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匀加速直线运动的 Kerr 黑洞的特征曲面*

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研究了作匀加速直线运动的变质量 Kerr 黑洞的三类特征曲面，即事件视界、表观视界和类时极限面（无限红移面）. 指出在变质量（存在吸积或蒸发）情况下，这类黑洞的三类特征曲面不再重合.

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1 引 言

近年来，黑洞的反作用已被大量地研究[1—3]，在文献[4]中研究了动态 Kerr 黑洞的类时极限面，表观视界和事件视界的位置，它们不再重合. 在动态情况下，黑洞外能层变厚，内能层变薄. 文献[5]中在 de Sitter 膨胀宇宙中研究了一个 Vaidya 蒸发黑洞模型，讨论了表观视界和事件视界，发现它们不再重合. 文献[6]中用 C 度规研究了作匀加速且旋转的稳态黑洞的量子热性质，由零超曲面方程决定出事件视界的位置. 而本文进一步研究了作匀加速直线运动的动态 Kerr 黑洞的三类特征曲面.

匀加速直线运动的 Kerr 黑洞的线元用超前爱丁顿坐标表示为[7]

$$
\begin{aligned}
\mathrm{d}s^2 = {} & (1-F)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r - 2fp^2\mathrm{d}v\mathrm{d}\theta + 2\sin^2\theta AF\mathrm{d}v\mathrm{d}\varphi \\
& + 2A\sin^2\theta\mathrm{d}r\mathrm{d}\varphi - p^2\mathrm{d}\theta + 2Afp^2\sin^2\theta\mathrm{d}\theta\mathrm{d}\varphi \\
& - \sin^2\theta[A^2F\sin^2\theta + A^2 + r^2]\mathrm{d}\varphi^2,
\end{aligned}
\tag{1}
$$

其中

$$
\begin{aligned}
& F \equiv 2ar\cos\theta + 2mr/p^2 + p^2f^2, \\
& f \equiv - a\sin\theta, \\
& p^2 \equiv r^2 + A^2\cos^2\theta.
\end{aligned}
\tag{2}
$$

转动轴的北极 $\theta=0$，指向加速度方向，A 为单位质量的角动量，是一个常量；a 为加

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速度,是一个常量; m 为源质量,是 v 的函数.

2　类时极限面(TLS)

类时极限面的定义为

$$g_{vv} = \delta_{ab}\left(\frac{\partial}{\partial v}\right)^a\left(\frac{\partial}{\partial v}\right)^b = 0. \tag{3}$$

由方程(1) 可知:

$$g_{vv} = 1 - F = 0, \tag{4}$$

即

$$-a^2\sin^2\theta r^4 - 2a\cos\theta r^3 + (1 - 2A^2a^2\sin^2\theta\cos^2\theta)r^2 - (2m + 2A^2a\cos^3\theta)r + A^2\cos^2\theta - A^4a^2\sin^2\theta\cos^4\theta = 0. \tag{5}$$

解方程(5)得 4 个根,其中 1 个是负根无物理意义,其余 3 个根为:

$$r_{\mathrm{TLS}_1} = -D + E + R - \frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{6}$$

$$r_{\mathrm{TLS}_2} = D - E + R - \frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{7}$$

$$r_{\mathrm{TLS}_3} = D + E - R - \frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{8}$$

其中

$$D = \frac{1}{\sqrt{6}\,a\sin\theta}[1 - \alpha\cos(\varphi/3 - \pi/3)]^{1/2}, \tag{9}$$

$$E = \frac{1}{\sqrt{6}\,a\sin\theta}[1 - \alpha\cos(\varphi/3 + \pi/3)]^{1/2}, \tag{10}$$

$$R = \frac{1}{\sqrt{6}\,a\sin\theta}[1 + \alpha\cos(\varphi/3)]^{1/2}, \tag{11}$$

$$\alpha = \left[\left(1 - 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\right)^2 - 12\left(\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^6\theta} + \frac{1}{4}\frac{\cos^2\theta}{a^2\sin^4\theta} + \frac{m\cos\theta}{a\sin^2\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta} + A^2\cos^2\theta - A^4a^2\sin^2\theta\cos^4\theta\right)\right]^{1/2}, \tag{12}$$

$$\varphi = \arccos(-\beta/\alpha^3). \tag{13}$$

当 $0 < \varphi < \pi$ 时,

$$\beta = \left(1 - 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\right)^3 + 30a^2\sin^2\theta\left(1 - 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\right)\left[\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^6\theta} + \frac{1}{4}\frac{\cos^2\theta}{a^2\sin^4\theta} + \frac{m\cos\theta}{a\sin^2\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta} + A^2\cos^2\theta - A^4a^2\sin^2\theta\cos^4\theta\right] - 27a^2\sin^2\theta\left(2m + \frac{\cos\theta}{a\sin^2\theta} + \frac{3}{2}\frac{\cos^3\theta}{a\sin^4\theta}\right)^2. \tag{14}$$

r_{TLS_1}为 Rindler 的类时极限面, r_{TLS_2}和 r_{TLS_3}分别为匀加速直线运动的 Kerr 黑洞的外部和内部类时极限面.

当 $A=0$, $m=0$ 时,方程(5)化为

$$a^2\sin^2\theta r^4 + 2a\cos\theta r^3 - r^2 = 0, \tag{15}$$

解之得 Rindler 的类时极限面

$$r = [a(1+\cos\theta)]^{-1}. \tag{16}$$

当 $a=0$ 时(即无加速静止不动的黑洞),方程(5)变为

$$r^2 - 2mr + A^2\cos^2\theta = 0, \tag{17}$$

解之得

$$r_{\mathrm{TLS}}^{\pm} = m \pm \sqrt{m^2 - A^2\cos^2\theta}. \tag{18}$$

此时的类时极限面与动态 Kerr 黑洞的类时极限面相一致.

对于蒸发黑洞 $\dot{m}=\frac{\mathrm{d}m}{\mathrm{d}v}<0$, 则不难看出外类时极限面逐渐收缩,而内类时极限面逐渐膨胀.

对于吸积黑洞 $\dot{m}=\frac{\mathrm{d}m}{\mathrm{d}v}>0$, 则有相反的结果.

3 表观视界(AH)

表观视界定义为对于出射光子的最外陷获面(俘获面),从它出发的两簇类光测地线汇的膨胀 Θ 为零.

由方程(1)可知其度规为

$$(g_{\mu\nu}) = \begin{pmatrix} 1-F & -1 & -fp^2 & AF\sin^2\theta \\ -1 & 0 & 0 & A\sin^2\theta \\ -fp^2 & 0 & -p^2 & Afp^2\sin^2\theta \\ AF\sin^2\theta & A\sin^2\theta & Afp^2\sin^2\theta & -\sin^2\theta[A^2F\sin^2\theta + A^2 + r^2] \end{pmatrix}. \tag{19}$$

不难算出行列式和逆变度规

$$g = -p^4\sin^2\theta, \tag{20}$$

$$(g^{\mu\nu}) = \begin{pmatrix} -\frac{A^2}{p^2}\sin^2\theta & -\left(1+\frac{A^2}{p^2}\sin^2\theta\right) & 0 & -\frac{A}{p^2} \\ -\left(1+\frac{A^2}{p^2}\sin^2\theta\right) & \left(F-1-f^2p^2-\frac{A^2}{p^2}\sin^2\theta\right) & f & -\frac{A}{p^2} \\ 0 & f & -\frac{1}{p^2} & 0 \\ -\frac{A}{p^2} & -\frac{A}{p^2} & 0 & -\frac{1}{p^2\sin^2\theta} \end{pmatrix}. \tag{21}$$

可算出不为零的联络(见附录),下面引入零标架.先将方程(1)对称化得

$$\begin{aligned} \mathrm{d}s^2 = {} & [\gamma(r^2+A^2)(\mathrm{d}v - A\sin^2\theta\mathrm{d}\varphi)]\left[\frac{1}{r^2+A^2}\left(\frac{\Delta}{2}\mathrm{d}v - \gamma^{-1}\mathrm{d}r\right.\right. \\ & \left.\left. - f\gamma^{-2}\mathrm{d}\theta - \frac{\Delta}{2}A\sin^2\theta\mathrm{d}\varphi\right)\right] + \left[\frac{1}{r^2+A^2}\left(\frac{\Delta}{2}\mathrm{d}v - \gamma^{-1}\mathrm{d}r - f\gamma^{-2}\mathrm{d}\theta\right.\right. \\ & \left.\left. - \frac{\Delta}{2}A\sin^2\theta\mathrm{d}\varphi\right)\right]\ [\gamma(r^2+A^2)(\mathrm{d}v - A\sin^2\theta\mathrm{d}\varphi)] - \sqrt{\frac{\gamma}{2}}\left[\mathrm{i}A\sin\theta\mathrm{d}v\right. \\ & \left. + \gamma^{-1}\mathrm{d}\theta - \mathrm{i}(r^2+A^2)\sin\theta\mathrm{d}\varphi\right]\sqrt{\frac{\gamma}{2}}[-\mathrm{i}A\sin\theta\mathrm{d}v + \gamma^{-1}\mathrm{d}\theta \end{aligned}$$

$$+\mathrm{i}(r^2+A^2)\sin\theta\mathrm{d}\varphi]-\sqrt{\frac{\gamma}{2}}[-\mathrm{i}A\sin\theta\mathrm{d}v+\gamma^{-1}\mathrm{d}\theta+\mathrm{i}(r^2+A^2)\sin\theta\mathrm{d}\varphi]$$

$$\times\sqrt{\frac{\gamma}{2}}[\mathrm{i}A\sin\theta\mathrm{d}v+\gamma^{-1}\mathrm{d}\theta-\mathrm{i}(r^2+A^2)\sin\theta\mathrm{d}\varphi], \tag{22}$$

其中
$$\gamma=1/p^2,\qquad \Delta=r^2+A^2-\gamma^{-1}F. \tag{23}$$

得零标架为(不为零的联络见附录)

$$\begin{aligned}
n_\mu&=\gamma(r^2+A^2)(1,0,0,-A\sin^2\theta),\\
l_\mu&=\frac{1}{r^2+A^2}\left(\frac{\Delta}{2},-\gamma^{-1},-f\gamma^{-2},-\frac{\Delta}{2}A\sin^2\theta\right),\\
m_\mu&=\sqrt{\frac{\gamma}{2}}(-\mathrm{i}A\sin\theta,0,\gamma^{-1},\mathrm{i}(r^2+A^2)\sin\theta),\\
\overline{m}_\mu&=\sqrt{\frac{\gamma}{2}}(\mathrm{i}A\sin\theta,0,\gamma^{-1},-\mathrm{i}(r^2+A^2)\sin\theta).
\end{aligned} \tag{24}$$

用度规升降指标得逆变零标架为

$$\begin{aligned}
n^\mu&=\gamma(r^2+A^2)(0,-1,0,0),\\
l^\mu&=\left(1,\frac{\Delta}{2(r^2+A^2)},0,\frac{A}{r^2+A^2}\right),\\
m^\mu&=\sqrt{\frac{\gamma}{2}}\left(\mathrm{i}A\sin\theta,0,1,\frac{\mathrm{i}}{\sin\theta}\right),\\
\overline{m}^\mu&=\sqrt{\frac{\gamma}{2}}\left(-\mathrm{i}A\sin\theta,0,1,-\frac{\mathrm{i}}{\sin\theta}\right).
\end{aligned} \tag{25}$$

不难验证,(24),(25)式满足零标架条件

$$g_{\mu\nu}=l_\mu n_\nu+n_\mu l_\nu-m_\mu\overline{m}_\nu-\overline{m}_\mu m_\nu, \tag{26}$$

$$\begin{aligned}
&n_\mu n^\mu=l_\mu l^\mu=m_\mu m^\mu=\overline{m}_\mu\overline{m}^\mu=0,\\
&n^\mu l_\mu=-m_\mu\overline{m}^\mu=1,\\
&n_\mu m^\mu=n_\mu\overline{m}^\mu=l_\mu m^\mu=l_\mu\overline{m}^\mu=0.
\end{aligned} \tag{27}$$

而膨胀
$$\Theta=l^\mu_{;\mu}-\kappa, \tag{28}$$

其中
$$\kappa=-\frac{1}{2}\frac{\mathrm{d}F}{\mathrm{d}r}+\frac{A^2\sin^2\theta}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{A^2Fr\sin^2\theta}{(r^2+A^2)^2}, \tag{29}$$

所以
$$\Theta=\frac{r\Delta}{p^2(r^2+A^2)}. \tag{30}$$

可见表观视界由 $\Delta=0$ 来定,即

$$\begin{aligned}
&-a^2\sin^2\theta r^4-2a\cos\theta r^3+(1-2A^2a^2\sin^2\theta\cos^2\theta)r^2\\
&-(2m+2A^2a\cos^3\theta)r-A^4a^2\sin^2\theta\cos^4\theta+A^2=0.
\end{aligned} \tag{31}$$

解方程(31)可得 4 个根,其中 1 个是负根无物理意义,其余 3 个根为

$$r_{\mathrm{AH}_1}=-D'+E'+R'-\frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{32}$$

$$r_{\mathrm{AH}_2}=D'-E'+R'-\frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{33}$$

$$r_{AH_3} = D' + E' - R' - \frac{1}{2}\frac{\cos\theta}{a\sin^2\theta}, \tag{34}$$

其中
$$D' = \frac{1}{\sqrt{6}\,a\sin\theta}[1 - \alpha'\cos(\varphi'/3 - \pi/3)]^{1/2}, \tag{35}$$

$$E' = \frac{1}{\sqrt{6}\,a\sin\theta}[1 - \alpha'\cos(\varphi'/3 + \pi/3)]^{1/2}, \tag{36}$$

$$R' = \frac{1}{\sqrt{6}\,a\sin\theta}[1 + \alpha'\cos(\varphi'/3)]^{1/2}, \tag{37}$$

$$\alpha' = \left[\left(1 - 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\right)^2 - 12\left(\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^6\theta} + \frac{1}{4}\frac{\cos^2\theta}{a^2\sin^4\theta}\right.\right.$$
$$\left.\left. + \frac{m\cos\theta}{a\sin^2\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta} + A^2 - A^4a^2\sin^2\theta\cos^4\theta\right)\right]^{1/2}, \tag{38}$$

$$\varphi' = \arccos(-\beta'/\alpha'^3). \tag{39}$$

当 $0<\varphi'<\pi$ 时,

$$\beta' = \left(1 - 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\right)^3 + 30a^2\sin^2\theta\left(1 - 2A^2a^2\sin^2\theta\cos^2\theta\right.$$
$$\left. + \frac{3}{2}\cot^2\theta\right)\left[\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^6\theta} + \frac{1}{4}\frac{\cos^2\theta}{a^2\sin^4\theta} + \frac{m\cos\theta}{a\sin^2\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta}\right.$$
$$\left. + A^2 - A^4a^2\sin^2\theta\cos^4\theta\right] - 27\left(2m + \frac{\cos\theta}{a\sin^2\theta} + \frac{3}{2}\frac{\cos^3\theta}{a\sin^4\theta}\right)^2 a^2\sin^2\theta. \tag{40}$$

r_{AH_1}为 Rindler 的表观视界, r_{AH_2}和 r_{AH_3}分别为匀加速直线运动的 Kerr 黑洞的外部和内部的表观视界.

当 $A=0$, $m=0$ 时得 Rindler 表观视界,方程(31)变为

$$a^2\sin^2\theta\, r^4 + 2a\cos\theta\, r^3 - r^2 = 0, \tag{41}$$

解之得

$$r = [a(1+\cos\theta)]^{-1}. \tag{42}$$

当 $a=0$ 时,则表观视界由 $\Delta = r^2 + A^2 - 2mr = 0$ 来确定,此时

$$r^{\pm}_{AH} = m \pm \sqrt{m^2 - A^2}. \tag{43}$$

也就是说此时的表观视界与不作加速运动的 Kerr 黑洞的表观视界相一致,即回到 Kerr 黑洞.

4 事件视界(EH)

确定动态时空中事件视界的位置是一个非常困难的问题. York[8]认为,事件视界是光子的黏着面,不允许光子从这里逃向远方,因此我们由

$$\frac{d}{dv} = l^a\nabla_a = \frac{\partial}{\partial v} + \frac{\Delta}{2(r^2 + A^2)}\frac{\partial}{\partial r} + \frac{A}{r^2 + A^2}\frac{\partial}{\partial\varphi} \tag{44}$$

可得

$$\dot{r} = \frac{dr}{dv} = \frac{\Delta}{2(r^2 + A^2)}, \tag{45}$$

$$\ddot{r}=-\frac{p^2}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}v}+\dot{r}\left(\frac{r(1-F)}{r^2+A^2}-\frac{p^2}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{2r\dot{r}}{r^2+A^2}\right).\tag{46}$$

由(45)和(46)式可知,当$\frac{\mathrm{d}F}{\mathrm{d}v}<0$时,在外表观视界处即 r_{AH}^{+}处

$$\dot{r}=0,\qquad \ddot{r}>0.\tag{47}$$

因此,光子可以从表观视界逃到远方,这不符合事件视界的定义.可见,对于匀加速直线运动的动态 Kerr 黑洞而言,表观视界与事件视界不再重合,根据 York 的"工作定义",即当把辐射量 $L=-\dot{m}$看作小量时,在一级近似下,1) 事件视界应严格类光;2) 光子应长时间滞留在事件视界上.这就是说,在 $O(L)$的数量级,事件视界应由

$$\dot{r}_{EH}\approx 0,\qquad \ddot{r}_{EH}\approx 0\tag{48}$$

决定,利用叠代法,先令(46)式中的 $\ddot{r}=0$ 得

$$-\frac{p^2}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}v}+\dot{r}\left(\frac{r(1-F)}{r^2+A^2}-\frac{p^2}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{2r\dot{r}}{r^2+A^2}\right)=0.\tag{49}$$

当$\frac{\mathrm{d}F}{\mathrm{d}v}=0$时,即稳态情况下,

$$\dot{r}\left(\frac{r(1-F)}{r^2+A^2}-\frac{p^2}{2(r^2+A^2)}\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{2r\dot{r}}{r^2+A^2}\right)=0.\tag{50}$$

解(50)式得

$$\dot{r}=\frac{1}{2}\left(1-2ar\cos\theta-\frac{2mr}{p^2}-p^2f^2\right)-\frac{p^2}{4}\left(2a\cos\theta+2rf^2+\frac{2mp^2-4mr}{p^4}\right).\tag{51}$$

这时的 r 应取 r_{AH}^{+},则

$$\dot{r}=\frac{1}{2}\left[1-2ar_{AH}^{+}\cos\theta-\frac{2mr_{AH}^{+}}{r_{AH}^{+2}+A^2\cos^2\theta}-A^2\sin^2\theta(r_{AH}^{+2}+A^2\cos^2\theta)\right]-\frac{1}{4}(r_{AH}^{+2}+A^2\cos^2\theta)$$
$$\times\left[2a\cos\theta+\frac{2m(r_{AH}^{+2}+A^2\cos^2\theta)-4mr_{AH}^{+}}{(r_{AH}^{+2}+A^2\cos^2\theta)^2}+2a^2\sin^2\theta\, r_{AH}^{+}\right].\tag{52}$$

将(52)式代入(45)式整理后得

$$-a^2\sin^2\theta\, r^4-2a\cos\theta\, r^3-(2A^2a^2\sin^2\theta\cos^2\theta+2\dot{r}-1)r^2$$
$$-(2m+2A^2\cos^3\theta\, a)r-2\dot{r}A^2+A^2-a^2\sin^2\theta A^4\cos^4\theta=0.\tag{53}$$

解(53)式得 4 个根,其中 1 个是负根无物理意义,其余 3 个根为

$$r_{EH_1}=-D''+E''+R''-\frac{1}{2}\frac{\cos\theta}{a\sin^2\theta},\tag{54}$$

$$r_{EH_2}=D''-E''+R''-\frac{1}{2}\frac{\cos\theta}{a\sin^2\theta},\tag{55}$$

$$r_{EH_3}=D''+E''-R''-\frac{1}{2}\frac{\cos\theta}{a\sin^2\theta},\tag{56}$$

其中

$$D''=\frac{1}{\sqrt{6}\,a\sin\theta}[1-\alpha''\cos(\varphi''/3-\pi/3)]^{1/2},\tag{57}$$

$$E''=\frac{1}{\sqrt{6}\,a\sin\theta}[1-\alpha''\cos(\varphi''/3+\pi/3)]^{1/2},\tag{58}$$

$$R'' = \frac{1}{\sqrt{6}\, a\sin\theta}[1 + \alpha''\cos(\varphi''/3)]^{1/2}, \tag{59}$$

$$\alpha'' = \Big[\Big(1 - r\dot{r} - 2A^2 a^2 \sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\Big)^2 - 12\Big(\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^6\theta} + \frac{m\cos\theta}{a\sin^2\theta}$$
$$+ \frac{1}{4}\frac{\cos^2\theta}{a^2\sin^4\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta} - 2\dot{r}A^2 + A^2 - a^2\sin^2\theta A^4\cos^4\theta\Big)\Big]^{1/2}, \tag{60}$$

$$\varphi'' = \arccos(-\beta''/\alpha''^3). \tag{61}$$

当 $0<\varphi''<\pi$ 时,

$$\beta'' = \Big(1 - 2\dot{r} - 2A^2 a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\Big)^3 + 30a^2\sin^2\theta\Big(1 - 2\dot{r}$$
$$- 2A^2a^2\sin^2\theta\cos^2\theta + \frac{3}{2}\cot^2\theta\Big)\Big(\frac{3}{16}\frac{\cos^4\theta}{a^2\sin^4\theta} + \frac{m\cos\theta}{a\sin^2\theta} + \frac{1}{2}\frac{A^2\cos^4\theta}{\sin^2\theta} - 2\dot{r}A^2$$
$$+ A^2 - A^4a^2\sin^2\theta\cos^4\theta\Big) - 27\Big(2m + \frac{\cos\theta}{a\sin^2\theta} + \frac{3}{2}\frac{\cos^3\theta}{a\sin^4\theta}\Big)^2 a^2\sin^2\theta. \tag{62}$$

r_{EH_1}为 Rindler 视界, r_{EH_2}和 r_{EH_3}分别为匀加速直线运动的 Kerr 黑洞的外部和内部事件视界.当 $m=0$, $A=0$, $\dot{r}=0$ 时,得 Rindler 事件视界,由方程(53)得

$$a^2\sin^2\theta r^2 + 2a\cos\theta r - 1 = 0. \tag{63}$$

解之得

$$r = [a(1+\cos\theta)]^{-1}. \tag{64}$$

恰为 Rindler 事件视界.由此可知,对于 Rindler 观测者而言,它的 3 个视界面是重合的.

当 $a=0$ 时,方程(53)变为

$$(1-2\dot{r})r^2 - 2mr + (1-2\dot{r})A^2 = 0, \tag{65}$$

解之得

$$r_{EH}^{\pm} = \frac{m \pm \sqrt{m^2 - A^2(1-2\dot{r})^2}}{(1-2\dot{r})}. \tag{66}$$

当 $\dot{r}=0$ 时,事件视界回到熟知的稳态 Kerr 视界的位置,

$$r_{EH}^{\pm} = m \pm \sqrt{m^2 - A^2}. \tag{67}$$

总之,我们求出了作匀加速直线运动的 Kerr 黑洞类时极限面、表观视界和事件视界的位置,它们不再重合,在动态情况下,黑洞的外能层变厚,内能层变薄.

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附 录

为了求表观视界,我们计算出了不为零的联络为

$$\Gamma^0_{00} = -\frac{A^2\sin^2\theta}{2p^2}\frac{dF}{dv} - \frac{1}{2}\Big(1 + \frac{A^2}{p^2}\sin^2\theta\Big)\frac{dF}{dr},$$

$$\Gamma^0_{02} = \Gamma^0_{20} = -\Big(1 + \frac{A^2}{p^2}\sin^2\theta\Big)rf - \frac{A^2F}{2p^2}\sin 2\theta,$$

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{2}\left(1+\frac{A^2}{p^2}\sin^2\theta\right)\left(A\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}r}\right)+\frac{A^3\sin^4\theta}{2p^2}\frac{\mathrm{d}F}{\mathrm{d}v},$$

$$\Gamma^0_{12} = \Gamma^0_{21} = -\frac{A^2\sin2\theta}{2p^2},\quad \Gamma^0_{13} = \Gamma^0_{31} = \frac{Ar\sin^2\theta}{p^2},$$

$$\Gamma^0_{22} = -\frac{(A^2+r^2)r}{p^2} - A^2 f\sin2\theta,$$

$$\Gamma^0_{23} = \Gamma^0_{32} = \frac{A^3\sin^2\theta}{2p^2}F\sin2\theta+\frac{(A^2+r^2)}{p^2}Afr\sin^2\theta,$$

$$\Gamma^0_{33} = -\frac{A^4\sin^6\theta}{2p^2}\left(\frac{\mathrm{d}F}{\mathrm{d}v}+\frac{\mathrm{d}F}{\mathrm{d}r}\right)-\frac{1}{2}A^2\sin^4\theta\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{(A^2+r^2)r\sin^2\theta}{p^2},$$

$$\Gamma^1_{00} = \frac{1}{2}\left(1-\frac{A^2}{p^2}\sin^2\theta\right)+\frac{\mathrm{d}F}{\mathrm{d}v}+\frac{1}{2}g^{11}\frac{\mathrm{d}F}{\mathrm{d}r}+\frac{1}{2}f\frac{\mathrm{d}F}{\mathrm{d}\theta},$$

$$\Gamma^1_{01} = \Gamma^1_{10} = \frac{1}{2}\frac{\mathrm{d}F}{\mathrm{d}r}-rf^2,$$

$$\Gamma^1_{02} = \Gamma^1_{20} = \frac{1}{2}\frac{\mathrm{d}F}{\mathrm{d}\theta}+g^{11}(rf)-\frac{A^2}{2p^2}F\sin2\theta,$$

$$\Gamma^1_{03} = \Gamma^1_{30} = -\frac{1}{2}g^{11}\left(A\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}r}\right)-\frac{1}{2}Af\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}\theta}-\frac{1}{2}AfF\sin2\theta+\frac{A^3\sin^4\theta}{2p^2}\frac{\mathrm{d}F}{\mathrm{d}v},$$

$$\Gamma^1_{12} = \Gamma^1_{21} = -\frac{A^2\sin2\theta}{2p^2},$$

$$\Gamma^1_{13} = \Gamma^1_{31} = -\frac{1}{2}A\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}r}+Arf^2\sin^2\theta-Af\sin\theta\cos\theta+\frac{Ar\sin^2\theta}{p^2},$$

$$\Gamma^1_{22} = fp^2\cot\theta-\frac{3}{2}A^2 f\sin2\theta+rg^{11},$$

$$\Gamma^1_{23} = \Gamma^1_{32} = -\frac{1}{2}A\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}\theta}-\frac{1}{2}f^2p^2A\sin2\theta-g^{11}Afr\sin^2\theta,$$

$$\Gamma^1_{33} = -\frac{1}{2}A^2\sin^4\theta\left(1+\frac{A^2}{p^2}\sin^2\theta\right)\frac{\mathrm{d}F}{\mathrm{d}v}+\frac{1}{2}g^{11}\left(A^2\sin^4\theta\frac{\mathrm{d}F}{\mathrm{d}r}+2r\sin^2\theta\right)$$
$$+\frac{1}{2}f\left(A^2\sin^4\theta\frac{\mathrm{d}F}{\mathrm{d}\theta}+2A^2F\sin^2\theta\sin2\theta+(A^2+r^2)\sin2\theta\right),$$

$$\Gamma^2_{00} = \frac{1}{2}f\frac{\mathrm{d}F}{\mathrm{d}r}-\frac{1}{2p^2}\frac{\mathrm{d}F}{\mathrm{d}\theta},$$

$$\Gamma^2_{01} = \Gamma^2_{10} = \frac{rf}{p^2},\quad \Gamma^2_{02} = \Gamma^2_{20} = rf^2$$

$$\Gamma^2_{03} = \Gamma^2_{30} = -\frac{1}{2}fA\sin^2\theta\frac{\mathrm{d}F}{\mathrm{d}r}+\frac{A\sin^2\theta}{2p^2}\frac{\mathrm{d}F}{\mathrm{d}\theta}+\frac{AF}{2p^2}\sin2\theta,$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{r}{p^2},\quad \Gamma^2_{13} = \Gamma^2_{31} = \frac{A\sin2\theta-2Arf\sin^2\theta}{2p^2},$$

$$\Gamma^2_{22} = rf-\frac{A^2\sin2\theta}{2p^2},\quad \Gamma^2_{23} = \Gamma^2_{32} = \frac{1}{2}Af\sin2\theta-Af^2r\sin^2\theta,$$

$$\Gamma^2_{33} = \frac{1}{2}f\left(A^2\sin^4\theta\frac{\mathrm{d}F}{\mathrm{d}r}+2r\sin^2\theta\right)-\frac{1}{2p^2}\left(A^2\sin^4\theta\frac{\mathrm{d}F}{\mathrm{d}\theta}\right.$$
$$\left.+2A^2F\sin^2\theta\sin2\theta+(A^2+r^2)\sin2\theta\right),$$

$$\Gamma^3_{00} = -\frac{A}{2p^2}\left[\frac{\mathrm{d}F}{\mathrm{d}r}+\frac{\mathrm{d}F}{\mathrm{d}v}\right],\quad \Gamma^3_{02} = \Gamma^3_{20} = -\frac{A}{p^2}(rf+F\cot\theta),$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{A^2\sin^2\theta}{2p^2}\left(\frac{\mathrm{d}F}{\mathrm{d}r}+\frac{dF}{dv}\right),\quad \Gamma^3_{12} = \Gamma^3_{21} = -\frac{A}{p^2}\cot\theta,$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{r}{p^2},\quad \Gamma^3_{22} = -2Af\cot\theta-\frac{Ar}{p^2},$$

$$\Gamma_{23}^{3} = \Gamma_{32}^{3} = \frac{(A^2 + r^2)}{p^2}\cot\theta + \frac{A^2 fr\sin^2\theta}{p^2} + \frac{A^2 F\sin 2\theta}{2p^2},$$

$$\Gamma_{33}^{3} = -\frac{A^3\sin^4\theta}{2p^2}\left(\frac{\mathrm{d}F}{\mathrm{d}v} + \frac{\mathrm{d}F}{\mathrm{d}r}\right) - \frac{Ar}{p^2}\sin^2\theta.$$

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CHARACTERISTIC SURFACES OF A UNIFORMLY RECTILINEARLY ACCELERATING KERR BLACK HOLE

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ABSTRACT

In this paper, we deal with three kinds of characteristic surfaces of a uniformly rectilinearly accelerating Kerr black hole whose mass is varying. i. e., the event horizon, the apparent horizon, and the time-like limit surface (the infinite red-shift surface). It is pointed out that the three kinds of characteristic surfaces no longer coincide with one other when the black hole mass is varying (accreting or evaporating).

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