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Einstein-Maxwell-Dilaton 黑膜的狄拉克粒子辐射*

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分析讨论了 Einstein-Maxwell-Dilaton 黑膜解的视界附近的狄拉克方程.证明了在乌龟坐标变换下,该黑膜视界附近确有狄拉克粒子辐射.

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1 引　言

文献[1]得出了带有 Dilaton 标量势和电磁场的有宇宙常数的场方程的平面对称解,其作用量为

$$S=\frac{1}{16\pi}\int d^4\sqrt{-g}\left[R-2(\nabla\Phi)^2-6\eta^2e^{2b\Phi}-e^{-2a\Phi}\boldsymbol{F}^2\right],\tag{1}$$

式中 a, b 为常数, g 为度规行列式.经变分后可得场方程为

$$\begin{aligned}
R_{\mu\nu}&=2\partial_\mu\Phi\partial_\nu\Phi+3\alpha^2\eta e^{2b\Phi}g_{\mu\nu}+2e^{-2a\Phi}\left(\boldsymbol{F}_{\mu\lambda}\boldsymbol{F}^\lambda_\nu-\frac{1}{4}g_{\mu\nu}\boldsymbol{F}^2\right),\\
0&=\partial_\mu\left(\sqrt{-g}e^{-2a\Phi}\boldsymbol{F}^{\mu\nu}\right),\\
\nabla^2\Phi&=3\alpha^2b\eta e^{2b\Phi}-\frac{a}{2}e^{-2a\Phi}\boldsymbol{F}^2,
\end{aligned}\tag{2}$$

式中 Φ 为 Dilaton 标量势, $\boldsymbol{F}_{\mu\nu}$ 为电磁场张量, $3\alpha^2$ 为宇宙常数, $\eta=1$ 代表宇宙常数的符号.(2)式的解为

$$ds^2=A(r)dt^2-B(r)dr^2-C(r)[dx^2+dy^2],\tag{3}$$

$$\begin{aligned}
&\Phi(r)=-\frac{\beta}{2}\ln r,\\
&F_{tr}=\frac{Q}{\sqrt{ABC}}e^{2a\Phi},\quad \beta=\sqrt{2N-N^2}\qquad 0<N<2,
\end{aligned}\tag{4}$$

式中
$$A(r)=B^{-1}(r)=-\frac{4\pi M}{N\alpha^N}r^{1-N}-\frac{6\alpha^2\eta}{N(2N-1)}r^N+\frac{2Q^2}{N\alpha^{2N}}r^{-N},\tag{5}$$

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$$C(r)=(\alpha r)^{N} \qquad r=|z|,$$

M, Q 分别为黑膜单位面积的质量和电荷, N 为解微分方程组的参数.

(2)式中含有参数 η, N, α, M, Q 等,其性质由于这些参数的取值不同而不同.

当 $\eta=-1$ 时,如果 $1/2<N<2$,该解为渐近反 de-Sitter 型解:不存在宇宙视界,其它视界由方程 $A(r)=0$ 决定.由于该方程一般有多重根,时空有多视界结构.特别地,如果此时 $M^2<3\alpha^2Q^2/\pi^2$,在 $r=0$ 处将出现裸奇点;如果 $0<N<1/2$,该解为渐近 de-Sitter 型解:存在宇宙视界,一般还有内外视界,它们都由方程 $A(r)=0$ 决定.在一些特殊情况下,如 $N=1,4$ 时,只有宇宙视界.

当 $\eta=+1$ 时,无论 N 取何值,(2)式都是渐近 de-Sitter 的:存在宇宙视界,一般还有内外视界,时空有多视界结构.特别地,如果此时 $N=1,1/4,3/2$,则只有宇宙视界.它们都满足方程 $A(r)=0$.故由文献[1]知,上述黑膜解的视界方程为

$$A(r)=0. \tag{5a}$$

而重力常数为

$$K=\frac{1}{2}\left.\frac{\partial A(r)}{\partial r}\right|_{r=r_H}=-\frac{2\pi M(1-N)}{N\alpha^N}r_H^{-N}-\frac{3\alpha^2\eta}{2N-1}r_H^{N-1}-\frac{Q^2}{\alpha^{2N}}r_H^{-N-1}. \tag{6}$$

如果存在宇宙视界,其方程及重力常数与一般视界满足同样的方程.因此,以下的讨论也同样适用于宇宙视界.

2 零标架的选取

将(3)式对称化,

$$\begin{aligned}ds^2=&[(1/2)A(dt+A^{-1}dr)](dt-A^{-1}dr)+(dt-A^{-1}dr)[(1/2)A(dt+A^{-1}dr)]\\&-[(2)^{-(1/2)}C^{1/2}(dx+idy)][(2)^{-(1/2)}C^{1/2}(dx-idy)]\\&-[(2)^{-(1/2)}C^{1/2}(dx-idy)][(2)^{-(1/2)}C^{1/2}(dx+idy)].\end{aligned}$$

设零标架的四个协变分量为

$$\begin{aligned}n_\mu&=(A/2,1/2,0,0),\\l_\mu&=(1,-A^{-1},0,0),\\m_\mu&=\left(0,0,\frac{\sqrt{2}}{2}C^{\frac{1}{2}},i\frac{\sqrt{2}}{2}C^{\frac{1}{2}}\right),\\\overline{m}_\mu&=\left(0,0,\frac{\sqrt{2}}{2}C^{\frac{1}{2}},-i\frac{\sqrt{2}}{2}C^{\frac{1}{2}}\right).\end{aligned} \tag{7}$$

由零标架的协变分量和度规可得它的逆变分量为

$$\begin{aligned}n^\mu&=\left(\frac{1}{2},-\frac{A}{2},0,0\right),\\l^\mu&=(1,-A^{-1},0,0),\\m^\mu&=\left(0,0,-\frac{\sqrt{2}}{2}C^{-\frac{1}{2}},-i\frac{\sqrt{2}}{2}C^{-\frac{1}{2}}\right),\end{aligned}$$

$$\overline{m}^{\mu}=\left(0,0,-\frac{\sqrt{2}}{2}C^{-\frac{1}{2}},\mathrm{i}\frac{\sqrt{2}}{2}C^{-\frac{1}{2}}\right). \tag{8}$$

容易验证它们满足零标架的定义式(参见文献[2]),故取零标架的协变形式如(7)式,逆变形式如(8)式.

3 旋系数的计算

根据文献[3]及(7),(8)式,得零标架下的旋系数为(下面“;”代表协变导数;“,”代表普通导数;希腊字母指标取 0,1,2,3,代表时空分量;重复指标表示求和;$\Gamma^{\lambda}_{\mu\nu}$为联络)

$$\kappa=l_{\mu;\nu}m^{\mu}l^{\nu}=0,$$

$$\varepsilon=\frac{1}{2}(l_{\mu;\nu}n^{\mu}l^{\nu}-m_{\mu;\nu}\overline{m}^{\mu}l^{\nu})=0,$$

$$\pi=-n_{\mu;\nu}\overline{m}^{\mu}l^{\nu}=0,$$

$$\rho=l_{\mu;\nu}m^{\mu}\overline{m}^{\nu}=-\frac{-1}{2}C^{-1}C',$$

$$\alpha=\frac{1}{2}(l_{\mu;\nu}n^{\mu}\overline{m}^{\nu}-m_{\mu;\nu}\overline{m}^{\mu}\overline{m}^{\nu})=0,$$

$$\lambda=-n_{\mu;\nu}\overline{m}^{\mu}\overline{m}^{\nu}=0,$$

$$\delta=l_{\mu;\nu}m^{\mu}m^{\nu}=0,$$

$$\beta=\frac{1}{2}(l_{\mu;\nu}n^{\mu}m^{\nu}-m_{\mu;\nu}\overline{m}^{\mu}m^{\nu})=0,$$

$$\mu=-n_{\mu;\nu}\overline{m}^{\mu}m^{\nu}=-\frac{1}{4}AC^{-1}C',$$

$$\tau=l_{\mu;\nu}m^{\mu}n^{\nu}=0,$$

$$\gamma=\frac{1}{2}(l_{\mu;\nu}n^{\mu}n^{\nu}-m_{\mu;\nu}\overline{m}^{\mu}n^{\nu})=\frac{1}{2}A',$$

$$\sigma=-n_{\mu;\nu}\overline{m}^{\mu}m^{\nu}=0.$$

以上各式中 A 代表 $A(r)$, A'代表 $A(r)$对 r 的导数, C 代表 $C(r)$, C'代表 $C(r)$对 r 的导数.

4 弯曲时空的狄拉克方程

弯曲时空的狄拉克方程为[3]

$$(D+\varepsilon-\rho+\mathrm{i}eA_{\mu}l^{\mu})F_1+(\overline{\delta}+\pi-\alpha+\mathrm{i}eA_{\mu}\overline{m}^{\mu})F_2=\frac{\mathrm{i}\mu_0}{\sqrt{2}}G_1,$$

$$(\Delta+\mu-\gamma+\mathrm{i}eA_{\mu}n^{\mu})F_2+(\delta+\beta-\tau+\mathrm{i}eA_{\mu}m^{\mu})F_1=\frac{\mathrm{i}\mu_0}{\sqrt{2}}G_2,$$

$$(D+\overline{\varepsilon}-\overline{\rho}+\mathrm{i}eA_{\mu}l^{\mu})G_2+(\delta+\overline{\pi}-\overline{\alpha}+\mathrm{i}eA_{\mu}m^{\mu})G_1=\frac{\mathrm{i}\mu_0}{\sqrt{2}}F_2,$$

$$(\Delta+\overline{\mu}-\overline{\nu}+\mathrm{i}eA_\mu n^\mu)G_1+(\overline{\delta}+\overline{\beta}-\overline{\tau}+\mathrm{i}eA_\mu\overline{m}^\mu)G_2=\frac{\mathrm{i}\mu_0}{\sqrt{2}}F_1, \tag{9}$$

式中

$$D=l^\mu\partial_\mu,\quad \Delta=n^\mu\partial_\mu,\quad \delta=m^\mu\partial_\mu,\quad \overline{\delta}=\overline{m}^\mu\partial_\mu, \tag{10}$$

$$\partial_\mu=\left(\frac{\partial}{\partial t},\frac{\partial}{\partial r},\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right). \tag{11}$$

字母上带"—"代表复共轭, A_μ 为 Einstein-Maxwell-Dilaton 时空中的电磁四矢, e 为狄拉克粒子的电荷, μ_0 为狄拉克粒子的静止质量, F_1, F_2, G_1, G_2 为波矢量的四个分量,都是时空坐标(t, r, x, y)的函数.

由(5)式可求得

$$A_\mu=(A_0,0,0,0)=\left(\frac{Q}{\alpha^N r},0,0,0\right). \tag{12}$$

将(8),(10),(11),(12)式代入(9)式,可得

$$D_t F_1+C^{\frac{-1}{2}}\overline{d}_t F_2=\frac{\mathrm{i}\mu_0}{\sqrt{2}}G_1,$$

$$\overline{D}_t F_2+C^{\frac{-1}{2}}d_t F_1=\frac{\mathrm{i}\mu_0}{\sqrt{2}}G_2,$$

$$D_t G_2-C^{\frac{-1}{2}}d_t G_1=\frac{\mathrm{i}\mu_0}{\sqrt{2}}F_2,$$

$$\overline{D}_t G_1-C^{\frac{-1}{2}}\overline{d}_t G_2=\frac{\mathrm{i}\mu_0}{\sqrt{2}}F_1, \tag{13}$$

式中

$$D_t=A^{-1}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\frac{1}{2}C^{-1}C'+\mathrm{i}eA_0A^{-1},$$

$$\overline{D}_t=\frac{1}{2}\frac{\partial}{\partial t}-\frac{A}{2}\frac{\partial}{\partial r}-\frac{1}{4}AC^{-1}C-\frac{1}{4}A'+\frac{1}{2}\mathrm{i}eA_0,$$

$$\overline{d}_t=\frac{-\sqrt{2}}{2}\frac{\partial}{\partial x}+\mathrm{i}\frac{\sqrt{2}}{2}\frac{\partial}{\partial y},$$

$$d_t=\frac{-\sqrt{2}}{2}\frac{\partial}{\partial x}-\mathrm{i}\frac{\sqrt{2}}{2}\frac{\partial}{\partial y}. \tag{14}$$

根据文献[4],可设

$$F_1=R_-(t,r)S_-(x,y),$$

$$F_2=R_+(t,r)S_+(x,y),$$

$$G_1=R_+(t,r)S_-(x,y),$$

$$G_2=R_-(t,r)S_+(x,y). \tag{15}$$

将(15)式代入(13)式,分离变量可得

$$C^{\frac{1}{2}}R_+^{-1}D_tR_--\frac{\mathrm{i}\mu_0}{\sqrt{2}}C^{\frac{1}{2}}=\lambda,$$

$$C^{\frac{1}{2}} R_-^{-1} \overline{D}_t R_+ - \frac{i\mu_0}{\sqrt{2}} C^{\frac{1}{2}} = -\lambda, \tag{16}$$

式中 λ 为常数.由(16)式,不难得到

$$\overline{D}_t \left[\left(\lambda C^{\frac{-1}{2}} + i\frac{\mu_0}{\sqrt{2}} \right)^{-1} D_t R_- \right] = \left(-\lambda C^{\frac{-1}{2}} + i\frac{\mu_0}{\sqrt{2}} \right) R_-,$$

$$D_t \left[\left(-\lambda C^{\frac{-1}{2}} + i\frac{\mu_0}{\sqrt{2}} \right)^{-1} \overline{D}_t R_+ \right] = \left(\lambda C^{\frac{-1}{2}} + i\frac{\mu_0}{\sqrt{2}} \right) R_+. \tag{17}$$

考虑到(14)式,此即

$$\begin{aligned} 0 = {} & \frac{\partial^2 R_-}{\partial t^2} + \left(\frac{1}{2} A' + 2ieA_0 + M^{-1} M'A \right) \frac{\partial R_-}{\partial t} - A^2 \frac{\partial^2 R_-}{\partial r^2} - \left(A^2 C^{-1} C' - A^2 M^{-1} M' \right. \\ & \left. + \frac{1}{2} A'A \right) \frac{\partial R_-}{\partial r} - \left[-\frac{1}{2} ieA_0 A' + e^2 A_0^2 + A^2 \frac{\partial}{\partial r} \left(\frac{1}{2} C^{-1} C' \right) + A \frac{\partial}{\partial r} (ieA_0) \right. \\ & - 2A \left(\lambda^2 C^{-1} + \frac{1}{2} \mu_0^2 \right) + \frac{1}{4} AA'C^{-1} C' + \frac{1}{2} ieA_0 AC^{-1} C' + \frac{1}{4} A^2 C^{-2} (C')^2 \\ & \left. - \frac{1}{2} A^2 C^{-1} C'M^{-1} M' - \frac{1}{2} C^{-1} C' ieA_0 A - ieA_0 AM^{-1} M' \right] R, \end{aligned} \tag{18}$$

式中 $M(r) = \lambda C^{\frac{-1}{2}} + \frac{i\mu_0}{\sqrt{2}}$.类似可得 R_+ 的方程,本文不作讨论.

5 乌龟坐标变换、解析延拓及辐射谱

参照文献[5]的方法,对(18)式作乌龟坐标变换

$$\frac{\partial}{\partial r} = \left(1 + \frac{1}{2\kappa(r - r_H)} \right) \frac{\partial}{\partial r^*},$$

$$\frac{\partial^2}{\partial r^2} = \left(1 + \frac{1}{2\kappa(r - r_H)} \right)^2 \frac{\partial^2}{\partial r^{*2}} - \frac{1}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r^*}.$$

让 $r \to r_H$, r_H 为视界位置, κ 为视界处的重力常数.考虑到:当 $r \to r_H$ 时,有 $A(r_H) \to 0$ (见(5a)式), $\frac{1}{2} A' \to \kappa$(见(6)式),而 $C(r_H)$, $M(r_H)$, $C'(r_H)$, $M'(r_H)$, $A_0(r_H)$等均为有限值.所以当 $r \to r_H$ 时,方程(18)可写为

$$-\frac{\partial^2 R_-}{\partial t^2} - (\kappa + 2ieA_0) \frac{\partial R_-}{\partial t} - \kappa \frac{\partial R_-}{\partial r^*} + \frac{\partial^2 R_-}{\partial r^{*2}} + (e^2 A_0^2 - i\kappa eA_0) R_- = 0. \tag{19}$$

设 $R_- = e^{-iat} R(r)$,代入(19)式得

$$\frac{\partial^2 R}{\partial r^{*2}} - \kappa \frac{\partial R}{\partial r^*} + (\omega - eA_0)^2 R + i\kappa(\omega - eA_0) R = 0. \tag{20}$$

解得 $R = \exp[i(\omega - eA_0) r^*]$,故得视界附近出射波函数的径向部分为

$$\begin{aligned} \Psi_{out} &= \exp\left[-i\omega \left(t - \frac{\omega - eA_0}{\omega} r^* \right) \right] \\ &= \exp(-iat) \exp\left[i(\omega_0 - eA_0) \frac{1}{2\kappa} \ln(r - r_H) \right]. \end{aligned} \tag{21}$$

容易看出此波函数在视界处不解析.按赵峥等[5]发展的 Damour-Ruffini 方法解析延拓,最后可得出射热谱为

$$N_{\omega}^{2} = \frac{1}{\exp\left(\frac{\omega - \omega_0}{k_B T}\right) + 1}, \tag{22}$$

$$T = \frac{\kappa}{2\pi k_B}, \tag{23}$$

$$\omega_0 = eA_0 = \frac{eQ}{\alpha^N r}, \tag{24}$$

式中 κ_B 为玻耳兹曼常数.

同理,对 R_+ 可以得到相似的结果.

6 结 论

求解 Einstein-Maxwell-Dilaton 黑膜时空中的狄拉克方程,在视界处得到了标准形式的出射波函数;得到了狄拉克粒子的 Hawking 辐射谱.可以看出,该谱由 Dilaton 标量势及黑膜上的电荷密度 Q 决定.

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DIRAC PARTICLE RADIATION FROM EINSTEIN-MAXWELL-DILATON BLACK PLANE

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ABSTRACT

Dirac equation near Einstein-Maxwell-Dilaton black plane's event horizon is analysed and discussed. Under tortoise coordinate translation, it can be verified that there exists thermal radiation of dirac particles from the event horizon.

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