第 47 卷 第 7 期 1998 年 7 月
1000-3290/98/47(7)/1090-11
物 理 学 报
ACTA PHYSICA SINICA
Vol. 47, No. 7, July, 1998

三维曲光轴坐标下的旁轴电子光学

聂世琦 林书铨 黄保麟
(上海交通大学 HDTV 研究所, 上海 200030)
(1997 年 6 月 23 日收到; 1998 年 1 月 4 日收到修改稿)

从费马原理出发, 应用微分几何的曲线坐标定义, 给出了三维曲光轴坐标中电子在旁轴下的一级和二级近似轨迹方程式. 借助于对电场和磁场的数值计算, 可以分析电磁场分布和电子运动轨迹复杂情况下的电子光学性质. 给出了对一种电子束扫描式扁平显像管电子光学系统的计算结果, 并给以定性说明.

PACC: 0777; 0780; 2920

1 以电子束中心轨迹为曲光轴的坐标系

在直角坐标系(x, y, z)中, 电子束中心电子轨迹方程为

$$
\begin{aligned}
m\ddot{x} &= -e\frac{\partial V}{\partial x} + e(\dot{y}B_z - \dot{z}B_y),\\
m\ddot{y} &= -e\frac{\partial V}{\partial y} + e(\dot{z}B_x - \dot{x}B_z),\\
m\ddot{z} &= -e\frac{\partial V}{\partial z} + e(\dot{x}B_y - \dot{y}B_x),
\end{aligned} \tag{1}
$$

由方程(1)所决定的曲线为

$$\boldsymbol{r} = \boldsymbol{r}(t) = (x(t), y(t), z(t)). \tag{2}$$

由此曲线作为曲光轴坐标系的三个轴的方向分别为 ω 轴方向为曲线的单位切向量方向、ξ 轴方向为曲线的主法向量方向、ψ 轴方向为曲线的从法向量方向. 见图 1.

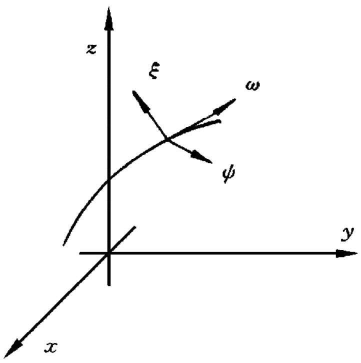

图 1 曲光轴坐标系

单位切向量为

$$\mathbf{T}(s) = \boldsymbol{r}'(s), \tag{3}$$

s 为曲线弧长参数. 主法向量为

$$\mathbf{N}(s) = \frac{\boldsymbol{r}''(s)}{|\boldsymbol{r}''(s)|}, \tag{4}$$

从法向量为

$$\mathbf{B}(s) = \mathbf{T}(s) \times \mathbf{N}(s). \tag{5}$$

在 x, y, z 坐标系下

$$
\begin{aligned}
\mathbf{T}(s) &= (x_\omega, y_\omega, z_\omega),\\
\mathbf{N}(s) &= (x_\xi, y_\xi, z_\xi),
\end{aligned}
$$

$$\mathbf{B}(s) = (x_\psi, y_\psi, z_\psi). \tag{6}$$

在 ω, ξ, ψ 坐标下

$$\mathbf{T} = (0,0,1), \quad \mathbf{N} = (1,0,0), \quad \mathbf{B} = (0,1,0).$$

矢量的各分量在两个坐标系之间的转换式为

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_\xi & x_\psi & x_\omega \\ y_\xi & y_\psi & y_\omega \\ z_\xi & z_\psi & z_\omega \end{bmatrix} \begin{bmatrix} \xi \\ \psi \\ \omega \end{bmatrix}, \tag{7}$$

$$\begin{bmatrix} A_\xi \\ A_\psi \\ A_\omega \end{bmatrix} = \begin{bmatrix} x_\xi & x_\psi & x_\omega \\ y_\xi & y_\psi & y_\omega \\ z_\xi & z_\psi & z_\omega \end{bmatrix}^{-1} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}, \tag{8}$$

式中含 $\alpha_i, \beta_i, \gamma_i$ 的矩阵为含 $x_\xi, y_\xi, z_\xi, \cdots$ 的矩阵的逆矩阵,前者各矩阵元可通过对后者求逆获得.用这两个矩阵,可得到磁矢位 $\mathbf{A}$ 的各分量在两个坐标系之间的关系.

2　曲光轴下的变分方程

曲光轴下的电子光学费马原理可表示为

$$\delta\int Q\mathrm{d}s = 0, \tag{9}$$

式中微分弧元为

$$\mathrm{d}s = \sqrt{\xi'^2 + \psi'^2 + (1+\xi h)^2}\,\mathrm{d}\omega,$$

折射率为

$$Q = V^{1/2} - \sqrt{\eta}\,\mathbf{A}\cdot\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s},$$

h 为曲率, V 为电位, $\mathbf{A}$ 为磁矢位. $m = Q\sqrt{\xi'^2 + \psi'^2 + (1+\xi h)^2}$, 上撇“′”为对 ω 的导数,则

$$\delta\int m\,\mathrm{d}\omega = \delta\int Q\sqrt{\xi'^2 + \psi'^2 + (1+\xi h)^2}\,\mathrm{d}\omega = 0.$$

设 $F = V^{1/2}$, $\sqrt{\eta}\,\mathbf{A} = (A_x, A_y, A_z)$, 将 F, A_x, A_y, A_z 展为 ξ, ψ 的乘积幂函数:

$$\begin{aligned} F &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{m,n}\,\xi^m\psi^n, \\ A_x &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{xm,n}\,\xi^m\psi^n, \\ A_y &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{ym,n}\,\xi^m\psi^n, \\ A_z &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{zm,n}\,\xi^m\psi^n. \end{aligned} \tag{10}$$

场量在轴附近展开,由于相应的场方程的要求,系数之间是相互联系的,下面是它们之间的相互关系.

首先,电位满足拉普拉斯方程,在曲光轴坐标系中

$$\nabla^2 V = \frac{1}{e_\omega}\left[\frac{\partial}{\partial \xi}\left(e_\omega \frac{\partial V}{\partial \xi}\right) + \frac{\partial}{\partial \psi}\left(e_\omega \frac{\partial V}{\partial \psi}\right) + \frac{\partial}{\partial \omega}\left(\frac{1}{e_\omega}\frac{\partial V}{\partial \omega}\right)\right] = 0,$$

$$e_\omega = 1 + h\xi\cos\phi + h\psi\sin\phi, \tag{11}$$

$$\phi = \int_\omega^{\omega+\Delta\omega}\tau(\omega)\mathrm{d}\omega,$$

$\tau(\omega)$为挠率,当 $\Delta\omega\to 0$ 时, $e_\omega = 1 + h\xi$. 将 V 展开

$$V(\xi, \psi, \omega) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_{mn}(\omega)\xi^m\psi^n. \tag{12}$$

代入拉普拉斯方程(11),可得

$$\begin{aligned}&(m+1)(3m+1)h\, a_{m+1,n} + (m+1)^2 h^2 a_{m-1,n} + m(3m+1)h^2 a_{m,n}\\&+(m+2)(m+1)a_{m+2,n} + (n+2)(n+1)a_{m,n+2} + (n+2)(n+1)h^3 a_{m-3,n+2}\\&+3(n+2)(n+1)h\, a_{m-1,n+2} + 3(n+2)(n+1)h^2 a_{m-2,n+2}\\&- h\tau a'_{m,n-1} - h' a'_{m-1,n} + a''_{m,n} + h a''_{m-1,n} = 0.\end{aligned} \tag{13}$$

磁矢位满足矢量形式的拉普拉斯方程为

$$\nabla^2 \mathbf{A} = 0. \tag{14}$$

在曲光轴坐标系中,与(14)式对应的三个具体方程为

$$\begin{aligned}&e_\omega^3\frac{\partial^2 A_\xi}{\partial\psi^2} + e_\omega^3\frac{\partial^2 A_\xi}{\partial\xi^2} + e_\omega\frac{\partial^2 A_\xi}{\partial\omega^2} - (h\psi\tau + h'\xi)\frac{\partial A_\xi}{\partial\omega} + h e_\omega^2\frac{\partial A_\xi}{\partial\xi} - 2h e_\omega\frac{\partial A_\omega}{\partial\omega}\\&+(h^2\psi\tau - h')A\omega = 0,\\&e_\omega^3\frac{\partial^2 A_\psi}{\partial\psi^2} + e_\omega\frac{\partial^2 A_\psi}{\partial\omega^2} + e_\omega^3\frac{\partial^2 A_\psi}{\partial\xi^2} - (h\psi\tau + h'\xi)\frac{\partial A_\psi}{\partial\omega} + h e_\omega^2\frac{\partial A_\psi}{\partial\xi} - h\tau e_\omega A_\omega = 0,\\&e_\omega^3\frac{\partial^2 A_\psi}{\partial\xi^2} + e_\omega^3\frac{\partial^2 A_\omega}{\partial\psi^2} + e_\omega\frac{\partial^2 A_\omega}{\partial\omega^2} + h e_\omega^2\frac{\partial A_\omega}{\partial\xi} - (h\psi\tau + h'\xi)\frac{\partial A_\omega}{\partial\omega} + 2h e_\omega\frac{\partial A_\xi}{\partial\omega}\\&- h^2 e_\omega A_\omega + h\tau e_\omega A_\psi + (h' - h^2\psi\tau)A_\xi = 0.\end{aligned} \tag{15}$$

磁矢位在曲光轴坐标系下的三个分量的级数形式为

$$\begin{aligned}A_\xi(\xi, \psi, \omega) &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} b_{mn}(\omega)\xi^m\psi^n,\\A_\psi(\xi, \psi, \omega) &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_{mn}(\omega)\xi^m\psi^n,\\A_\omega(\xi, \psi, \omega) &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} d_{mn}(\omega)\xi^m\psi^n.\end{aligned} \tag{16}$$

代入方程(15),得到系数之间的关系为

$$\begin{aligned}&(n+2)(n+1)b_{m,n+2} + h^3(n+2)(n+1)b_{m-3,n+2} + 3h^2(n+2)(n+1)b_{m-2,n+2}\\&+ b''_{m,n} + h b''_{m-1,n} + 3h(n+2)(n+1)b_{m-1,n+2} + (m+2)(m+1)b_{m+2,n}\\&+ h^3(m^2 - 2m + 2)b_{m-1,n} - h\tau b'_{m,n-1} - h' b'_{m-1,n} + h^2 m(3m-1)b_{m,n}\end{aligned}$$

$$
\begin{aligned}
&+ h(m+1)(3m+1)\,b_{m+1,\,n} - 2h\,d'_{m,\,n} - 2h^2 d'_{m-1,\,n}\\
&+ h^2\tau\,d_{m,\,n-1} - h'd_{m,\,n} = 0,\\
&(n+2)(n+1)\,c_{m,\,n+2} + h^3(n+2)(n+1)\,c_{m-3,\,n+2} + 3h^2(n+2)(n+1)\,c_{m-2,\,n+2}\\
&+ c''_{m,\,n} + h\,c''_{m-1,\,n} + 3h(n+2)(n+1)\,c_{m-1,\,n+2} + (m+2)(m+1)\,c_{m+2,\,n}\\
&+ h^3(m-1)^2 c_{m-1,\,n} - h\tau\,c'_{m,\,n-1} - h'c'_{m-1,\,n} + h^2 m(3m-1)\,c_{m,\,n}\\
&+ h(m+1)(3m+1)\,c_{m+1,\,n} - h\tau\,d_{m,\,n} - h^2\tau\,d_{m-1,\,n} = 0,\\
&(m+2)(m+1)\,d_{m+2,\,n} + h^3(m-1)(m-2)\,d_{m-1,\,n} + 3h^2 m(m-1)\,d_{m,\,n}\\
&+ h(m+1)(3m+1)\,d_{m+1,\,n} + (n+2)(n+1)\,d_{m,\,n+2} + h^3(n+2)(n+1)\,d_{m-3,\,n+2}\\
&+ 3h^2(n-2)(n-3)\,d_{m-2,\,n+2} + h\,d''_{m-1,\,n} + d''_{m,\,n} + 3h(n+2)(n+1)\,d_{m-1,\,n+2}\\
&+ h^2(2m-1)\,d_{m,\,n} + h^3(m-2)\,d_{m-1,\,n} - h\tau\,d'_{m,\,n-1} - h'd'_{m-1,\,n} + 2h\,b'_{m,\,n}\\
&+ 2h^2 b'_{m-1,\,n} + h\tau\,c_{m,\,n} + h^2\tau\,c_{m-1,\,n} + h'b_{m,\,n} - h^2\tau\,b_{m,\,n-1} = 0.
\end{aligned}
\tag{17}
$$

可以看到在曲光轴坐标系里,由于曲率和挠率的因素,级数展开式系数 $a_{m,\,n}$, $b_{m,\,n}$, $c_{m,\,n}$, $d_{m,\,n}$之间的关系与直角坐标系、圆柱坐标系情况下相比是比较复杂.下面为 $a_{m,\,n}$, $b_{m,\,n}$, $c_{m,\,n}$, $d_{m,\,n}$与 $\mathbf{F}_{m,\,n}$, $\mathbf{A}_{xm,\,n}$, $\mathbf{A}_{ym,\,n}$, $\mathbf{A}_{zm,\,n}$之间的关系式:

$$
\begin{aligned}
&a_{m,\,n} = \sum_{\substack{n_1+n_2=n\\ m_1+m_2=m}} \mathbf{F}_{m_1,\,n_1}\mathbf{F}_{m_2,\,n_2},\\
&\begin{bmatrix} b_{m,\,n}\\ c_{m,\,n}\\ d_{m,\,n}\end{bmatrix} = \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{bmatrix}\begin{bmatrix}\mathbf{A}_{xm,\,n}\\ \mathbf{A}_{ym,\,n}\\ \mathbf{A}_{zm,\,n}\end{bmatrix}.
\end{aligned}
\tag{18}
$$

借助(18)式和 $a_{m,\,n}$, $b_{m,\,n}$, $c_{m,\,n}$, $d_{m,\,n}$之间的关系式(13)和(17),即可得到 $\mathbf{F}_{m,\,n}$, $\mathbf{A}_{xm,\,n}$, $\mathbf{A}_{ym,\,n}$, $\mathbf{A}_{zm,\,n}$之间的关系.

下面以 $\mathbf{A}_{xm,\,n}$为例,说明 $\mathbf{F}_{m,\,n}$, $\mathbf{A}_{xm,\,n}$, $\mathbf{A}_{ym,\,n}$, $\mathbf{A}_{zm,\,n}$前三级的实际确定方法.

$$
\begin{aligned}
&\mathbf{A}_{x00} = \mathbf{A}_x(\omega),\ \mathbf{A}_{x10} = \frac{\partial \mathbf{A}_x}{\partial \xi}(\omega),\ \mathbf{A}_{x01} = \frac{\partial \mathbf{A}_x}{\partial \psi}(\omega),\ \mathbf{A}_{x20} = \frac{1}{2}\frac{\partial^2 \mathbf{A}_x}{\partial \xi^2}(\omega),\\
&\mathbf{A}_{x02} = \frac{1}{2}\frac{\partial^2 \mathbf{A}_x}{\partial \psi^2}(\omega),\ \mathbf{A}_{x11} = \frac{\partial^2 \mathbf{A}_x}{\partial \xi\partial \psi}(\omega),\ \mathbf{A}_{x30} = \frac{1}{6}\frac{\partial^3 \mathbf{A}_x}{\partial \xi^3}(\omega),\ \mathbf{A}_{x03} = \frac{1}{6}\frac{\partial^3 \mathbf{A}_x}{\partial \psi^3}(\omega),\\
&\mathbf{A}_{x21} = \frac{1}{2}\frac{\partial^3 \mathbf{A}_x}{\partial^2 \xi\partial \psi}(\omega),\ \mathbf{A}_{x12} = \frac{1}{2}\frac{\partial^3 \mathbf{A}_x}{\partial^2 \psi\partial \xi}(\omega).
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\frac{\partial \mathbf{A}_x}{\partial \xi} &= \frac{\partial \mathbf{A}_x}{\partial x}x_\xi + \frac{\partial \mathbf{A}_x}{\partial y}y_\xi + \frac{\partial \mathbf{A}_x}{\partial z}z_\xi,\\
\frac{\partial^2 \mathbf{A}_x}{\partial \xi\partial \psi} &= \frac{\partial^2 \mathbf{A}_x}{\partial x^2}x_\xi x_\psi + \frac{\partial^2 \mathbf{A}_x}{\partial y^2}y_\xi y_\psi + \frac{\partial^2 \mathbf{A}_x}{\partial z^2}z_\xi z_\psi + \frac{\partial^2 \mathbf{A}_x}{\partial x\partial y}(y_\xi x_\psi + x_\xi y_\psi)\\
&\quad + \frac{\partial^2 \mathbf{A}_x}{\partial z\partial x}(z_\xi x_\psi + x_\xi z_\psi) + \frac{\partial^2 \mathbf{A}_x}{\partial z\partial y}(z_\xi y_\psi + y_\xi z_\psi).
\end{aligned}
\tag{20}
$$

$$\frac{\partial^3 A_x}{\partial^2 \xi \partial \psi} = \frac{\partial^3 A_x}{\partial x^3} x_\xi^2 x_\psi + \frac{\partial^3 A_x}{\partial y^3} y_\xi^2 y_\psi + \frac{\partial^3 A_x}{\partial z^3} z_\xi^2 z_\psi + \frac{\partial^3 A_x}{\partial y^2 \partial x}(y_\xi^2 x_\psi + 2 x_\xi y_\xi y_\psi)$$
$$+ \frac{\partial^3 A_x}{\partial x^2 \partial y}(2 x_\xi y_\xi x_\psi + x_\xi^2 y_\psi) + \frac{\partial^3 A_x}{\partial z^2 \partial x}(z_\xi^2 x_\psi + 2 z_\xi x_\xi z_\psi)$$
$$+ \frac{\partial^3 A_x}{\partial z \partial x^2}(2 z_\xi x_\xi x_\psi + x_\xi^2 z_\psi) + \frac{\partial^3 A_x}{\partial z^2 \partial y}(z_\xi^2 y_\psi + 2 z_\xi y_\xi z_\psi)$$
$$+ \frac{\partial^3 A_x}{\partial y^2 \partial z}(2 z_\xi y_\xi y_\psi + y_\xi^2 z_\psi) + \frac{\partial^3 A_x}{\partial z \partial y \partial x}(2 z_\xi y_\xi x_\psi + 2 z_\xi x_\xi y_\psi + 2 x_\xi y_\xi z_\psi). \tag{21}$$

将(20)和(21)式中含 ξ, ψ 下标处代以相应的 ξ, ψ，即可得到关于 ξ, ψ 的一、二、三阶偏导的其余几个式子．

将 m 展为 ξ, ψ, ξ', ψ' 的乘积幂函数：

$$m = m_0 + m_1 + m_2 + m_3 + \cdots$$
$$m_0 = F_{00} + H_{00}^{00},$$
$$m_1 = (F_{10} + F_{00} h)\xi + F_{01}\psi + H_{10}^{00}\xi + H_{01}^{00}\psi + H_{00}^{10}\xi' + H_{00}^{01}\psi', \tag{22}$$
$$m_2 = (F_{20} + F_{10} h)\xi^2 + F_{02}\psi^2 + (F_{11} + F_{01} h)\xi\psi + \frac{1}{2}F_{00}\xi'^2 + \frac{1}{2}F_{00}\psi'^2 + H_{20}^{00}\xi^2$$
$$+ H_{02}^{00}\psi^2 + H_{11}^{00}\xi\psi + H_{10}^{10}\xi\xi' + H_{10}^{01}\xi\psi' + H_{01}^{10}\psi\xi' + H_{01}^{01}\psi\psi',$$
$$m_3 = \xi^3(F_{30} + F_{20} h) + \psi^3 F_{03} + \xi^2\psi(F_{21} + F_{11} h) + \xi\psi^2(F_{12} + F_{02} h) + \frac{1}{2}\psi\xi'^2 F_{01}$$
$$+ \frac{1}{2}\xi\xi'^2(F_{10} - F_{00} h) + \frac{1}{2}\xi\psi'^2(F_{10} - F_{00} h) + \frac{1}{2}\psi\psi'^2 F_{01} + \xi^3 H_{30}^{00} + \psi^3 H_{03}^{00}$$
$$+ \xi\psi^2 H_{12}^{00} + \xi^2\xi' H_{20}^{10} + \xi^2\psi H_{21}^{00} + \xi^2\psi' H_{02}^{10} + \xi'\psi^2 H_{02}^{10} + \psi^2\psi' H_{02}^{01} + \xi\xi'\psi H_{11}^{10}$$
$$+ \xi\psi\psi' H_{11}^{01},$$

式中系数 $H_{mn}^{m'n'}$ 表示 ξ, ψ, ξ', ψ' 乘积项系数中含磁矢位的部分，与磁矢位的具体关系如下：

$$H_{00}^{00} = r_1 A_{x00} + r_2 A_{y00} + r_3 A_{z00},$$
$$H_{10}^{00} = r_1 A_{x10} + r_2 A_{y10} + r_3 A_{z10},$$
$$H_{01}^{00} = r_1 A_{x01} + r_2 A_{y01} + r_3 A_{z01},$$
$$H_{00}^{10} = \alpha_1 A_{x00} + \alpha_2 A_{y00} + \alpha_3 A_{z00},$$
$$H_{00}^{01} = \beta_1 A_{x00} + \beta_2 A_{y00} + \beta_3 A_{z00},$$
$$H_{20}^{00} = r_1 A_{x20} + r_2 A_{y20} + r_3 A_{z20} + h(r_1 A_{x10} + r_2 A_{y10} + r_3 A_{z10}),$$
$$H_{02}^{00} = r_1 A_{x02} + r_2 A_{y02} + r_3 A_{z02},$$
$$H_{11}^{00} = r_1 A_{x11} + r_2 A_{y11} + r_3 A_{z11} + h(r_1 A_{x01} + r_2 A_{y01} + r_3 A_{z01}),$$
$$H_{10}^{10} = \alpha_1 A_{x10} + \alpha_2 A_{y10} + \alpha_3 A_{z10},$$
$$H_{10}^{01} = \beta_1 A_{x10} + \beta_2 A_{y10} + \beta_3 A_{z10},$$

$$
\begin{aligned}
H_{01}^{10} &= \alpha_1 A_{x01} + \alpha_2 A_{y01} + \alpha_3 A_{z01},\\
H_{01}^{01} &= \beta_1 A_{x01} + \beta_2 A_{y01} + \beta_3 A_{z01},\\
H_{30}^{00} &= r_1 A_{x30} + r_2 A_{y30} + r_3 A_{z30} + h(r_1 A_{x20} + r_2 A_{y20} + r_3 A_{z20}),\\
H_{03}^{00} &= r_1 A_{x03} + r_2 A_{y03} + r_3 A_{z03},\\
H_{21}^{00} &= r_1 A_{x21} + r_2 A_{y21} + r_3 A_{z21} + h(r_1 A_{x11} + r_2 A_{y11} + r_3 A_{z11}),\\
H_{12}^{00} &= r_1 A_{x12} + r_2 A_{y12} + r_3 A_{z12} + h(r_1 A_{x02} + r_2 A_{y02} + r_3 A_{z02}),\\
H_{20}^{10} &= \alpha_1 A_{x20} + \alpha_2 A_{y20} + \alpha_3 A_{z20},\\
H_{20}^{01} &= \beta_1 A_{x20} + \beta_2 A_{y20} + \beta_3 A_{z20},\\
H_{02}^{10} &= \alpha_1 A_{x02} + \alpha_2 A_{y02} + \alpha_3 A_{z02},\\
H_{02}^{01} &= \beta_1 A_{x02} + \beta_2 A_{y02} + \beta_3 A_{z02},\\
H_{11}^{10} &= \alpha_1 A_{x11} + \alpha_2 A_{y11} + \alpha_3 A_{z11},\\
H_{11}^{01} &= \beta_1 A_{x11} + \beta_2 A_{y11} + \beta_3 A_{z11},
\end{aligned}
\tag{23}
$$

式中 $\alpha_i, \beta_i, \gamma_i (i=1,2,3)$ 为(8)式转换矩阵中的元素.

变分方程等效于欧拉方程:

$$
\frac{d}{d\omega}\left(\frac{\partial m}{\partial \xi'}\right) = \frac{\partial m}{\partial \xi}, \quad \frac{d}{d\omega}\left(\frac{\partial m}{\partial \psi'}\right) = \frac{\partial m}{\partial \psi}. \tag{24}
$$

作为一级近似,即取 $m = m_0 + m_1 + m_2$ 时,可得

$$
\frac{d}{d\omega}\begin{bmatrix} \xi \\ \xi_1 \\ \psi \\ \psi_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \xi \\ \xi_1 \\ \psi \\ \psi_1 \end{bmatrix}, \tag{25}
$$

式中 $a_{11}=0,\ a_{12}=1,\ a_{13}=0,\ a_{14}=0,\ a_{31}=0,\ a_{32}=0,\ a_{33}=0,\ a_{34}=1,$

$$
\begin{aligned}
a_{21} &= -\frac{(H_{10}^{10'} - 2F_{20} - 2F_{10}h - 2H_{20}^{00})}{F_{00}}, \quad a_{22} = -\frac{F_{00}'}{F_{00}},\\
a_{23} &= \frac{F_{11} + F_{01}h + H_{11}^{00} - H_{01}^{10'}}{F_{00}}, \quad a_{24} = \frac{H_{10}^{01} - H_{01}^{10}}{F_{00}},\\
a_{41} &= \frac{F_{11} + F_{01}h + H_{11}^{00} - H_{10}^{01'}}{F_{00}}, \quad a_{42} = \frac{H_{01}^{10} - H_{10}^{01}}{F_{00}},\\
a_{43} &= -\frac{H_{01}^{01'} - 2F_{02} - 2H_{02}^{00}}{F_{00}}, \quad a_{44} = -\frac{F_{00}'}{F_{00}}.
\end{aligned}
\tag{26}
$$

解可以表示为

$$
\begin{aligned}
&[Y] = \xi_0[Y_a] + \xi_{10}[Y_b] + \psi_0[Y_c] + \psi_{10}[Y_d],\\
&[Y_a] = \begin{bmatrix} \xi_a \\ \xi_{1a} \\ \psi_a \\ \psi_{1a} \end{bmatrix}, \quad [Y_b] = \begin{bmatrix} \xi_b \\ \xi_{1b} \\ \psi_b \\ \psi_{1b} \end{bmatrix}, \quad [Y_c] = \begin{bmatrix} \xi_c \\ \xi_{1c} \\ \psi_c \\ \psi_{1c} \end{bmatrix}, \quad [Y_d] = \begin{bmatrix} \xi_d \\ \xi_{1d} \\ \psi_d \\ \psi_{1d} \end{bmatrix},
\end{aligned}
\tag{27}
$$

$[Y_a]$, $[Y_b]$, $[Y_c]$, $[Y_d]$分别为

$$\xi_a(\omega_0)=1,\quad \xi_{1a}(\omega_0)=0,\quad \psi_a(\omega_0)=0,\quad \psi_{1a}(\omega_0)=0;$$
$$\xi_b(\omega_0)=0,\quad \xi_{1b}(\omega_0)=1,\quad \psi_b(\omega_0)=0,\quad \psi_{1b}(\omega_0)=0;$$
$$\xi_c(\omega_0)=0,\quad \xi_{1c}(\omega_0)=0,\quad \psi_c(\omega_0)=1,\quad \psi_{1c}(\omega_0)=0;$$
$$\xi_d(\omega_0)=0,\quad \xi_{1d}(\omega_0)=0,\quad \psi_d(\omega_0)=0,\quad \psi_{1d}(\omega_0)=1.$$

四组初始条件下的方程的特解,坐标的表达式为

$$\begin{aligned}\xi&=\xi_0\,\xi_a(\omega)+\xi_{10}\,\xi_b(\omega)+\psi_0\,\xi_c(\omega)+\psi_{10}\,\xi_d(\omega),\\ \psi&=\xi_0\,\psi_a(\omega)+\xi_{10}\,\psi_b(\omega)+\psi_0\,\psi_c(\omega)+\psi_{10}\,\psi_d(\omega).\end{aligned}\tag{28}$$

当 $\xi_{10}=0$, $\psi_0=0$, $\psi_{10}=0$ 时,

$$\sqrt{\xi^2+\psi^2}=\xi_0\sqrt{\xi_a^2(\omega)+\psi_a^2(\omega)},$$
$$\frac{\xi}{\psi}=\frac{\xi_a(\omega)}{\psi_a(\omega)}.$$

这说明,初始在 $\xi\omega$ 面内平行于 ω 轴的电子轨迹在曲光轴坐标系中,沿着光轴旋转的角度相同,且距光轴的距离与初始条件成正比.

当 $\xi_0=0$, $\psi_0=0$, $\psi_{10}=0$ 时,

$$\sqrt{\xi^2+\psi^2}=\xi_{10}\sqrt{\xi_b^2(\omega)+\psi_b^2(\omega)},$$
$$\frac{\xi}{\psi}=\frac{\xi_b(\omega)}{\psi_b(\omega)}.$$

这说明,初始在 $\xi\omega$ 平面内自 ω 轴上以不同角度出发的电子轨迹在曲光轴坐标系中沿着光轴旋转的角度相同,且距光轴的距离与初始角度成正比.

下面可以看到,当满足$\dfrac{\xi_a(\omega)}{\psi_a(\omega)}=\dfrac{\xi_b(\omega)}{\psi_b(\omega)}$时,初始在 $\xi\omega$ 平面内的电子束具备成像条件.

当 $\psi_0=0$, $\psi_{10}=0$ 时,

$$\begin{aligned}\xi^2+\psi^2=&\ \xi_0^2(\xi_a^2(\omega)+\psi_a^2(\omega))+\xi_{10}^2(\xi_b^2(\omega)+\psi_b^2(\omega))\\ &+2\,\xi_0\,\xi_{10}(\xi_a(\omega)\,\xi_b(\omega)+\psi_a(\omega)\,\psi_b(\omega)),\end{aligned}$$
$$\frac{\xi}{\psi}=\frac{\xi_0\,\xi_a(\omega)+\xi_{10}\,\xi_b(\omega)}{\xi_0\,\psi_a(\omega)+\xi_{10}\,\psi_b(\omega)}.$$

设

$$G(\omega)=\frac{\xi_a(\omega)}{\psi_a(\omega)}=\frac{\xi_b(\omega)}{\psi_b(\omega)},$$

则

$$\sqrt{\xi^2+\psi^2}=\sqrt{1+G(\omega)}\,(\xi_0\,\xi_a(\omega)+\xi_{10}\,\xi_b(\omega)),$$
$$\frac{\xi}{\psi}=G(\omega).$$

上式表明在这个旋转面内具有成像特性.

当取 $m=m_0+m_1+m_2+m_3$ 时,可得

$$\frac{\mathrm{d}}{\mathrm{d}\,\omega}\begin{bmatrix}\xi\\ \xi_1\\ \psi\\ \psi_1\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\ a_{21}&a_{22}&a_{23}&a_{24}\\ a_{31}&a_{32}&a_{33}&a_{34}\\ a_{41}&a_{42}&a_{43}&a_{44}\end{bmatrix}\begin{bmatrix}\xi\\ \xi_1\\ \psi\\ \psi_1\end{bmatrix}+\begin{bmatrix}0\\ f_1(\omega)\\ 0\\ f_2(\omega)\end{bmatrix},\tag{29}$$

$$\begin{aligned}f_1(\omega)=&3\xi^2(F_{30}+F_{20}h+H_{30}^{00})+2\xi\psi(F_{21}+F_{11}h+H_{21}^{00})+\psi^2(F_{12}+F_{02}h+H_{12}^{00})\\&+\xi'^2\left(\frac{1}{2}F_{10}-\frac{1}{2}F_{00}h\right)+\psi'^2\left(\frac{1}{2}F_{10}-\frac{1}{2}F_{00}h\right)+2\xi\xi'H_{20}^{10}+2\xi\psi'H_{20}^{01}+\xi'\psi H_{11}^{10}\\&+\psi\psi'H_{11}^{01}-\frac{\mathrm{d}}{\mathrm{d}\,\omega}(\xi\xi'(F_{10}-F_{00}h)+\psi\xi'F_{01}+\xi^2H_{20}^{10}+\psi^2H_{02}^{10}+\xi\psi H_{11}^{10}),\\f_2(\omega)=&3\psi^2(F_{03}+H_{03}^{00})+\xi^2(F_{21}+F_{11}h+H_{21}^{00})+2\xi\psi(F_{12}+F_{02}h+H_{12}^{00})\\&+\frac{1}{2}\xi'^2F_{01}+\frac{1}{2}\psi'^2F_{01}+2\xi'\psi H_{02}^{10}+2\psi\psi'H_{02}^{01}+\xi\xi'H_{11}^{10}+\xi\psi'H_{11}^{01}\\&-\frac{\mathrm{d}}{\mathrm{d}\,\omega}(\psi'\xi(F_{10}-F_{00}h)+\psi\psi'F_{01}+\xi^2H_{20}^{01}+\psi^2H_{02}^{01}+\xi\psi H_{11}^{01}).\end{aligned}\tag{30}$$

方程(29)和(30)的解为线性微分方程组的解与一个特解的和,特解为

$$\begin{aligned}&[\bar{\mathrm{Y}}]=c_1(\omega)[\mathrm{Y}_a]+c_2(\omega)[\mathrm{Y}_b]+c_3(\omega)[\mathrm{Y}_c]+c_4(\omega)[\mathrm{Y}_d],\\&c_i=\int c_i'(\omega)\mathrm{d}\,\omega(i=1,2,3,4),\end{aligned}\tag{31}$$

式中

$$\begin{aligned}&c_1'(\omega)\xi_a(\omega)+c_2'(\omega)\xi_b(\omega)+c_3'(\omega)\xi_c(\omega)+c_4'(\omega)\xi_d(\omega)=0,\\&c_1'(\omega)\xi_{1a}(\omega)+c_2'(\omega)\xi_{1b}(\omega)+c_3'(\omega)\xi_{1c}(\omega)+c_4'(\omega)\xi_{1d}(\omega)=f_1(\omega),\\&c_1'(\omega)\psi_a(\omega)+c_2'(\omega)\psi_b(\omega)+c_3'(\omega)\psi_c(\omega)+c_4'(\omega)\psi_d(\omega)=0,\\&c_1'(\omega)\psi_{1a}(\omega)+c_2'(\omega)\psi_{1b}(\omega)+c_3'(\omega)\psi_{1c}(\omega)+c_4'(\omega)\psi_{1d}(\omega)=f_1(\omega).\end{aligned}\tag{32}$$

二次近似轨迹 ξ,ψ 为

$$\begin{aligned}\xi=&(\xi_0+c_1(\omega))\xi_a(\omega)+(\xi_{10}+c_2(\omega))\xi_b(\omega)+(\psi_0+c_3(\omega))\xi_c(\omega)\\&+(\psi_{10}+c_4(\omega))\xi_d(\omega),\\\psi=&(\xi_0+c_1(\omega))\psi_a(\omega)+(\xi_{10}+c_2(\omega))\psi_b(\omega)+(\psi_0+c_3(\omega))\psi_c(\omega)\\&+(\psi_{10}+c_4(\omega))\psi_d(\omega).\end{aligned}\tag{33}$$

3　扁平阴极射线管电子光学系统曲光轴坐标系下旁轴一级近似的数值计算结果

扁平阴极射线管与常规阴极射线管的主要区别在于:常规阴极射线管中,电子束出电子枪后,经过一次磁偏转上屏;扁平阴极射线管中的电子束则是偏转数次后上屏.在设计和试制过程中,计算主要是两个目的,一是计算电子束上屏时光点的大小,与实验结果参照,分析工艺过程中电磁结构实际装配的影响,二是分析电磁场对电子束的影响,以便进一步改善电磁结构.基本电磁结构如图 2 所示.偏转三次上屏的电子束主轨迹坐标之间的关系如图 3 和图 4 所示.图 5 为中心面上磁场的分布.电子束在出了电子枪后,首先进入帧扫描磁场,偏转两次,当磁场强度变化就会使电子束在帧方向上产生扫描,接着电子束

进入行电场扫描区域，电子束在电场作用下偏转打到显示屏上，当电极板电压改变时，就会产生行方向上的扫描．在众多的扁平管方案中，电子束通常要偏转数次，当电子束到达

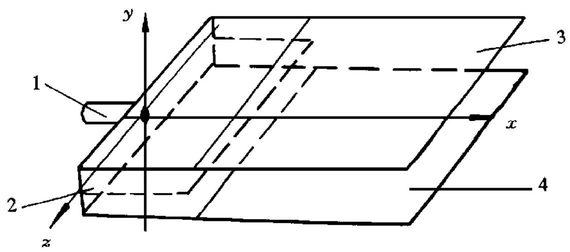

图 2 基本电磁结构 1 为电子枪；2 为磁场中心面；3 为屏；4 为反射电极

屏时有可能已经散焦．

在某一尺寸结构下的电磁场分布中计算出主轨迹，以此主轨迹为曲光轴，计算出沿曲光轴的各系数和各特解．在此仅给出一级近似下几个特解曲光轴坐标下的结果，见图 6 至图 13．

首先，可得到束斑大小．电子束出枪进入磁场偏转时，束半径约为 0.5 mm，张角约为 0.002 rad，结合计算结果可知电

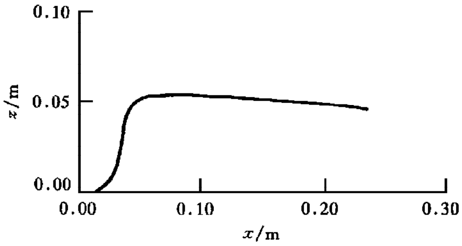

图 3 主轨迹在 z-x 平面的投影

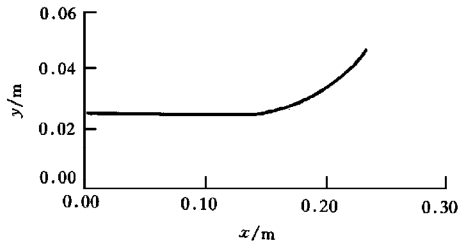

图 4 主轨迹在 y-x 平面的投影

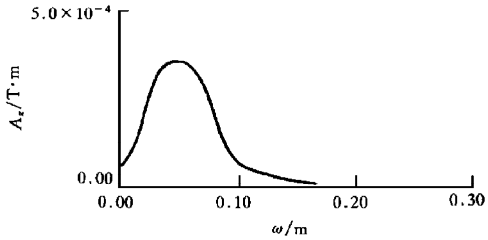

图 5 磁矢位 z 分量沿曲光轴的分布 数据为计算值

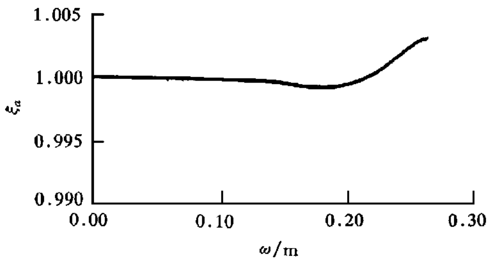

图 6 ξ_a 沿曲光轴的分布

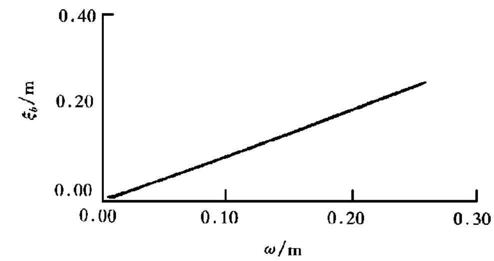

图 7 ξ_b 沿曲光轴的分布

子束到达屏时的束斑大小约为 2 mm.

其次,由计算的结果可以看出,电子束在通过这种结构的磁场时,所受到的聚散焦作用远小于进入偏转电场后所受到的作用,因此进一步优化电场的分布比优化磁场的分布显得更为重要.

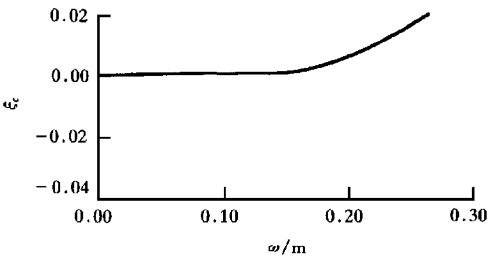

图 8　ξ_c 沿曲光轴的分布

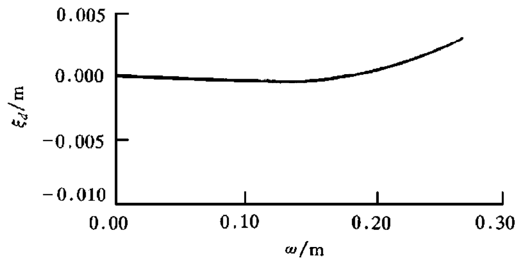

图 9　ξ_d 沿曲光轴的分布

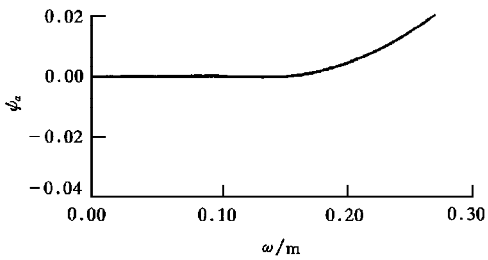

图 10　ψ_a 沿曲光轴的分布

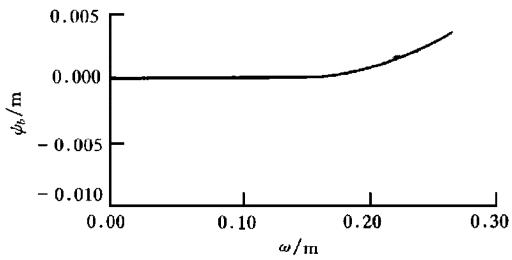

图 11　ψ_b 沿曲光轴的分布

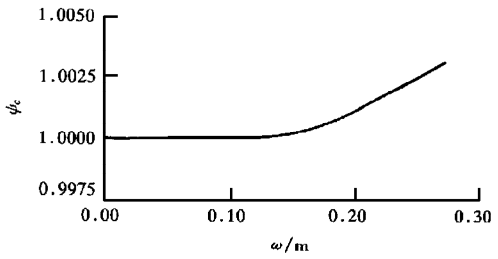

图 12　ψ_c 沿曲光轴的分布

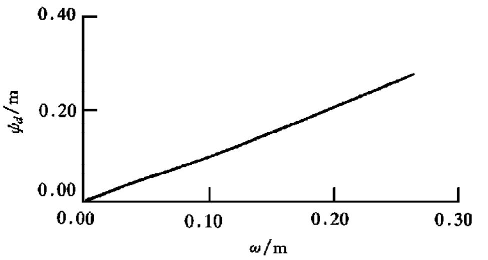

图 13　ψ_d 沿曲光轴的分布

再者,由结果可以看出,曲光轴附近的电子束,在与曲光轴垂直的两个方向上的运动相互不是独立的,用矩阵描述将比较合适.

[1] Ximen Ji-ye, Theory of Electron and Ion Optic Principle and Aberration (Science Press, Beijing, 1983), p. 15 (in Chinese).

[2] Su Bu-qing, Differential Geometry (People Education Press, Beijing, 1980), p. 10 (in Chinese).

[3] Ximen Ji-ye, *Acta Physica Sinica*, **29** (1980), 330 (in Chinese).

THE PARAXIAL ELECTRON OPTICS IN THE THREE-DIMENSIONAL CURVILINEAR COORDINATE SYSTEM

NIE SHI-QI LIN SHU-QUAN HUANG BAO-LIN
(HDTV Institute, Shanghai Jiaotong University, Shanghai 200030)
(Received 23 June 1997; revised manuscript received 4 January 1998)

ABSTRACT

By applying the definitions of curvilinear coordinate in differential geometry and using the main electron trajectory as the curve axis, the first order and second order paraxial electron trajectory equations in the three-dimensional coordinate system are obtained. Some numerical results of these equations applied to the analysis of an electron-beam scanning flat cathode ray tube are given.

PACC: 0777; 0780; 2920

更 正

本刊第 47 卷第 1 期“低掺杂 $La_{1-x}Sr_xMnO_3$ 结构相变对其电磁特性的影响”一文,由于作者疏忽,造成数处错误,现更正如下:

1. 第 112 页引言第 9 至 10 行:“反应锰离子最外层巡游电子跃迁能力的交换积分可表示为 $t=$……”应改为“反应晶格畸变大小可定量地用公差因子 t 来描述,且 $t=$……”

2. 第 113 页最后一句:“反应巡游电子跃迁能力的交换积分可表示为 $t=$……”应改为“与 e_g 电子跃迁能力相关的量可表示为 $t=$……”

3. 图 4 应改为

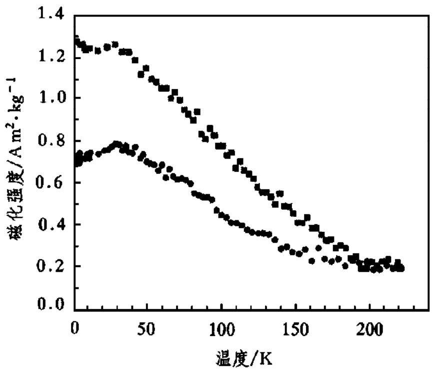

图 4 用提拉法测得 $La_{1-x}Sr_xMnO_3$ ($x=0.15$)样品的磁化强度-温度曲线 ●为零场冷却;■为场冷;磁场约为 4000 A/m