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强激光短脉冲的引力效应

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研究了强激光短脉冲产生的引力场中氢原子能级的移动.指出在目前高功率激光脉冲条件下,高激发态氢原子的能级移动大小将达到可观察值.

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1 引　　言

广义相对论和其他度规理论都预言引力表现为时空的弯曲.但是,由于引力多涉及大尺度时空,引力实验相对其他学科而言少得多.然而随着高功率激光技术的发展,强激光场的强度可达到原子单位(大于 3×10^{16} W/cm^2)数量级,甚至可高达 10^{19} W/cm^2 数量级.目前的诊断技术也已能提供微米尺度的空间分辨和皮秒范围的时间分辨,因而探索强激光脉冲的引力效应及检测方法引起了不少兴趣.例如:Scully 用广义相对论方法处理了激光束之间的引力耦合[1],讨论了探针脉冲的引力偏离和相移效应;Parker 的研究表明,由于原子受局域时空曲率的影响,处于弯曲时空区域中的原子能级将移动,这种效应在原理上可以用作检验广义相对论的正确与否[2].本工作将利用文献[1]得到的强脉冲产生的引力度规,按文献[2]的方法,讨论单电子原子的能级移动,探索检验引力效应的途径.

2 强激光场的引力度规

假定激光脉冲的电场和磁场具有如下形式[1]:

$$E_2(\boldsymbol{r},t)=\mathscr{E}(\boldsymbol{r},t)\sin(\omega t-kx), \tag{1a}$$

$$B_3(\boldsymbol{r},t)=\frac{v}{c^2}\mathscr{E}(\boldsymbol{r},t)\sin(\omega t-kx), \tag{1b}$$

$$B_1(\boldsymbol{r},t)=\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}\frac{\mathscr{E}(\boldsymbol{r},t)}{c}\cos(\omega t-kx), \tag{1c}$$

其中 $\mathscr{E}(\boldsymbol{r},t)$为脉冲的包络,

$$\mathscr{E}^2(\boldsymbol{r},t)=E_0^2A[\theta(v(t+T_0)-x)-\theta(vt-x)]\delta(y)\delta(z), \tag{2}$$

E_0 为脉冲振幅, A 为光束横截面, v 为脉冲运动速度, T_0 为脉冲持续时间,步函数

$$\theta(x)=\begin{cases}1 & x>0,\\ 0 & x<0.\end{cases}$$

通过求解线性化的 Einstein 场方程,得到引力度规

$$g_{\mu\nu}(\boldsymbol{r}, t) = \eta_{\mu\nu} + h_{\mu\nu}(\boldsymbol{r}, t), \tag{3}$$

其中

$$h_{\mu\nu}(\boldsymbol{r}, t) = h(\boldsymbol{r}, t) M_{\mu\nu}. \tag{4}$$

而

$$h(\boldsymbol{r}, t) = -(2G\epsilon_0 E_0^2 A/c^2) \times \ln\left\{\frac{v(t+T_0) - x + \left[(v(t+T_0) - x)^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right]^{1/2}}{vt - x + \left[(vt - x)^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right]^{1/2}}\right\} \tag{5}$$

和

$$M_{\mu v} = \begin{pmatrix} 1 & -\frac{v}{c^2} & 0 & 0 \\ -\frac{v}{c^2} & \frac{v^2}{c^4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2}\left[1 - \left(\frac{v}{c}\right)^2\right] \end{pmatrix}. \tag{6}$$

在短脉冲近似下,(5)式简化为

$$h(\boldsymbol{r}, t) = \frac{-(2\epsilon_0 GE_0^2 A v T_0/c^2)}{\left[(x - vt)^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right]^{1/2}}. \tag{5a}$$

我们认为实际的光束截面受测不准关系限制,因此 $A_0 \leqslant y^2 + z^2 \leqslant A$, A_0 为由测不准关系决定的光束截面, A 为有效光束截面. 鉴于对 y 方向探针脉冲的偏转,文献[1]已作了详细分析,本文只研究 x 方向测地运动,用 $A_1 = (y^2 + z^2)$代入(5a)式,得

$$h(x, t) = \frac{-(2\epsilon_0 GE_0^2 A v T_0/c^2)}{\left[(x - vt)^2 + \left(1 - \frac{v^2}{c^2}\right) A_1\right]^{1/2}}, \tag{5b}$$

于是短脉冲所占据时空的特性可用下列线元描述:

$$\mathrm{d}s^2 = \left(1 + \frac{h}{c^2}\right) c^2 \mathrm{d}t^2 - 2\frac{v}{c^3} h(c\,\mathrm{d}t)\mathrm{d}x + \left(-1 + \frac{v^2}{c^4} h\right)\mathrm{d}x^2 - \mathrm{d}y^2 + \left[-1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]\mathrm{d}z^2. \tag{7}$$

设 $\bar{x} = x - vt$, $\bar{t} = t - F(\bar{x})$, $\bar{y} = y$, $\bar{z} = z$, 适当选择 $F(\bar{x})$, 使

$$\frac{\partial F}{\partial \bar{x}} = \frac{v + \frac{v}{c^2} h - \frac{v^3}{c^4} h}{c^2 - v^2 + h - 2\frac{v^2}{c^2} h + \frac{v^4}{c^4} h}, \tag{8}$$

可消去线元中的交叉项$(c\mathrm{d}t)\mathrm{d}x$, 于是(7)式可以写成

$$ds^2=\left(1-\frac{v^2}{c^2}\right)\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right](c\mathrm{d}\bar{t})^2-\left(1-\frac{v^2}{c^2}\right)^{-1}(\mathrm{d}\bar{x})^2$$

$$-(\mathrm{d}\bar{y})^2-\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right](\mathrm{d}\bar{z})^2, \tag{9}$$

因此,在新坐标系($c\bar{t}$, $\bar{x}$, $\bar{y}$, $\bar{z}$)中的引力度规系数为

$$\bar{g}_{00}=\left(1-\frac{v^2}{c^2}\right)\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right],$$

$$\bar{g}_{11}=-\left(1-\frac{v^2}{c^2}\right)^{-1},$$

$$\bar{g}_{22}=-1$$

$$\bar{g}_{33}=-\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right],$$

其中

$$h=\frac{-2\varepsilon_0 GE_0^2 AvT_0/c^2}{\left[\bar{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]^{1/2}}.$$

3 测地运动与 Fermi 标准坐标

由于氢原子能级的引力移动是在 Fermi 标准坐标中给出[2],因而我们要研究由度规(9)式决定的测地运动,然后建立相应的 Fermi 标准坐标.由度规(9)式,可知其相应的拉格朗日函数为

$$L=\left(1-\frac{v^2}{c^2}\right)\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right](c\dot{\bar{t}})^2-\left(1-\frac{v^2}{c^2}\right)^{-1}(\dot{\bar{x}})^2$$

$$-(\dot{\bar{y}})^2-\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right](\dot{\bar{z}})^2, \tag{10}$$

拉氏方程为

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial L}{\partial\dot{\bar{x}}^\mu}\right)-\frac{\partial L}{\partial\bar{x}^\mu}=0 \qquad (\mu=0,1,2,3), \tag{11}$$

其中 $\dot{\bar{x}}^\mu=\frac{\mathrm{d}\bar{x}^\mu}{c\mathrm{d}\tau}$, τ 为固有时间,$\mathrm{d}s=c\mathrm{d}\tau$.求解各分量的拉氏方程,得

$$c\dot{\bar{t}}=\frac{C_0}{\left(1-\frac{v^2}{c^2}\right)\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]}, \tag{12}$$

$$\dot{\bar{y}}=C_y, \tag{13}$$

$$\dot{\bar{z}}=\frac{C_z}{1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}} \tag{14}$$

和

$$\dot{\bar{x}} = \pm\left(1-\frac{v^2}{c^2}\right)^{1/2}\left\{\frac{C_0^2}{\left(1-\frac{v^2}{c^2}\right)\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]} - 1 - (C_y)^2 - \frac{C_z^2}{1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}}\right\}^{1/2}. \tag{15}$$

(12)—(15)式中 C_0, C_y 和 C_z 为积分常数,其物理图像是当一激光脉冲沿 $\bar{x}$方向传播时,处于该激光引力场中的原子将以 $\dot{\bar{x}}$, $\dot{\bar{y}}$和 $\dot{\bar{z}}$速度作测地运动.

假定在 $\bar{x}\to\pm\infty$ ($h\to 0$)时, y 方向、z 方向和 x 方向的速度为零,那么可求得积分常数 $C_y = C_z = 0$,而 $C_0^2 = \left(1-\frac{v^2}{c^2}\right)$. 于是(12)和(15)式可改写为

$$c\dot{\bar{t}} = \frac{\pm 1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]} \tag{12$'$}$$

和

$$\dot{\bar{x}} = \pm\left(1-\frac{v^2}{c^2}\right)^{1/2}\left\{\frac{-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}}{1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}}\right\}^{1/2}. \tag{15$'$}$$

从(12′)和(15′)式,可求得粒子在 $\bar{x}$方向上的速度为

$$V_{\bar{x}} = \frac{\mathrm{d}\bar{x}}{c\mathrm{d}\bar{t}} = \left(1-\frac{v^2}{c^2}\right)\left[-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{1/2}\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{1/2}. \tag{16}$$

非零的仿射联络为

$$\Gamma_{01}^0 = \frac{-\left(1-\frac{v^2}{c^2}\right)\bar{x}h}{2c^2\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]\left[\bar{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]}, \tag{17}$$

$$\Gamma_{00}^1 = -\frac{\left(1-\frac{v^2}{c^2}\right)^3\bar{x}h}{2c^2\left[\bar{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]}, \tag{18}$$

$$\Gamma_{33}^1 = -\frac{\left(1-\frac{v^2}{c^2}\right)^2\bar{x}h}{2c^2\left[\bar{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]}, \tag{19}$$

$$\Gamma_{31}^3 = \frac{\left(1-\frac{v^2}{c^2}\right)\bar{x}h}{2c^2\left[\bar{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]}. \tag{20}$$

非零的黎曼曲率张量为

$$R_{1010} = R_{0101} = -R_{1001} = -R_{0110}$$

$$= \frac{4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)hc^2 + \left(3\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)h^2 - 2\left(1-\frac{v^2}{c^2}\right)^4 A_1 h^2}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1+\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]}, \tag{21}$$

$$R_{3030} = R_{0303} = -R_{3003} = -R_{0330}$$

$$= -\frac{\left(1-\frac{v^2}{c^2}\right)^4 \bar{x}^2 h^2}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2}, \tag{22}$$

$$R_{1313} = R_{3131} = -R_{3113} = -R_{1331}$$

$$= \frac{4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)hc^2 - \left(3\bar{x}^2 + \frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)h^2 + 2\left(1-\frac{v^2}{c^2}\right)^3 A_1 h^2}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]}, \tag{23}$$

$$R_{00} = R^1_{010} + R^3_{030}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]}\left[-4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^3 hc^2\right.$$

$$\left. + 2\left(\bar{x}^2 + \frac{A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^4 h^2 + 4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^5\frac{h^3}{c^2} - 2\left(1-\frac{v^2}{c^2}\right)^6 A_1\frac{h^3}{c^2}\right], \tag{24}$$

$$R_{11} = R^0_{101} + R^3_{131}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}\left[-10\bar{x}^2\left(1-\frac{v^2}{c^2}\right)h^2\right.$$

$$\left. + 4\left(1-\frac{v^2}{c^2}\right)^3 A_1 h^2 + 6\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^4\frac{h^4}{c^4} - 4\left(1-\frac{v^2}{c^2}\right)^5 A_1\frac{h^4}{c^4}\right], \tag{25}$$

$$R_{33} = R^0_{303} + R^1_{313}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^3}{c^4}\right]}\left[-4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^2 hc^2\right.$$

$$\left. + 2\left(\frac{A_1c^2}{h} - \bar{x}^2\right)\left(1-\frac{v^2}{c^2}\right)^3 h^2 + 4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^4\frac{h^3}{c^2} - 2\left(1-\frac{v^2}{c^2}\right)^5 A_1\frac{h^3}{c^2}\right], \tag{26}$$

$$R = R^0_0 + R^1_1 + R^3_3$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}\left[22\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^3 h^2\right.$$

$$\left. - 8\left(1-\frac{v^2}{c^2}\right)^4 A_1 h^2 - 14\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4} + 8\left(1-\frac{v^2}{c^2}\right)^6 A_1\frac{h^4}{c^4}\right]. \tag{27}$$

沿着 $\bar{x}$方向的测地线,其 Fermi 标准基[3]可取

$$\hat{e}_0 = c\dot{\bar{t}}\,\frac{\partial}{c\partial\bar{t}} + \dot{\bar{x}}\,\frac{\partial}{\partial\bar{x}}, \tag{28}$$

$$\hat{e}_1 = \alpha\,\frac{\partial}{c\partial\bar{t}} + \beta\,\frac{\partial}{\partial\bar{x}}, \tag{29}$$

$$\hat{e}_2 = \frac{\partial}{\partial\bar{y}}, \tag{30}$$

$$\hat{e}_3 = \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{-1/2}\frac{\partial}{\partial\bar{z}}. \tag{31}$$

显见,

$$\hat{e}_0^{\bar{t}} = c\dot{\bar{t}},\quad \hat{e}_0^{\bar{x}} = \dot{\bar{x}},\quad \hat{e}_1^{\bar{t}} = \alpha,\quad \hat{e}_1^{\bar{x}} = \beta,\quad \hat{e}_2^{\bar{y}} = 1,\quad \hat{e}_3^{\bar{z}} = \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{-1/2}, \tag{32}$$

其中

$$\alpha^2 = \frac{-h}{c^2\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^2}, \tag{33}$$

$$\beta^2 = \frac{1 - \frac{v^2}{c^2}}{1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}}. \tag{34}$$

相应地,在 Fermi 标准基中算得的黎曼曲率张量为

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}} = R_{\hat{1}\hat{0}\hat{1}\hat{0}} = -R_{\hat{0}\hat{1}\hat{1}\hat{0}} = -R_{\hat{1}\hat{0}\hat{0}\hat{1}}$$
$$= \frac{4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + \left(3\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 - 2\left(1 - \frac{v^2}{c^2}\right)^4 A_1 h^2}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^2}, \tag{35}$$

$$R_{\hat{0}\hat{3}\hat{0}\hat{3}} = R_{\hat{3}\hat{0}\hat{3}\hat{0}} = -R_{\hat{0}\hat{3}\hat{3}\hat{0}} = -R_{\hat{3}\hat{0}\hat{0}\hat{3}}$$
$$= \frac{-5\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)h^2 + 2\left(1 - \frac{v^2}{c^2}\right)A_1h^2 + 3\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^5\frac{h^4}{c^4} - 2\left(1 - \frac{v^2}{c^2}\right)^6 A_1\frac{h^4}{c^4}}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}, \tag{36}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}} = R_{\hat{3}\hat{1}\hat{3}\hat{1}} = -R_{\hat{1}\hat{3}\hat{3}\hat{1}} = -R_{\hat{3}\hat{1}\hat{1}\hat{3}}$$
$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}\Bigg[4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2$$
$$+ \left(\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 - 2\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^4\frac{h^3}{c^2}$$

$$+\left(-\overline{x}^2+\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4}\Big], \tag{37}$$

$$R_{\hat{0}\hat{0}}=R^{\hat{1}}_{\hat{0}\hat{1}\hat{0}}+R^{\hat{3}}_{\hat{0}\hat{3}\hat{0}}=-R_{\hat{1}\hat{0}\hat{1}\hat{0}}-R_{\hat{3}\hat{0}\hat{3}\hat{0}}$$
$$=\frac{1}{4c^4\left[\overline{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}\Big\{-4\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^2hc^2$$
$$+\left(10\overline{x}^2+\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^3h^2+\left(2\overline{x}^2-\frac{4A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^4\frac{h^3}{c^2}$$
$$+\left(-6\overline{x}^2-\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4}+4\left(1-\frac{v^2}{c^2}\right)^6A_1\frac{h^4}{c^4}\Big\}, \tag{38}$$

$$R_{\hat{1}\hat{1}}=R^{\hat{0}}_{\hat{1}\hat{0}\hat{1}}+R^{\hat{3}}_{\hat{1}\hat{3}\hat{1}}=R_{\hat{0}\hat{1}\hat{0}\hat{1}}-R_{\hat{1}\hat{3}\hat{1}\hat{3}}$$
$$=\frac{1}{4c^4\left[\overline{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2}\Big\{-6\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^3h^2$$
$$+2\left(1-\frac{v^2}{c^2}\right)^4A_1h^2+4\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4}-2\left(1-\frac{v^2}{c^2}\right)^6A_1\frac{h^4}{c^4}\Big\}, \tag{39}$$

$$R_{\hat{3}\hat{3}}=R^{\hat{0}}_{\hat{3}\hat{0}\hat{3}}+R^{\hat{1}}_{\hat{3}\hat{1}\hat{3}}=R_{\hat{0}\hat{3}\hat{0}\hat{3}}-R_{\hat{1}\hat{3}\hat{1}\hat{3}}$$
$$=\frac{1}{4c^4\left[\overline{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^2}\right]^2}\Big\{-4\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^2hc^2$$
$$+\left(-6\overline{x}^2+\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^3h^2+\left(2\overline{x}^2+\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^4\frac{h^3}{c^2}$$
$$+\left(4\overline{x}^2-\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4}-2\left(1-\frac{v^2}{c^2}\right)^6A_1\frac{h^4}{c^4}\Big\}, \tag{40}$$

$$R=R^{\hat{0}}_{\hat{0}}+R^{\hat{1}}_{\hat{1}}+R^{\hat{3}}_{\hat{3}}=R_{\hat{0}\hat{0}}-R_{\hat{1}\hat{1}}-R_{\hat{3}\hat{3}}$$
$$=\frac{1}{4c^4\left[\overline{x}^2+\left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h^2}{c^4}\right]^2}\Big\{22\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^3h^2$$
$$-8\left(1-\frac{v^2}{c^2}\right)^4A_1h^2-14\overline{x}^2\left(1-\frac{v^2}{c^2}\right)^5\frac{h^4}{c^4}+8\left(1-\frac{v^2}{c^2}\right)^6A_1\frac{h^4}{c^4}\Big\}. \tag{41}$$

(41)式是 Fermi 标准基中算得的黎曼曲率标量，显然，它与(27)式相同.

4 氢原子能级的引力移动

有了标准基中的黎曼曲率张量的表示式，不难根据文献[2]给出的氢原子能级的公式，估计其引力移动.

由(5a)式可知，$h(\boldsymbol{r}, t)$具有速度平方的量纲，且在 $\bar{x}^2=0$ 处，有最大值. 我们估计在该处可达到氢原子能级有可观察移动所要求的光场强度. 由于 $h/c^2 \ll 1$，可以只保留最低阶项. 于是在 $\bar{x}^2=0$ 处，有

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}} = R_{\hat{1}\hat{0}\hat{1}\hat{0}} = - R_{\hat{0}\hat{1}\hat{1}\hat{0}} = - R_{\hat{1}\hat{0}\hat{0}\hat{1}} \simeq - \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h, \tag{35'}$$

$$R_{\hat{0}\hat{3}\hat{0}\hat{3}} = R_{\hat{3}\hat{0}\hat{3}\hat{0}} = - R_{\hat{0}\hat{3}\hat{3}\hat{0}} = - R_{\hat{3}\hat{0}\hat{0}\hat{3}} \simeq 0, \tag{36'}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}} = R_{\hat{3}\hat{1}\hat{3}\hat{1}} = - R_{\hat{1}\hat{3}\hat{3}\hat{1}} = - R_{\hat{3}\hat{1}\hat{1}\hat{3}} \simeq - \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h, \tag{37'}$$

$$R_{\hat{0}\hat{0}} \simeq \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h, \tag{38'}$$

$$R_{\hat{1}\hat{1}} \simeq 0, \tag{39'}$$

$$R_{\hat{3}\hat{3}} \simeq \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h, \tag{40'}$$

$$R \simeq 0. \tag{41'}$$

将以上曲率张量代入能级移动公式[2]

$$E^{(1)} = \mathscr{A} R_{\hat{0}\hat{0}} + \mathscr{B} R + \sum_{i=1}^{3} \mathscr{C}^{ii} R_{\hat{0}\hat{i}\hat{0}\hat{i}},$$

得

$$E^{(1)} = \mathscr{A} R_{\hat{0}\hat{0}} + \mathscr{C}^{11} R_{\hat{0}\hat{1}\hat{0}\hat{1}} = \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h[\mathscr{A} - \mathscr{C}^{11}], \tag{42}$$

$\mathscr{A}$ 和 $\mathscr{C}^{11}$ 为依赖于精细结构常数和电子能量的常数，对于高激发或 Rydberg 态原子，有[2]

$$E_n^{(1)} \simeq 2.5\, \frac{\hbar^2}{\alpha^2 m_e}\left[\frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right) h\right] n^4, \tag{43}$$

其中 α 为精细结构常数，而

$$E_n^{(0)} = - 2\pi \hbar c R_y n^{-2},$$

其中 R_y 为 Rydberg 常数. 引力移动相对于能级之比为

$$\frac{E_{n+1}^{(1)} - E_n^{(1)}}{E_{n+1}^{(0)} - E_n^{(0)}} \simeq 5.26 \times 10^{-16}\, \frac{G \epsilon_0 E_0^2 A v T_0}{c^4 A_1^{3/2}}\left(1 - \frac{v^2}{c^2}\right)^{1/2} n^6.$$

考虑到高功率激光的单位脉冲输出能量可达 10^6 J，持续时间为 10^{-13} s，选择如下参数：

$$\frac{1}{2}\epsilon_0 E_0^2 A v T_0 = 2.5 \times 10^6\ \mathrm{J}, \quad v = 0.9c, \quad n = 10^2.$$

考虑到测不准关系的约束，对于激光波长为 10^{-7} m，取 $A_1 = 10^{-12}\ \mathrm{m}^2$，则

$$\frac{\Delta E_n^{(1)}}{\Delta E_n^{(0)}} \simeq 10^{-24}.$$

由文献[1]知，目前的实验技术能达到这一灵敏度.

5 讨　　论

本文借助文献[1]给出的激光引力场，研究了氢原子光谱的引力扰动[2]，初步估计了

得到可观察引力效应所需要的高功率激光脉冲的能量输出和速度.正如(42)和(43)式所指出,当 $v=c$ 时,能级移动为零,将观察不到引力效应.这与文献[1]的结论一致.与文献[1]不同的是,我们引进了参量 A_1 来代替(5a)式中的(y^2+z^2).由于受测不准关系的约束,光束强度实际上不可能在 x轴($y=z=0$)上达到无限大.而当我们研究沿着 x轴上进行测地运动的原子时,可以认为(y^2+z^2)是常数 A_1.值得注意的是,A_1 与 A 不同,A_1 是我们为研究问题人为引入的,而 A 是光束的实际有效截面,是一个可测的客观物理量.调节各参量,如 v, T_0, A_1, A 和 E_0^2,可以得到更合理更精确的光谱移动的估计.至少可以根据我们上面给出的各参量的依赖关系,在实验上对光谱的移动做出定性的研究,借以检验强激光场的引力效应.

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GRAVITATIONAL EFFECT OF THE STRONG LASER PULSE

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ABSTRACT

The energy-level shifts of hydrogen spectrum in the curved space-time caused by the strong short laser pulse are studied. It is shown that under the present conditions of high-power laser pulse, the magnitude of the energy-level shifts of highly excited hydrogen atom will be observable.

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