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Vaidya-Bonner-de Sitter 时空中荷电 Dirac 粒子的 Hawking 辐射

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在 Vaidya-Bonner-de Sitter 时空中, 准确地给出了黑洞内外视界及宇宙视界的位置, 研究了该时空中黑洞外视界和宇宙视界附近荷电 Dirac 粒子的 Hawking 辐射, 确定三个视界面中任何两个都不可能重合.

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1 引 言

文献[1]给出的 Vaidya-Bonner-de Sitter 时空线元用超前爱丁顿坐标表示为

$$ds^2 = \left[1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} - \chi^2 r^2\right] dv^2 - 2dv dr - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1}$$

这里采用号差(+1, −1, −1, −1), 此度规行列式为 $g = -r^4\sin^2\theta$, 非零逆变分量为

$$g^{01} = g^{10} = -1, \quad g^{11} = -\frac{B}{r^2}, \quad g^{22} = -\frac{1}{r^2}, \quad g^{33} = -\frac{1}{r^2\sin^2\theta}, \tag{2}$$

其中 $B = r^2 - 2Mr + Q^2 - \chi^2 r^4$.

本文在该时空中, 给出了黑洞内外视界和宇宙视界准确位置, 研究了荷电 Dirac 粒子在黑洞外视界和宇宙视界附近的 Hawking 辐射, 得到这两个视界准确的 Hawking 温度和 Hawking 辐射谱, 确定上述三个视界面中任何两个都不可能重合.

2 视界面

现在用零曲面条件

$$g^{\mu\nu}\frac{\partial F}{\partial \chi^\mu}\frac{\partial F}{\partial \chi^\nu} = 0 \tag{3}$$

来寻找上述时空的局部事件视界, 考虑到该时空球对称性, 曲面方程

$$F = F(v, r) = 0. \tag{4}$$

由(2)—(4)式可得视界面方程

$$-\chi^2 r_H^4 + r_H^2(1 - 2\dot{r}_H) - 2Mr_H + Q^2 = 0. \tag{5}$$

当 r_H 为黑洞外视界时，由(5)式得

$$r_H \equiv r_+ = \sqrt{Z_1^+} - \sqrt{Z_2^+} + \sqrt{Z_3^+}, \tag{6}$$

其中

$$\sqrt{Z_1^+} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_+) + \sqrt{(1-2\dot{r}_+)^2 - 12Q^2\chi^2}\cos\frac{\alpha_+}{3}\right]^{1/2},$$

$$\sqrt{Z_2^+} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_+) - \sqrt{(1-2\dot{r}_+)^2 - 12Q^2\chi^2}\cos\left(\frac{\alpha_+}{3} + \frac{\pi}{3}\right)\right]^{1/2},$$

$$\sqrt{Z_3^+} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_+) - \sqrt{(1-2\dot{r}_+)^2 - 12Q^2\chi^2}\cos\left(\frac{\alpha_+}{3} - \frac{\pi}{3}\right)\right]^{1/2},$$

$$\alpha_+ = \arccos\left[-\frac{1 - 18\chi^2\left(\dfrac{3M^2}{(1-2\dot{r}_+)^3} - \dfrac{2Q^2}{(1-2\dot{r}_+)^2}\right)}{\left(1 - \dfrac{12Q^2\chi^2}{(1-2\dot{r}_+)^2}\right)^{3/2}}\right]. \tag{7}$$

当 r_H 为黑洞内视界时，由(5)式得

$$r_H \equiv r_- = -\sqrt{Z_1^-} + \sqrt{Z_2^-} + \sqrt{Z_3^-}, \tag{8}$$

其中

$$\sqrt{Z_1^-} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_-) + \sqrt{(1-2\dot{r}_-)^2 - 12Q^2\chi^2}\cos\frac{\alpha_-}{3}\right]^{1/2},$$

$$\sqrt{Z_2^-} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_-) - \sqrt{(1-2\dot{r}_-)^2 - 12Q^2\chi^2}\cos\left(\frac{\alpha_-}{3} + \frac{\pi}{3}\right)\right]^{1/2},$$

$$\sqrt{Z_3^-} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_-) - \sqrt{(1-2\dot{r}_-)^2 - 12Q^2\chi^2}\cos\left(\frac{\alpha_-}{3} - \frac{\pi}{3}\right)\right]^{1/2},$$

$$\alpha_- = \arccos\left[-\frac{1 - 18\chi^2\left(\dfrac{3M^2}{(1-2\dot{r}_-)^3} - \dfrac{2Q^2}{(1-2\dot{r}_-)^2}\right)}{\left(1 - \dfrac{12Q^2\chi^2}{(1-2\dot{r}_-)^2}\right)^{3/2}}\right]. \tag{9}$$

当 r_H 为宇宙视界时，同样由(5)式得

$$r_H \equiv r_c = \sqrt{Z_1^c} + \sqrt{Z_2^c} - \sqrt{Z_3^c}, \tag{10}$$

其中

$$\sqrt{Z_1^c} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_c) + \sqrt{(1-2\dot{r}_c)^2 - 12Q^2\chi^2}\cos\frac{\alpha_c}{3}\right]^{1/2},$$

$$\sqrt{Z_2^c} = \frac{1}{\sqrt{6}\chi}\left[(1-2\dot{r}_c) - \sqrt{(1-2\dot{r}_c)^2 - 12Q^2\chi^2}\cos\left(\frac{\alpha_c}{3} + \frac{\pi}{3}\right)\right]^{1/2},$$

$$\sqrt{Z_3^c}=\frac{1}{\sqrt{6}\,\chi}\left[(1-2\dot{r}_c)-\sqrt{(1-2\dot{r}_c)^2-12Q^2\chi^2}\cos\left(\frac{\alpha_c}{3}-\frac{\pi}{3}\right)\right]^{1/2},$$

$$\alpha_c=\arccos\left[-\frac{1-18\chi^2\left(\frac{3M^2}{(1-2\dot{r}_c)^3}-\frac{2Q^2}{(1-2\dot{r}_c)^2}\right)}{\left(1-\frac{12Q^2\chi^2}{(1-2\dot{r}_c)^2}\right)^{3/2}}\right], \tag{11}$$

式中 r_+, r_-, r_c 分别代表黑洞外视界、内视界和宇宙视界的精确值. $\dot{r}_H=\frac{\partial r_H}{\partial v}$, $\dot{r}_+=\frac{\partial r_+}{\partial v}$, $\dot{r}_-=\frac{\partial r_-}{\partial v}$, $\dot{r}=\frac{\partial r_c}{\partial v}$.

3 荷电 Dirac 粒子的 Hawking 辐射

弯曲时空中荷电粒子 Dirac 方程可写为[2]

$$\begin{aligned}
&(D+\varepsilon-\rho+\mathrm{i}eA_\mu l^\mu)F_1+(\bar{\delta}+\pi-\alpha+\mathrm{i}eA_\mu\bar{m}^\mu)F_2-\frac{1}{\sqrt{2}}\mathrm{i}\mu_0G_1=0,\\
&(\Delta+\mu-\gamma+\mathrm{i}eA_\mu n^\mu)F_2+(\delta+\beta-\tau+\mathrm{i}eA_\mu m^\mu)F_1-\frac{1}{\sqrt{2}}\mathrm{i}\mu_0G_2=0,\\
&(D+\varepsilon^*-\rho^*+\mathrm{i}eA_\mu l^\mu)G_2-(\delta+\pi^*-\alpha^*+\mathrm{i}eA_\mu m^\mu)G_1-\frac{1}{\sqrt{2}}\mathrm{i}\mu_0F_2=0,\\
&(\Delta+\mu^*-\gamma^*+\mathrm{i}eA_\mu n^\mu)G_1-(\bar{\delta}+\beta^*-\tau^*+\mathrm{i}eA_\mu\bar{m}^\mu)G_2-\frac{1}{\sqrt{2}}\mathrm{i}\mu_0F_1=0,
\end{aligned} \tag{12}$$

μ_0, e 分别为 Dirac 粒子的静质量和电荷, 其中

$$F_1=P^0,\quad F_2=P^1,\quad G_1=Q^{*1'},\quad G_2=-Q^{*0'},$$

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$$A_\mu=\left(\frac{Q}{r},0,0,0\right). \tag{13}$$

在 Vaidya-Bonner-de Sitter 时空中, 构造如下零标架:

$$\begin{aligned}
&l_\mu=\left(\frac{1}{2B}\right)^{1/2}\left[\frac{B}{r},0,0,0\right],\quad n_\mu=\left(\frac{1}{2B}\right)^{1/2}\left[\frac{B}{r},-2r,0,0\right],\\
&m_\mu=\frac{r}{\sqrt{2}}[0,0,1,\mathrm{i}\sin\theta],\quad \bar{m}_\mu=\frac{r}{\sqrt{2}}[0,0,1,-\mathrm{i}\sin\theta].
\end{aligned} \tag{14}$$

由(14)式得

$$\begin{aligned}
&l^\mu=\left(\frac{1}{2B}\right)^{1/2}\left[0,-\frac{B}{r},0,0\right],\quad n^\mu=\left(\frac{1}{2B}\right)^{1/2}\left[2r,\frac{B}{r},0,0\right],\\
&m^\mu=\frac{1}{\sqrt{2}r}\left[0,0,-1,-\frac{\mathrm{i}}{\sin\theta}\right],\quad \bar{m}^\mu=\frac{1}{\sqrt{2}r}\left[0,0,-1,\frac{\mathrm{i}}{\sin\theta}\right].
\end{aligned} \tag{15}$$

不难验证零标架(14)和(15)式满足[3]

$$
\begin{aligned}
&l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \overline{m}_\mu \overline{m}^\mu = 0,\\
&l_\mu n^\mu = - m_\mu \overline{m}^\mu = 1,\\
&l_\mu m^\mu = l_\mu \overline{m}^\mu = n_\mu m^\mu = n_\mu \overline{m}^\mu = 0,\\
&g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \overline{m}_\nu - \overline{m}_\mu m_\nu.
\end{aligned}
\tag{16}
$$

按照 Newman 和 Penrose 的方法[3],计算得旋系数如下:

$$
\begin{aligned}
&\varepsilon = -\frac{Mr - Q^2 - \chi^2 r^4}{2 r^2 \sqrt{2\mathrm{B}}}, \quad \mu = \rho = \frac{\mathrm{B}}{r^2 \sqrt{2\mathrm{B}}},\\
&\alpha = -\beta = \frac{1}{2\sqrt{2} r}\cot\theta,\\
&\gamma = \frac{r}{\sqrt{2\mathrm{B}}}\left(\frac{Q\dot{Q} - \dot{M} r}{\mathrm{B}} - \frac{Mr - Q^2 - \chi^2 r^4}{2 r^3}\right),\\
&\kappa = \pi = \lambda = \sigma = \nu = \tau = 0.
\end{aligned}
\tag{17}
$$

由(15)式可得微分算子如下:

$$
\begin{aligned}
&\mathrm{D} = l^\mu \partial_\mu = -\frac{\mathrm{B}}{r\sqrt{2\mathrm{B}}}\frac{\partial}{\partial r}, \quad \Delta = n^\mu \partial_\mu = \frac{1}{\sqrt{2\mathrm{B}}}\left(2r\frac{\partial}{\partial v} + \frac{\mathrm{B}}{r}\frac{\partial}{\partial r}\right),\\
&\delta = m^\mu \partial_\mu = -\frac{1}{\sqrt{2} r}\left(\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right),\\
&\bar{\delta} = \overline{m}^\mu \partial_\mu = -\frac{1}{\sqrt{2} r}\left(\frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right).
\end{aligned}
\tag{18}
$$

将(13),(15),(17)和(18)式代入(12)式,然后对所得方程作如下形式的分离变量:

$$
\begin{aligned}
&F_1 = \mathrm{e}^{\mathrm{i} m\varphi} f_1(v, r, \theta), \quad F_2 = \mathrm{e}^{\mathrm{i} m\varphi} f_2(v, r, \theta),\\
&G_1 = \mathrm{e}^{\mathrm{i} m\varphi} g_1(v, r, \theta), \quad G_2 = \mathrm{e}^{\mathrm{i} m\varphi} g_2(v, r, \theta),
\end{aligned}
$$

则(12)式化为

$$
\begin{aligned}
&\sqrt{\mathrm{B}}\mathscr{D}_0 f_1 + \mathscr{L}_+ f_2 + \mathrm{i}\mu_0 r g_1 = 0,\\
&\sqrt{\mathrm{B}}\mathscr{D}_1 f_2 - \mathscr{L}_- f_1 - \mathrm{i}\mu_0 r g_2 = 0,\\
&\sqrt{\mathrm{B}}\mathscr{D}_0 g_2 - \mathscr{L}_- g_1 + \mathrm{i}\mu_0 r f_2 = 0,\\
&\sqrt{\mathrm{B}}\mathscr{D}_1 g_1 + \mathscr{L}_+ g_2 - \mathrm{i}\mu_0 r f_1 = 0,
\end{aligned}
\tag{19}
$$

其中

$$
\begin{aligned}
&\mathscr{D}_0 = \frac{\partial}{\partial r} + \frac{1}{\mathrm{B}}\frac{2r^2 - 3Mr + Q^2 - 3\chi^2 r^4}{2r},\\
&\mathscr{D}_1 = \frac{2r^2}{\mathrm{B}}\frac{\partial}{\partial v} + \mathscr{D}_0 - \frac{r}{\mathrm{B}}\left[\frac{r}{\mathrm{B}}(Q\dot{Q} - \dot{M} r) - 2\mathrm{i} eQ\right],\\
&\mathscr{L}_\pm = \frac{\partial}{\partial\theta} \pm \frac{m}{\sin\theta} + \frac{1}{2}\cot\theta,
\end{aligned}
\tag{20}
$$

再分离变量,设

$$
\begin{aligned}
&f_1 = R_-(v, r) S_-(\theta), \quad f_2 = R_+(v, r) S_+(\theta),\\
&g_1 = R_+(v, r) S_-(\theta), \quad g_2 = R_-(v, r) S_+(\theta),
\end{aligned}
\tag{21}
$$

代入(19)式,得

$$\mathscr{D}_0 R_- + \frac{1}{\sqrt{B}}(\mathrm{i}\mu_0 r + \lambda) R_+ = 0, \tag{22a}$$

$$\mathscr{D}_1 R_+ - \frac{1}{\sqrt{B}}(\mathrm{i}\mu_0 r - \lambda) R_- = 0, \tag{22b}$$

$$\mathscr{L}_+ S_+ = \lambda S_-, \tag{22c}$$

$$\mathscr{L}_- S_- = -\lambda S_+, \tag{22d}$$

式中 m, λ 为分离变量时引进的常数.(22)式可以完全退耦,但我们关心的是 Dirac 方程径向部分.将(22b)式代入(22a)式,可得

$$\mathscr{D}_0\mathscr{D}_1 R_+ + \left[-\frac{\mu_0^2 r - \mathrm{i}\mu_0\lambda}{\lambda^2 + \mu_0^2 r^2} + \frac{1}{B}(r - M - 2\chi^2 r^3)\right]\mathscr{D}_1 R_+ - \frac{1}{B}(\lambda^2 + \mu_0^2 r^2) R_+ = 0. \tag{23}$$

引入 Tortoise 坐标变换[4,5]:

$$r_* = r + \frac{1}{2\kappa}\ln[r - r_{\mathrm{H}}(v)], \quad v_* = v - v_0, \tag{24}$$

其中 $\kappa = \kappa(v_0)$ 为调节参数,后面将看到它即是温度函数,v_0 为一常数.由(24)式可得

$$\begin{aligned}
&\frac{\partial}{\partial r} = \left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right]\frac{\partial}{\partial r_*}, \quad \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{\dot{r}_{\mathrm{H}}}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial}{\partial r_*},\\
&\frac{\partial^2}{\partial r^2} = \left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right]^2\frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r - r_{\mathrm{H}})^2}\frac{\partial}{\partial r_*},\\
&\frac{\partial^2}{\partial v\partial r} = \left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right]\frac{\partial^2}{\partial v_*\partial r_*} - \left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right]\frac{\dot{r}_{\mathrm{H}}}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial^2}{\partial r_*^2}\\
&\qquad + \frac{\dot{r}_{\mathrm{H}}}{2\kappa(r - r_{\mathrm{H}})^2}\frac{\partial}{\partial r_*}.
\end{aligned} \tag{25}$$

将(20)式代入(23)式,展开后再把(25)式代入(23)式的展开式,则(23)式变为

$$\begin{aligned}
&\left\{\frac{B}{r^2}\left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right] - \frac{2\dot{r}_{\mathrm{H}}}{2\kappa(r - r_{\mathrm{H}})}\right\}\frac{\partial^2 R_+}{\partial r_*^2} + 2\frac{\partial^2 R_+}{\partial v_*\partial r_*}\\
&+ \frac{B}{r^2}\left[1 + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\right]^{-1}\left\{\frac{r}{B^2}(2r^2 - 7Mr + 5Q^2 + \chi^2 r^4)\right.\\
&\left. + \frac{2r^2}{B}\left[-\frac{\mu_0^2 r - \mathrm{i}\mu_0\lambda}{\lambda^2 + \mu_0^2 r^2} + \frac{1}{B}(r - M - 2\chi^2 r^3)\right]\right\}\frac{\partial R_+}{\partial v_*}\\
&+ \left\{-\frac{r^2 - 2Mr + Q^2 - \chi^2 r^4 - 2r^2\dot{r}_{\mathrm{H}}}{r^2(r - r_{\mathrm{H}})[2\kappa(r - r_{\mathrm{H}}) + 1]}\right.\\
&- \frac{B\dot{r}_{\mathrm{H}}}{r^2[2\kappa(r - r_{\mathrm{H}}) + 1]}\left[\frac{r}{B^2}(2r^2 - 7Mr + 5Q^2 + \chi^2 r^4)\right.
\end{aligned}$$

$$
+\frac{2r^2}{B}\left(-\frac{\mu_0^2 r-\mathrm{i}\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{r-M-2\chi^2 r^3}{B}\right)\Bigg]+\frac{B}{r^2}\Bigg[-\frac{\mu_0^2 r-\mathrm{i}\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}
$$

$$
+\frac{r-M-2\chi^2 r^3}{B}+\frac{2}{B}\frac{2r^2-3Mr+Q^2-3\chi^2 r^4}{2r}-\frac{r^2}{B^2}(Q\dot{Q}-\dot{M}r)+2\mathrm{i}eQ\frac{r}{B}\Bigg]\Bigg\}\frac{\partial R_+}{\partial r_*}
$$

$$
+\frac{B}{r^2}\left[1+\frac{1}{2\kappa(r-r_{\mathrm{H}})}\right]^{-1}\Bigg\{\left[-\frac{\mu_0^2 r-\mathrm{i}\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{1}{B}(r-M-2\chi^2 r^3)\right]
$$

$$
\times\left[\frac{1}{B}\frac{2r^2-3Mr+Q^2-3\chi^2 r^4}{2r}-\frac{r^2}{B^2}(Q\dot{Q}-\dot{M}r)+2\mathrm{i}eQ\frac{r}{B}\right]
$$

$$
-\frac{2}{B^2}(r-M-2\chi^2 r^3)\frac{1}{2r}(2r^2-3Mr+Q^2-3\chi^2 r^4)+\frac{1}{B}\frac{2r^2-Q^2-9\chi^2 r^4}{2r^2}
$$

$$
+\frac{4r^2}{B^3}(r-M-2\chi^2 r^3)(Q\dot{Q}-\dot{M}r)-\frac{r}{B^2}(2Q\dot{Q}-3\dot{M}r)
$$

$$
+\frac{2\mathrm{i}eQ}{B^2}(Q^2-r^2-3\chi^2 r^4)+\frac{1}{B}\frac{2r^2-3Mr+Q^2-3\chi^2 r^4}{2r}\left[\frac{1}{B}\frac{2r^2-3Mr+Q^2-3\chi^2 r^4}{2r}\right.
$$

$$
\left.-\frac{r^2}{B^2}(Q\dot{Q}-\dot{M}r)+2\mathrm{i}eQ\frac{r}{B}\right]-\frac{1}{B}(\lambda^2+\mu_0^2 r^2)\Bigg\}R_+=0. \tag{26}
$$

由(5)式知，(26)式第一项系数在 $r\to r_{\mathrm{H}}$ 时为 $\frac{0}{0}$ 不定型．令该系数在 $r\to r_{\mathrm{H}}$ 时的极限为 A，即

$$
A=\lim_{\substack{r\to r_{\mathrm{H}}\\ v\to v_0}}\frac{(r^2-2Mr+Q^2-\chi^2 r^4)[2\kappa(r-r_{\mathrm{H}})+1]-2r^2\dot{r}_{\mathrm{H}}}{2\kappa r^2(r-r_{\mathrm{H}})}, \tag{27}
$$

对(27)式运用 L′Hospital 法则，当 $r_{\mathrm{H}}=r_+$ 时，选择调节参数

$$
\kappa_+(v_0)=\frac{-2\chi^2 r_+^3+(1-2\dot{r}_+)r_+-M}{2Mr_++\chi^2 r_+^4-Q^2}, \tag{28}
$$

则 $A=1$．

同样，当 $r_{\mathrm{H}}=r_c$ 时，选择调节参数

$$
\kappa_c(v_0)=\frac{-2\chi^2 r_c^3+(1-2\dot{r}_c)r_c-M}{2Mr_c+\chi^2 r_c^4-Q^2}, \tag{29}
$$

也使 $A=1$．

于是当 $r\to r_{\mathrm{H}}$ 时，即在黑洞外视界和宇宙视界附近，(26)式化为

$$
\frac{\partial^2 R_+}{\partial r_*^2}+2\frac{\partial^2 R_+}{\partial v_*\partial r_*}+(\alpha_0+2\mathrm{i}\omega_0)\frac{\partial R_+}{\partial r_*}=0, \tag{30}
$$

其中

$$
\alpha_0=\frac{1}{2r_{\mathrm{H}}^3}(2r_{\mathrm{H}}^2-3Mr_{\mathrm{H}}+Q^2-3\chi^2 r_{\mathrm{H}}^4)-\frac{Q\dot{Q}-\dot{M}r_{\mathrm{H}}}{2r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}},
$$

$$
\omega_0=\frac{eQ}{r_{\mathrm{H}}}. \tag{31}
$$

(30)式两个线性独立解为

$$
\begin{aligned}
R_{+}^{\text{in}} &= \mathrm{e}^{-\mathrm{i}\omega v_*}, \\
R_{+}^{\text{out}} &= \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0)r_*}\mathrm{e}^{-\alpha_0 r_*}.
\end{aligned} \tag{32}
$$

从(32)式知,在黑洞外视界 $r_{\mathrm{H}}=r_+$ 上,入射波 R_{+}^{in} 解析,但出射波 R_{+}^{out} 有对数奇异性,将 R_{+}^{out} 解析延拓到黑洞外视界内,得

$$
\overline{P}_{\omega}^{+} = \begin{cases} \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0^+)r_*}\mathrm{e}^{-\alpha_0^+ r_*} & (r>r_+), \\ \mathrm{e}^{\pi(\omega-\omega_0^+)/\kappa_+}\mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0^+)r_*}\mathrm{e}^{-\alpha_0^+ r_*}\mathrm{e}^{\mathrm{i}\pi\alpha_0^+/2\kappa_+} & (r<r_+), \end{cases} \tag{33}
$$

其中 ω_0^+ 和 α_0^+ 为(31)式中 r_{H} 取为 r_+ 时 ω_0 和 α_0 的值.按照 Damour, Ruffini 和 Sannan 的方法[6,7],可得出射波的黑体谱为

$$
N_{\omega}^{+} = \frac{1}{\mathrm{e}^{(\omega-\omega_0^+)/T_+}+1}, \tag{34}
$$

其中

$$
T_+ = \frac{\kappa_+}{2\pi} = \frac{1}{2\pi}\frac{(1-2\dot{r}_+)r_+ - 2\chi^2 r_+^3 - M}{2Mr_+ + \chi^2 r_+^4 - Q^2} \tag{35}
$$

为黑洞的 Hawking 温度, ω_0^+ 起化学势的作用.同样从(32)式知,在宇宙视界 $r_{\mathrm{H}}=r_c$ 上, R_{+}^{in} 解析, R_{+}^{out} 有对数奇异性,将 R_{+}^{out} 解析延拓得

$$
\overline{P}_{\omega}^{c} = \begin{cases} \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0^c)r_*}\mathrm{e}^{-\alpha_0^c r_*} & (r<r_c), \\ \mathrm{e}^{\pi(\omega-\omega_0^c)/\kappa_c}\mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0^c)r_*}\mathrm{e}^{-\alpha_0^c r_*}\mathrm{e}^{\mathrm{i}\pi\alpha_0^c/2\kappa_c} & (r>r_c), \end{cases} \tag{36}
$$

其中 ω_0^c 和 α_0^c 为(31)式中 r_{H} 取为 r_c 时 ω_0 和 α_0 的值.黑体谱为

$$
N_{\omega}^{c} = \frac{1}{\mathrm{e}^{(\omega-\omega_0^c)/T_c}+1}, \tag{37}
$$

其中

$$
T_c = \frac{\kappa_c}{2\pi} = \frac{1}{2\pi}\frac{(1-2\dot{r}_c)r_c - 2\chi^2 r_c^3 - M}{2Mr_c + \chi^2 r_c^4 - Q^2} \tag{38}
$$

为宇宙视界 Hawking 温度.Dirac 粒子在视界上的电势能 ω_0^+, ω_0^c 起化学势作用.

4 讨 论

当 $\dot{M}=\dot{Q}=0$ 时,由(6)—(11)式知

$$
\begin{aligned}
r_+ &= \sqrt{Z_1} - \sqrt{Z_2} + \sqrt{Z_3}, \\
r_c &= \sqrt{Z_1} + \sqrt{Z_2} - \sqrt{Z_3}, \\
r_- &= -\sqrt{Z_1} + \sqrt{Z_2} + \sqrt{Z_3},
\end{aligned} \tag{39}
$$

其中

$$
\sqrt{Z_1} = \frac{1}{\sqrt{6}\chi}\left[1+\sqrt{1-12Q^2\chi^2}\cos\frac{\alpha}{3}\right]^{1/2},
$$

$$\sqrt{Z_2} = \frac{1}{\sqrt{6}\,\chi}\left[1 - \sqrt{1 - 12\,Q^2\chi^2}\cos\left(\frac{\alpha}{3} + \frac{\pi}{3}\right)\right]^{1/2},$$

$$\sqrt{Z_3} = \frac{1}{\sqrt{6}\,\chi}\left[1 - \sqrt{1 - 12\,Q^2\chi^2}\cos\left(\frac{\alpha}{3} - \frac{\pi}{3}\right)\right]^{1/2},$$

$$\alpha = \arccos\left[-\frac{1 - 18\,\chi^2(3\,M^2 - 2\,Q^2)}{(1 - 12\,Q^2\chi^2)^{3/2}}\right]. \tag{40}$$

由(35)和(38)式知

$$T_+ = \frac{1}{2\pi}\frac{r_+ - 2\chi^2 r_+^3 - M}{2\,Mr_+ + \chi^2 r_+^4 - Q^2},$$

$$T_c = \frac{1}{2\pi}\frac{r_c - 2\chi^2 r_c^3 - M}{2\,Mr_c + \chi^2 r_c^4 - Q^2}. \tag{41}$$

此时黑洞内外视界和宇宙视界、黑洞温度和宇宙视界的温度均回到稳态 Reissner-Nordström-de Sitter时空的情况.

当 $Q=0$ 时，由(6)—(11)式可得

$$r_+ = 2\sqrt{\frac{(1 - 2\,\dot{r}_+)}{\lambda}}\cos\left(\alpha_1 + \frac{\pi}{3}\right), \tag{42}$$

其中
$$\alpha_1 = \frac{1}{3}\arccos\left[\frac{3\,M\lambda^{1/2}}{(1 - 2\,\dot{r}_+)^{3/2}}\right],$$

$$r_c = 2\sqrt{\frac{(1 - 2\,\dot{r}_c)}{\lambda}}\cos\left(\alpha_2 - \frac{\pi}{3}\right), \tag{43}$$

其中
$$\alpha_2 = \frac{1}{3}\arccos\left[\frac{3\,M\lambda^{1/2}}{(1 - 2\,\dot{r}_c)^{3/2}}\right],$$

$$r_- = 0. \tag{44}$$

由(35)和(38)式知

$$T_+ = \frac{1}{2\pi}\frac{1 - \lambda r_+^2 - 2\,\dot{r}_+}{2\left(2\,M + \frac{1}{3}\lambda r_+^3\right)}, \tag{45}$$

$$T_c = \frac{1}{2\pi}\frac{1 - \lambda r_c^2 - 2\,\dot{r}_c}{2\left(2\,M + \frac{1}{3}\lambda r_c^3\right)}, \tag{46}$$

其中用了 $\lambda = 3\chi^2$，这正是 Vaidya-Schwarzschild-de Sitter 时空的情况[8].

当 $r_+ = r_c$ 时，由(6)，(7)，(10)和(11)式知 $\alpha_+ = \alpha_c \equiv \alpha'$，

$$\cos\left(\frac{\alpha'}{3} + \frac{\pi}{3}\right) = \cos\left(\frac{\alpha'}{3} - \frac{\pi}{3}\right), \tag{47}$$

结合考虑 Vaidya-Schwarzschild-de Sitter 时空情况，由(47)式可确定

$$\alpha' = 6\,n\pi \qquad (n = 0,\ \pm 1,\ \pm 2, \cdots), \tag{48}$$

从而得到黑洞外视界和宇宙视界重合时

$$r_+ = \frac{1}{\sqrt{6}\,\chi}\left[(1-2\dot{r}_+)+\sqrt{(1-2\dot{r}_+)^2-12Q^2\chi^2}\right]^{1/2}, \tag{49}$$

$$r_c = \frac{1}{\sqrt{6}\,\chi}\left[(1-2\dot{r}_c)+\sqrt{(1-2\dot{r}_c)^2-12Q^2\chi^2}\right]^{1/2}, \tag{50}$$

其中 $\dot{r}_+ = \dot{r}_c$. 将(49)和(50)式分别代入(35)和(38)式, 得

$$T_+ = 0, \qquad T_c = 0. \tag{51}$$

即当黑洞外视界与宇宙视界重合时, 违背黑洞热力学第三定律, 故 Vaidya-Bonner-de Sitter时空中黑洞外视界和宇宙视界永不重合.

类似地可求得当黑洞内视界和外视界重合时, 也有 $T_+ = 0$, 所以该黑洞内外视界也永不重合.

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HAWKING RADIATION OF CHARGED DIRAC PARTICLES IN VAIDYA-BONNER-DE SITTER SPACE-TIME

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ABSTRACT

The exact locations of the universe horizon and the outer and inner horizons of the black hole are given in the Vaidya-Bonner-de Sitter space-time. Hawking radiation of charged Dirac particles near the outer horizon and the universe horizon are studied. It is determined that any two of the three horizons cannot coincide.

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