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轴对称荷电动态黑洞的特征曲面

吕君丽　王永久
(湖南师范大学物理系,物理研究所,长沙　410081)
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讨论了轴对称荷电动态黑洞的特征曲面,即类时极限面、表观视界和事件视界.分析了黑洞蒸发或吸收质量,以及黑洞电荷发生变化诸种情况.

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1　引　　言

在文献[1—3]中对几种轴对称动态黑洞的三类特征曲面进行了研究.在动态情况下,这些特征曲面的位置不再重合,且外能层变厚,内能层变薄.本文对另一种轴对称动态黑洞的三类特征曲面进行讨论.

轴对称荷电动态黑洞的时空线元由文献[4]给出:

$$\begin{aligned}\mathrm{d}s^2 = &[1-(2mr-Q^2)\rho\bar{\rho}\mathrm{d}u^2+2\mathrm{d}u\mathrm{d}r+2a(2mr-Q^2)\rho\bar{\rho}\sin^2\theta\mathrm{d}u\mathrm{d}\varphi \\ &-2a\sin^2\theta\mathrm{d}r\mathrm{d}\varphi-\frac{1}{\rho\bar{\rho}}\mathrm{d}\theta^2+\left[(Q^2-2mr)a^2\rho\bar{\rho}-\frac{a^2+r^2}{\sin^2\theta}\right]\sin^4\theta\mathrm{d}\varphi^2,\end{aligned} \tag{1}$$

其中 u 为超前爱丁顿坐标,质量 $m=m(u)$,电荷 $Q=Q(u)$,$\rho=-(r-\mathrm{i}a\cos\theta)^{-1}$,$a$ 为比角动量.

2　类时极限面(TLS)

由类时极限面的定义

$$g_{uu}=g_{ab}\left(\frac{\partial}{\partial u}\right)^a\left(\frac{\partial}{\partial u}\right)^b=0, \tag{2}$$

可得

$$r^2-2mr+(Q^2+a^2\cos^2\theta)=0. \tag{3}$$

解方程(3),求出黑洞的类时极限面为

$$r_{\mathrm{TLS}}^{\pm}=m\pm\sqrt{m^2-(Q^2+a^2\cos^2\theta)}, \tag{4}$$

r_{TLS}^{+}和 r_{TLS}^{-}分别为黑洞的外类时极限面和内类时极限面.在稳态情况下,它们即为 Kerr-Newman 黑洞的类时极限面.下面对其动态过程进行讨论.

1) 当 $m=m(u)$, $Q=\mathrm{const}$,

$$\frac{\mathrm{d}r_{\mathrm{TLS}}^{\pm}}{\mathrm{d}u}=\frac{\mathrm{d}m}{\mathrm{d}u}\left[1\pm\frac{m}{\sqrt{m^2-(Q^2+a^2\cos^2\theta)}}\right]. \tag{5}$$

可见，黑洞蒸发质量时($\mathrm{d}m/\mathrm{d}u<0$)，黑洞的外类时极限面收缩，内类时极限面膨胀；黑洞吸收质量时($\mathrm{d}m/\mathrm{d}u>0$)，黑洞的外类时极限面膨胀，内类时极限面收缩.

2) 当 $Q=Q(u)$, $m=\mathrm{const}$,

$$\frac{\mathrm{d}r_{\mathrm{TLS}}^{\pm}}{\mathrm{d}u}=\mp\frac{Q}{\sqrt{m^2-(Q^2+a^2\cos^2\theta)}}\frac{\mathrm{d}Q}{\mathrm{d}u}. \tag{6}$$

不难看出，黑洞电荷的减少，将使外类时极限面膨胀，内类时极限面收缩；黑洞电荷的增加，将使外类时极限面收缩，内类时极限面膨胀.

3) 当 $m=m(u)$, $Q=Q(u)$,

$$\frac{\mathrm{d}r_{\mathrm{TLS}}^{\pm}}{\mathrm{d}u}=\frac{\mathrm{d}m}{\mathrm{d}u}\pm\frac{m\frac{\mathrm{d}m}{\mathrm{d}u}-Q\frac{\mathrm{d}Q}{\mathrm{d}u}}{\sqrt{m^2-(Q^2+a^2\cos^2\theta)}}. \tag{7}$$

此时黑洞类时极限面位置的变化与质量 m 和电荷 Q 变化速率的比值$\frac{\mathrm{d}Q}{\mathrm{d}u}\Big/\frac{\mathrm{d}m}{\mathrm{d}u}$有关.但是只要它们的比值满足

$$\frac{\mathrm{d}Q}{\mathrm{d}u}\Big/\frac{\mathrm{d}m}{\mathrm{d}u}<\frac{m-\sqrt{m^2-(Q^2+a^2\cos^2\theta)}}{Q}, \tag{8}$$

则黑洞类时极限面位置变化与仅仅只有质量变化时的情形类似.即黑洞处于吸收状态时，其外类时极限面膨胀，内类时极限面收缩；黑洞处于蒸发状态时，其外类时极限面收缩，内类时极限面膨胀.

一般情况下，$[m-\sqrt{m^2-(Q^2+a^2\cos^2\theta)}]/Q$ 小于 1，故(8)式可写为

$$\frac{\mathrm{d}Q}{\mathrm{d}u}<\frac{\mathrm{d}m}{\mathrm{d}u}. \tag{8'}$$

3 表观视界(AH)

表观视界被定义为出射光子陷获面的外边界，从它出发的两簇类光测地线汇的膨胀 Θ 为零，即

$$\Theta = n^{\mu}_{;\mu}-k=0, \tag{9}$$

其中

$$k = n_{\mu;\nu}l^{\mu}n^{\nu}. \tag{10}$$

引入零标架

$$\begin{aligned}
l_\mu &= \delta^0_\mu - a\sin^2\theta\delta^3_\mu,\\
n_\mu &= \rho\bar{\rho}\left[\frac{r^2+a^2+Q^2-2mr}{2}\delta^0_\mu+\frac{1}{\rho\bar{\rho}}\delta^1_\mu+\frac{2mr-Q^2-r^2-a^2}{2}a\sin^2\theta\delta^3_\mu\right],\\
m_\mu &= -\frac{\bar{\rho}}{\sqrt{2}}\left[\mathrm{i}a\sin\theta\delta^0_\mu-\frac{1}{\rho\bar{\rho}}\delta^2_\mu-\mathrm{i}(r^2+a^2)\sin\theta\delta^3_\mu\right],\\
\bar{m}_\mu &= -\frac{\rho}{\sqrt{2}}\left[-\mathrm{i}a\sin\theta\delta^0_\mu-\frac{1}{\rho\bar{\rho}}\delta^2_\mu+\mathrm{i}(r^2+a^2)\sin\theta\delta^3_\mu\right],
\end{aligned} \tag{11}$$

其逆变分量为

$$
\begin{aligned}
l^\mu &= \delta_1^\mu, \\
n^\mu &= \rho\bar{\rho}\left[(r^2+a^2)\delta_0^\mu + \frac{2mr-Q^2-r^2-a^2}{2}\delta_1^\mu + a\delta_3^\mu\right], \\
m^\mu &= -\frac{\bar{\rho}}{\sqrt{2}}\left[\mathrm{i}a\sin\theta\delta_0^\mu + \delta_2^\mu + \frac{\mathrm{i}}{\sin\theta}\delta_3^\mu\right], \\
\overline{m}^\mu &= -\frac{\rho}{2}\left[-\mathrm{i}a\sin\theta\delta_0^\mu + \delta_2^\mu - \frac{\mathrm{i}}{\sin\theta}\delta_3^\mu\right].
\end{aligned} \tag{12}
$$

不难验证,(11)和(12)式满足度规条件、零矢条件和伪正交关系:

$$
\begin{aligned}
g_{\mu\nu} &= l_\mu n_\nu + n_\mu l_\nu - m_\mu \overline{m}_\nu - \overline{m}_\mu m_\nu, \\
l_\mu n^\mu &= - m_\mu \overline{m}^\mu = 1, \\
l_\mu m^\mu &= l_\mu \overline{m}^\mu = n_\mu m^\mu = n_\mu \overline{m}^\mu = 0, \\
l_\mu l^\mu &= n_\mu n^\mu = m_\mu m^\mu = \overline{m}_\mu \overline{m}^\mu = 0.
\end{aligned}
$$

利用(11)和(12)式及联络(见附录),经计算得膨胀 Θ 为

$$\Theta = (2mr - Q^2 - r^2 - a^2) r\rho^2\bar{\rho}^2. \tag{13}$$

由表观视界的定义 $\Theta=0$,有

$$2mr - Q^2 - r^2 - a^2 = 0. \tag{14}$$

由此得到表观视界

$$r_{\mathrm{AH}}^{\pm} = m \pm \sqrt{m^2 - (a^2 + Q^2)}. \tag{15}$$

在稳态情况下,它就是 Kerr-Newman 黑洞的视界位置,其表观视界与事件视界重合,显然它是一个球面.在动态情况下,该黑洞表观视界的位置随黑洞的吸收和蒸发发生变化,其讨论与类时极限面相似.下面还将看到,动态情况下表观视界与事件视界不再重合,事件视界也不再是一个球面.

4 事件视界(EH)

按照 York 关于事件视界的定义,事件视界应是光子的粘着面,不允许有光子从这里逃向远方.当把 $L=-\dot{m}$ 看作小量时,在一级近似下,1) 事件视界应严格类光;2) 光子应长时间滞留在事件视界上.即在 $O(L)$ 量级,事件视界应由

$$\dot{r}_{\mathrm{EH}} \approx 0, \quad \ddot{r}_{\mathrm{EH}} \approx 0$$

决定.现在考察径向出射类光测地线:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}u} &= n^a \nabla_a \\
&= \rho\bar{\rho}(r^2+a^2)\frac{\partial}{\partial u} + \frac{1}{2}(2mr - Q^2 - r^2 - a^2)\rho\bar{\rho}\frac{\partial}{\partial r} + a\rho\bar{\rho}\frac{\partial}{\partial \varphi}.
\end{aligned}
$$

由上式得

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}u} = \frac{1}{2}(2mr - Q^2 - r^2 - a^2)\rho\bar{\rho}, \tag{16}$$

$$\ddot{r}=\frac{\mathrm{d}^2 r}{\mathrm{d}u^2}=\rho\bar{\rho}(r^2+a^2)\frac{\partial\dot{r}}{\partial u}+\frac{1}{2}(2mr-Q^2-r^2-a^2)\rho\bar{\rho}\frac{\partial\dot{r}}{\partial u}$$

$$=(r^2+a^2)(\dot{m}r-Q\dot{Q})\rho^2\bar{\rho}^2$$

$$+\dot{r}[(m-r)-(2mr-Q^2-r^2-a^2)r\rho\bar{\rho}]\rho\bar{\rho}. \tag{17}$$

从(16)和(17)式可知,当$(\dot{m}r-Q\dot{Q})>0$时,在外表观视界处

$$\dot{r}_{\mathrm{AH}}=0,\quad \ddot{r}_{\mathrm{AH}}>0,$$

因此,光子可以从表观视界处逃逸到远方.这与事件视界的定义不符,所以对于轴对称荷电动态黑洞,其表观视界不再与事件视界重合.

为了求出事件视界的位置,令(17)式为零,得

$$\dot{r}=\frac{(r^2+a^2)(\dot{m}r-Q\dot{Q})\rho\bar{\rho}}{(2mr-Q^2-r^2-a^2)r\rho\bar{\rho}+(r-m)}. \tag{18}$$

由于 r_{AH}与 r_{EH}接近,故

$$\dot{r}\approx\dot{r}|_{r=r_{\mathrm{AH}}^{+}}=\frac{[2m(m+\sqrt{m^2-a^2-Q^2})-Q^2][\dot{m}(m+\sqrt{m^2-a^2-Q^2})-Q\dot{Q}]}{\sqrt{m^2-a^2-Q^2}[2m(m+\sqrt{m^2-Q^2-a^2})-Q^2-a^2\sin^2\theta]}$$

$$\equiv D. \tag{19}$$

将(19)式代入(16)式,得

$$(1+2D)r^2-2mr+Q^2+a^2(1+2D\cos^2\theta)=0. \tag{20}$$

(20)式即为在动态情况下,轴对称荷电黑洞的事件视界表达式.解(20)式,可得

$$r_{\mathrm{EH}}^{\pm}=\frac{m\pm[m^2-(1+2D)(Q^2+a^2+2Da^2\cos^2\theta)]^{1/2}}{(1+2D)}, \tag{21}$$

r_{EH}^{+}和 r_{EH}^{-}分别为该黑洞的外事件视界和内事件视界.它们与 θ 有关,因此黑洞事件视界不是一个球面.

1) 当 $\dot{m}=0$, $\dot{Q}=0$ 时,由(19)式得 $D=0$.

$$r_{\mathrm{EH}}^{\pm}=m\pm\sqrt{m^2-Q^2-a^2}. \tag{22}$$

事件视界回到我们熟悉的稳态 Kerr-Newman 黑洞视界的位置.这时表观视界和事件视界重合.

2) 当 $a=0$ 时,由(19)式得

$$D=[\dot{m}(m+\sqrt{m^2-Q^2})-Q\dot{Q}]/\sqrt{m^2-Q^2}. \tag{23}$$

此时的事件视界位置为

$$r_{\mathrm{EH}}^{\pm}=\frac{m\pm[m^2-(1+2D)Q^2]^{1/2}}{(1+2D)}. \tag{24}$$

这就是球对称荷电动态黑洞的事件视界.显然,在静态情况下($D=0$),它回到 Reissner-Nordstorm 黑洞的事件视界位置 $r_{\mathrm{EH}}^{\pm}=m\pm\sqrt{m^2-Q^2}$.

3) 当 $\dot{m}<0$, $\dot{Q}=0$ 时,由(19)式得

$$D=\frac{[2m(m+\sqrt{m^2-a^2-Q^2})-Q^2](m+\sqrt{m^2-a^2-Q^2})}{\sqrt{m^2-a^2-Q^2}[2m(m+\sqrt{m^2-Q^2-a^2})-Q^2-a^2\sin^2\theta]}\dot{m}<0. \tag{25}$$

在 $O(L)$量级,可将(21)式表示为

$$r_{EH}^{\pm} \approx (1-2D)\, r_{AH}^{\pm} \mp \frac{Q^2 + a^2(1+\cos^2\theta)}{\sqrt{m^2 - Q^2 - a^2}} D, \tag{26}$$

其中 $r_{AH}^{\pm} = m \pm \sqrt{m^2 - a^2 - Q^2}$为事件的表观视界. 由(26)式可知, $r_{EH}^{+} > (1-2D)\, r_{AH}^{+} \approx r_{AH}^{+}$, $r_{EH}^{-} < (1-2D)\, r_{AH}^{-} \approx r_{AH}^{-}$, 即内事件视界收缩, 外事件视界膨胀. 因此, 黑洞的内外能层都变薄.

4) 当 $\dot{m}>0$, $\dot{Q}=0$ 时, 由(25)式得 $D>0$.

同上讨论, 由(26)式可知 $r_{EH}^{+} < r_{AH}^{+}$, $r_{EH}^{-} > r_{AH}^{-}$, 即内事件视界膨胀, 外事件视界收缩. 因此, 黑洞的内外能层都变厚.

5) 若 $\dot{Q}\neq 0$, $\dot{m}=0$, 则事件视界的变化正好与 3) 4) 的结论相反.

附 录

联络的非零分量为

$$\Gamma_{00}^{0} = -(\dot{m}\, r - Q\dot{Q})\, a^2\sin^2\theta\rho^2\bar{\rho}^2 - (mr^2 - Q^2 r)\,\rho^2\bar{\rho}^2 + ma^2\rho^2\bar{\rho}^2 - (2mr - Q^2)\, ra^2\sin^2\theta\rho^3\bar{\rho}^3,$$

$$\Gamma_{02}^{0} = -(2mr - Q^2)\, a^2\sin\theta\cos\theta\rho^2\bar{\rho}^2,$$

$$\begin{aligned}\Gamma_{03}^{0} =\ & (2mr - Q^2 r)\, a^3\sin^4\theta\rho^3\bar{\rho}^3 + (2mr - Q^2)\, ar\sin^2\theta\rho^2\bar{\rho}^2 - ma^3\sin^4\theta\rho^2\bar{\rho}^2 \\ & + (\dot{m}\, r - Q\dot{Q})\, a^3\sin^4\theta\rho^2\bar{\rho}^2 - ma\sin^2\theta\rho\bar{\rho},\end{aligned}$$

$$\Gamma_{12}^{0} = a^2\sin\theta\cos\theta\rho\bar{\rho},$$

$$\Gamma_{13}^{0} = ra\sin^2\theta\rho\bar{\rho},$$

$$\Gamma_{22}^{0} = r(r^2 + a^2)\,\rho\bar{\rho},$$

$$\Gamma_{23}^{0} = (2mr - Q^2)\, a^3\sin^3\theta\cos\theta\rho^2\bar{\rho}^2,$$

$$\begin{aligned}\Gamma_{33}^{0} =\ & -(mr^4 - Q^2 r^3 - Q^2 ra^2)\, a^2\sin^4\theta\rho^3\bar{\rho}^3 - mr^2a^4\sin^6\theta\rho^3\bar{\rho}^3 \\ & + ma^6\sin^4\theta\cos^2\theta\rho^3\bar{\rho}^3 - (\dot{m}\, r - Q\dot{Q})\, a^4\sin^6\theta\rho^2\bar{\rho}^2 + ra^2\sin^4\theta\rho\bar{\rho} + r\sin^2\theta,\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^{1} =\ & (2mr^2 - Q^2 r)\, a^2\sin^2\theta\rho^3\bar{\rho}^3 - (2mr - Q^2)^2 r\rho^3\bar{\rho}^3 + (\dot{m}\, r - Q\dot{Q})\, a^2\sin^2\theta\rho^2\bar{\rho}^2 \\ & + (2mr - Q^2)\, m\rho^2\bar{\rho}^2 + (2mr^2 - Q^2 r)\,\rho^2\bar{\rho}^2 - ma^2\sin^2\theta\rho^2\bar{\rho}^2 - (\dot{m}\, r - Q\dot{Q})\,\rho\bar{\rho} - m\rho\bar{\rho},\end{aligned}$$

$$\Gamma_{01}^{1} = (2mr - Q^2)\, r\rho^2\bar{\rho}^2 - m\rho\bar{\rho},$$

$$\begin{aligned}\Gamma_{03}^{1} =\ & (2mr - Q^2)^2 ra\sin^2\theta\rho^3\bar{\rho}^3 - (2mr - Q^2)^2 ra^3\sin^4\theta\rho^3\bar{\rho}^3 - (2mr - Q^2)\, ma\sin^2\theta\rho^2\bar{\rho}^2 \\ & - (2mr - Q^2)\, ra\sin^2\theta\rho^2\bar{\rho}^2 + ma^3\sin^4\theta\rho^2\bar{\rho}^2 - (\dot{m}\, r - Q\dot{Q})\, a^3\sin^4\theta\rho^2\bar{\rho}^2 + ma\sin^2\theta\rho\bar{\rho},\end{aligned}$$

$$\Gamma_{12}^{1} = -a^2\sin\theta\cos\theta\rho\bar{\rho},$$

$$\Gamma_{13}^{1} = -(2mr - Q^2)\, ra\sin^2\theta\rho^2\bar{\rho}^2 + ma\sin^2\theta\rho\bar{\rho} - ra\sin^2\theta\rho\bar{\rho},$$

$$\Gamma_{22}^{1} = (2mr - Q^2)\, r\rho\bar{\rho} - ra\sin^2\theta\rho\bar{\rho} - r,$$

$$\begin{aligned}\Gamma_{33}^{1} =\ & -(2mr - Q^2)^2 ra^2\sin^4\theta\rho^3\bar{\rho}^3 + (2mr - Q^2)\, ra^4\sin^2\theta\rho^3\bar{\rho}^3 \\ & + (\dot{m}\, r - Q\dot{Q})\, a^4\sin^6\theta\rho^2\bar{\rho}^2 + (2mr - Q^2)\, ra^2\sin^4\theta\rho^2\bar{\rho}^2 \\ & + (2mr - Q^2)\, ma^2\sin^4\theta\rho^2\bar{\rho}^2 - ma^4\sin^6\theta\rho^2\bar{\rho}^2 + (\dot{m}\, r - Q\dot{Q})\, a^2\sin^4\theta\rho\bar{\rho} \\ & - ma^2\sin^4\theta\rho\bar{\rho} + (2mr - Q^2)\, r\sin^2\theta\rho\bar{\rho} - ra^2\sin^4\theta\rho\bar{\rho} - r\sin^2\theta,\end{aligned}$$

$$\Gamma_{00}^{2} = -(2mr - Q^2)\, a^2\sin\theta\cos\theta\rho^3\bar{\rho}^3,$$

$$\Gamma^2_{03} = (2mr - Q^2) r^2 a\sin\theta\cos\theta\rho^3\bar{\rho}^3 + (2mr - Q^2) a^3\sin\theta\cos\theta\rho^3\bar{\rho}^3,$$

$$\Gamma^2_{12} = r\rho\bar{\rho},$$

$$\Gamma^2_{13} = -a\sin\theta\cos\theta\rho\bar{\rho},$$

$$\Gamma^2_{22} = -a^2\sin\theta\cos\theta\rho\bar{\rho},$$

$$\Gamma^2_{33} = -(2mr - Q^2) a^4\sin^5\theta\cos\theta\rho^3\bar{\rho}^3 - 2(2mr - Q^2) a^2\sin^3\theta\cos\theta\rho^2\bar{\rho}^2 - (r^2 + a^2)\sin\theta\cos\theta\rho\bar{\rho},$$

$$\Gamma^3_{00} = -(\dot{m}r - Q\dot{Q}) a\rho^2\bar{\rho}^2 + ma\rho^2\bar{\rho}^2 - (2mr - Q^2) ra\rho^3\bar{\rho}^3,$$

$$\Gamma^3_{02} = -(2mr - Q^2) a\cos\theta\mathrm{cosec}\,\theta\rho^2\bar{\rho}^2,$$

$$\Gamma^3_{03} = (2mr - Q^2) ra^2\sin^2\theta\rho^3\bar{\rho}^3 - ma^2\sin^2\theta\rho^2\bar{\rho}^2 + (\dot{m}r - Q\dot{Q}) a^2\sin^2\theta\rho^2\bar{\rho}^2,$$

$$\Gamma^3_{12} = a\cos\theta\mathrm{cosec}\,\theta\rho\bar{\rho},$$

$$\Gamma^3_{13} = r\rho\bar{\rho},$$

$$\Gamma^3_{22} = ra\rho\bar{\rho},$$

$$\Gamma^3_{23} = (2mr - Q^2) a^2\sin\theta\cos\theta\rho^2\bar{\rho}^2 + \cos\theta\mathrm{cosec}\,\theta,$$

$$\Gamma^3_{33} = -(2mr - Q^2) ra^3\sin^4\theta\rho^3\bar{\rho}^3 - (\dot{m}r - Q\dot{Q}) a^3\sin^4\theta\rho^2\bar{\rho}^2 + ma^3\sin^4\theta\rho^2\bar{\rho}^2 + ra\sin^2\theta\rho\bar{\rho}.$$

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CHARCTERISTIC SURFACES OF THE CHARGED AXIALLY NONSTATIONARY BLACK HOLE

L绊 JUN-LI WANG YONG-JIU

(*Department of Physics, Institute of Physics, Hunan Normal University, Changsha* 410081)

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ABSTRACT

Three kinds of characteristic surfaces of the charged axially nonstationary black hole are studied, namely, the event horizon, the apparent horizon and the time-like limit surface. The new results are shown when mass and charge of the black hole vary.

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