} + \{r(1-2r_H) - 2M[2\kappa(r-r_H) + 1]\}e^{-2\kappa r_*}}. \quad (55)$$

类似地,可认为存在时间尺度变换,其补偿场的规范势为

$$A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^\alpha}{\partial x'^\mu} \right) \quad (56)$$

$$\text{或} \quad A'_\mu = A_\nu \frac{\partial x^\nu}{\partial x'^\mu} + \frac{\partial \ln \Omega}{\partial x'^\mu}, \quad (57)$$

其中 $x^\mu = (V, R)$, $x'^\mu = (v_*, r_*)$. 利用(55)式可以算出纯规范势 $\frac{\partial \ln \Omega}{\partial x'^\mu}$. 从(41)式可知

$$\frac{\partial r}{\partial r_*} = \frac{2\kappa(r-r_H)}{2\kappa(r-r_H) + 1}, \quad \frac{\partial r}{\partial v_*} = \frac{\dot{r}_H}{2\kappa(r-r_H) + 1}, \quad \frac{\partial v}{\partial r_*} = 0, \quad \frac{\partial v}{\partial v_*} = 1. \quad (58)$$

$$\text{所以} \quad \lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{\partial \ln \Omega}{\partial r_*} = \kappa. \quad (59)$$

此处用了(40), (47)式及 $\kappa = (1-2r_H)/4M$, $r_H = 2M/(1-2r_H)$. 同样可算得

$$\lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{\partial \ln \Omega}{\partial v_*} = -\kappa \dot{r}_H (1-2r_H). \quad (60)$$

这里用了(58)式及 $2M = r_H(1-2r_H)$, $2\dot{M} = \dot{r}_H(1-2r_H) - 2r_H \dot{r}_H$, $r - r_H = r_0 e^{2\kappa(r_* - r)}$, 还使用了洛必大法则.

我们看到, Vaidya 黑洞与史瓦西黑洞不同, 补偿场纯规范势的两个分量均不为零. 如果仍然把黑洞事件视界面积 A 看作黑洞熵 S ,

$$S = A/4 = \pi r_H^2, \quad (61)$$

$$\text{那么} \quad \sigma = \frac{1}{4} \frac{d \ln S}{d v_0} = \frac{\dot{r}_H}{2 r_H} = \kappa \dot{r}_H, \quad (62)$$

于是有

$$\kappa = \lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{\partial \ln \Omega}{\partial r_*}, \quad (63)$$

$$\tilde{\sigma} \equiv \sigma(1 - 2r_H) = - \lim_{\substack{r \rightarrow r_H \\ v \rightarrow v_0}} \frac{\partial \ln \Omega}{\partial v_*}, \quad (64)$$

κ 决定黑洞的温度, σ 决定黑洞熵的相对变化率, $\tilde{\sigma}$ 反映黑洞的熵变化.

5 结论与讨论

黑洞的量子热效应可以看作时间尺度变换的补偿效应. 黑洞的温度与熵变化都以补偿场纯规范势的形式出现.

值得注意的是, 动态黑洞补偿场的纯规范势, 仅在视界上取值时才反映黑洞的温度和熵变化. 而静态情况, 反映黑洞温度的纯规范势则不需要“限定”在视界上取值. 这可能是由于目前采用的动态情况的乌龟坐标变换式(40), 仅在视界上精确成立的缘故.

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BOTH THE ENTROPY CHANGE AND THE TEMPERATURE OF BLACK HOLES ARE THE COMPENSATE EFFECT UNDER THE TIME-SCALE TRANSFORMATION

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ABSTRACT

The compensate effect under the time-scale transformation for the Schwarzschild black hole and Vaidya black hole are studied. The gauge potential is the contraction of the affine connection. Both the Hawking temperature and the change of entropy of black holes can be regarded as the pure-gauge potentials of the compensate field.

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