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一般球对称带电蒸发黑洞的温度

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从视界附近的 Klein-Gordon 方程出发,准确地定出了一般球对称带电蒸发黑洞的 Hawking 温度和辐射谱,同时计算出事件视界方程.所得结果与用零曲面方程得到的结果一致.

关键词:蒸发黑洞,视界,辐射

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1. 引　　言

1974 年,Hawking 发现黑洞存在温度为 $T=\kappa/2\pi$ 的量子辐射[1],其中 κ 是黑洞视界面上的表面重力,T 叫做 Hawking 温度.此后人们在黑洞热力学的研究上做了大量的工作,特别是在计算黑洞的 Hawking 温度与其辐射谱方面,取得了许多有价值的成果.继 Hawking 之后,人们又采用各种各样的方法来研究 Hawking 温度.主要的方法有路径积分法[2]、温度格林函数法[3]、Damour-Ruffini 方法[4]、Unruh 法[5]和赵峥等人在 1991 年提出的决定动态黑洞热效应的新方法[6,7]等等.值得说明的是,赵峥等人提出的这种新方法,在计算黑洞的温度方面,具有简单、精确和适应性强等特点,它同时可用于稳态和动态黑洞,包括球对称黑洞、非球对称黑洞以及非渐近平直时空中的黑洞等等.人们用这种新方法已经成功地计算了一些动态黑洞的温度与视界[8—11].本文采用这种方法计算一般球对称带电蒸发黑洞的事件视界和温度,并采用文献[12,13]中的方法,计算黑洞的辐射谱.得到的结果可以返回原来已知的一般球对称蒸发黑洞和球对称带电蒸发黑洞的结果.

2. 一般球对称带电蒸发黑洞的时空线元

一般球对称带电蒸发黑洞的时空线元[14]为

$$\mathrm{d}s^2=-e^{2\psi}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}v^2+2e^{\psi}\mathrm{d}v\mathrm{d}r+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2),\tag{1}$$

式中 $\psi=\psi(r,v)$,v 是超前爱丁顿坐标,$M=M(r,v)$为黑洞的质量,$Q=Q(v)$为黑洞的电荷.

由(1)式容易求出度规的行列式和不为零的逆变度规分量为

$$g=-e^{2\psi}r^4\sin^2\theta,\tag{2}$$

$$g^{01}=g^{10}=e^{-\psi},g^{11}=1-\frac{2M}{r}+\frac{Q^2}{r^2},$$
$$g^{22}=r^{-2},g^{33}=(r\sin\theta)^{-2}.\tag{3}$$

3. 一般球对称带电蒸发黑洞的视界与温度

把度规行列式 g 和逆变度规 $g^{\mu\nu}$ 的各分量代入带电粒子的 Klein-Gordon 方程

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^{\mu}}-\mathrm{i}eA_{\mu}\right)\times\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-\mathrm{i}eA_{\nu}\right)\Phi\right]-\mu_0^2\Phi=0,\tag{4}$$

考虑到时空的球对称性,(1)式中的 A_{μ} 为

$$A_{\mu}=(A_0,A_1,A_2,A_3)=\left(-\frac{Q}{r},0,0,0\right),\tag{5}$$

代入(4)式得

$$\frac{2}{e^{\psi}}\frac{\partial^2\Phi}{\partial v\partial r}+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2\Phi}{\partial r^2}+\frac{2}{e^{\psi}r}\frac{\partial\Phi}{\partial v}$$

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$$+\left[\psi'\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)+\frac{2}{r^2}(r-M-M'r)\right.$$
$$\left.+\frac{2\mathrm{i}eQ}{e^{\psi}r}\right]\frac{\partial\Phi}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2}-\left(\mu_0^2-\frac{\mathrm{i}eQ}{e^{\psi}r^2}\right)\Phi=0. \tag{6}$$

其中

$$\psi'=\frac{\partial\psi}{\partial r},M'=\frac{\partial M}{\partial r}. \tag{7}$$

分离变量,令

$$\Phi=\frac{1}{r}\rho(r,v)Y_{lm}(\theta,\varphi), \tag{8}$$

则(6)式转化为

$$\frac{2Y}{e^{\psi}}\frac{\partial^2}{\partial v\partial r}\left(\frac{\rho}{r}\right)+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)Y\frac{\partial^2}{\partial r^2}\left(\frac{\rho}{r}\right)$$
$$+\frac{2Y}{e^{\psi}r}\frac{\partial}{\partial v}\left(\frac{\rho}{r}\right)+\left[\psi'\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\right.$$
$$\left.+\frac{2}{r^2}(r-M-M'r)+\frac{2\mathrm{i}eQ}{e^{\psi}r}\right]Y\frac{\partial}{\partial r}\left(\frac{\rho}{r}\right)$$
$$+\frac{\rho}{r^3\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{\rho}{r^3\sin^2\theta}\frac{\partial^2Y}{\partial\varphi^2}$$
$$-\left(\mu_0^2-\frac{\mathrm{i}eQ}{e^{\psi}r^2}\right)\frac{\rho}{r}Y=0.$$

由此可得径向方程

$$\frac{2r}{e^{\psi}}\frac{\partial^2}{\partial v\partial r}\left(\frac{\rho}{r}\right)+r\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2}{\partial r^2}\left(\frac{\rho}{r}\right)$$
$$+\frac{2}{e^{\psi}}\frac{\partial}{\partial v}\left(\frac{\rho}{r}\right)+\left[\psi'\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\right.$$
$$\left.+\frac{2}{r^2}(r-M-M'r)+\frac{2\mathrm{i}eQ}{e^{\psi}r}\right]r\frac{\partial}{\partial r}\left(\frac{\rho}{r}\right)$$
$$-\left[\mu_0^2-\frac{\mathrm{i}eQ}{e^{\psi}r^2}+\frac{l(l+1)}{r^2}\right]\rho=0. \tag{9}$$

利用文献[13]中已经推导的公式

$$\frac{\partial}{\partial r}\left(\frac{\rho}{r}\right)=\frac{1}{r}\left(-\frac{\rho}{r}+\frac{\partial\rho}{\partial r}\right),$$
$$\frac{\partial^2}{\partial v\partial r}\left(\frac{\rho}{r}\right)=\frac{1}{r}\left(-\frac{1}{r}\frac{\partial\rho}{\partial v}+\frac{\partial^2\rho}{\partial v\partial r}\right),$$
$$\frac{\partial^2}{\partial r^2}\left(\frac{\rho}{r}\right)=\frac{1}{r}\left(\frac{2\rho}{r^2}-\frac{2}{r}\frac{\partial\rho}{\partial r}+\frac{\partial^2\rho}{\partial r^2}\right), \tag{10}$$

将(10)式代入(9)式得

$$\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2\rho}{\partial r^2}+\frac{2}{e^{\psi}}\frac{\partial^2\rho}{\partial v\partial r}+\left[\psi'\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\right.$$
$$\left.+\frac{2}{r^2}\left(M-M'r-\frac{Q^2}{r}\right)+\frac{2\mathrm{i}eQ}{e^{\psi}r}\right]\frac{\partial\rho}{\partial r}$$
$$-\left\{\frac{\psi'}{r}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)+\frac{2}{r^3}(r-M-M'r)\right.$$
$$\left.+\left[\mu_0^2+\frac{\mathrm{i}eQ}{e^{\psi}r^2}+\frac{l(l+1)}{r^2}\right]\right\}\rho=0. \tag{11}$$

定义广义的乌龟坐标

$$r_*=r+\frac{1}{2\kappa}\ln[r-r_H(v)],$$
$$v_*=v-v_0, \tag{12}$$

其中 κ 为待定参数,在乌龟坐标变换下是一个常数.由(12)式可以得到[13]

$$\frac{\partial}{\partial r}=\left[1+\frac{1}{2\kappa(r-r_H)}\right]\frac{\partial}{\partial r_*},$$
$$\frac{\partial}{\partial v}=\frac{\partial}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},$$
$$\frac{\partial^2}{\partial r^2}=\left[1+\frac{1}{2\kappa(r-r_H)}\right]^2\frac{\partial^2}{\partial r_*^2}-\frac{1}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*},$$
$$\frac{\partial^2}{\partial v\partial r}=\left[1+\frac{1}{2\kappa(r-r_H)}\right]\frac{\partial^2}{\partial v_*\partial r_*}-\frac{\dot r_H}{2\kappa(r-r_H)}$$
$$\times\left[1+\frac{1}{2\kappa(r-r_H)}\right]\frac{\partial^2}{\partial r_*^2}+\frac{\dot r_H}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*}, \tag{13}$$

其中

$$\dot r_H=\frac{\partial r_H}{\partial v}. \tag{14}$$

把(13)式代入(11)式得

$$\left\{e^{\psi}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left[1+\frac{1}{2\kappa(r-r_H)}\right]\right.$$
$$\left.-\frac{2\dot r_H}{2\kappa(r-r_H)}\right\}\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial v_*\partial r_*}$$
$$+\left[1+\frac{1}{2\kappa(r-r_H)}\right]^{-1}e^{\psi}\left\{\frac{2\dot r_H}{e^{\psi}2\kappa(r-r_H)^2}\right.$$
$$-\frac{1}{2\kappa}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{1}{(r-r_H)^2}$$
$$+\left[1+\frac{1}{2\kappa(r-r_H)}\right]\left[\psi'\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\right.$$
$$\left.\left.+\frac{2}{r^2}\left(M-M'r-\frac{Q^2}{r}\right)+\frac{2\mathrm{i}eQ}{e^{\psi}r}\right]\right\}\frac{\partial\rho}{\partial r_*}$$
$$-e^{\psi}\left[1+\frac{1}{2\kappa(r-r_H)}\right]^{-1}\left\{\frac{\psi'}{r}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\right.$$
$$+\frac{2}{r^3}(r-M-M'r)$$
$$\left.+\left[\mu_0^2+\frac{\mathrm{i}eQ}{e^{\psi}r^2}+\frac{l(l+1)}{r^2}\right]\right\}\rho=0. \tag{15}$$

令 $\frac{\partial^2\rho}{\partial r_*^2}$ 项的系数为 A,则

$$A=e^{\psi}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left[1+\frac{1}{2\kappa(r-r_H)}\right]-\frac{2\dot r_H}{2\kappa(r-r_H)}$$

$$= \frac{e^{\psi}(r^2 - 2Mr + Q^2)[2\kappa(r - r_H) + 1] - 2r^2 \dot{r}_H}{r^2 2\kappa(r - r_H)}. \tag{16}$$

由于当 $r \to r_H$ 时 A 的分母为 0,因此要求当 $r \to r_H$ 时 A 的分子也为 0,即

$$\lim_{\substack{r \to r_H \\ v \to v_0}} \{e^{\psi}(r^2 - 2Mr + Q^2)[2\kappa(r - r_H) + 1] - 2r^2 \dot{r}_H\} = 0, \tag{17}$$

解之得

$$r_H = \frac{M + [M^2 - Q^2(1 - 2\dot{r}_H e^{-\psi})]^{\frac{1}{2}}}{1 - 2\dot{r}_H e^{-\psi}}. \tag{18}$$

从(16)式可以看出,当 $r \to r_H$ 时,A 的极限属于 $\frac{0}{0}$ 型,可以用罗比塔法则求极限,并调节参数 κ,使 A 的极限为 1,即

$$\lim_{\substack{r \to r_H \\ v \to v_0}} \frac{e^{\psi}(r^2 - 2Mr + Q^2)[2\kappa(r - r_H) + 1] - 2r^2 \dot{r}_H}{r^2 2\kappa(r - r_H)} = 1,$$

解之得

$$\kappa = \frac{\psi'(r_H^2 - 2Mr_H + Q^2) + 2(r_H - M - M'r_H) - 4r_H \dot{r}_H e^{-\psi}}{4Mr_H + 2r_H^2(e^{-\psi} - 1) - 2Q^2}. \tag{19}$$

同理可得当 $r \to r_H$ 时,(15)式中 $\frac{\partial \rho}{\partial r_*}$ 项的系数为 $\frac{2ieQ}{r_H}$,而 ρ 项的系数为 0.这样(15)式可化为

$$\frac{\partial^2 \rho}{\partial r_*^2} + 2\frac{\partial^2 \rho}{\partial v_* \partial r_*} + \frac{2ieQ}{r_H}\frac{\partial \rho}{\partial r_*} = 0. \tag{20}$$

容易得到(20)式的线性独立的入射波解和出射波解

$$\rho_\omega^{\mathrm{in}} = \exp(-i\omega v_*), \tag{21}$$

$$\rho_\omega^{\mathrm{out}} = \exp(-i\omega v_*)\exp[2i(\omega - \omega_0) r_*], \tag{22}$$

其中 $\omega_0 = eV$,$V = \frac{Q}{r_H}$ 为黑洞外视界上两极处的静电势.采用文献[13]中的方法,对 $\rho_\omega^{\mathrm{out}}$ 进行解析延拓,可以得到视界内的出射波解为

$$\rho_\omega^{\mathrm{out}} = e^{2(\omega - \omega_0)\pi/2\kappa} e^{2i(\omega - \omega_0) r_*} e^{-i\omega v_*} \quad (r < r_H), \tag{23}$$

其中 $r_* = r + \ln[r_H(v) - r]$,而视界外的出射波解由(22)式描述,即

$$\rho_\omega^{\mathrm{out}} = e^{2i(\omega - \omega_0) r_*} e^{-i\omega v_*} \quad (r > r_H). \tag{24}$$

最后得到一般球对称带电蒸发黑洞向外辐射的热谱为

$$N_\omega^2 = \frac{1}{e^{4\pi(\omega - \omega_0)/2\kappa} \pm 1}, \tag{25}$$

其中'+'号对应费米子;'-'号对应玻色子.

在与黑体辐射的公式进行比较后,可以把(25)式改写为

$$N_\omega^2 = \frac{1}{e^{(\omega - \omega_0)/T} \pm 1}, \tag{26}$$

其中

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi}\frac{\psi'(r_H^2 - 2Mr_H + Q^2) + 2(r_H - M - M'r_H) - 4r_H \dot{r}_H e^{-\psi}}{4Mr_H + 2r_H^2(e^{-\psi} - 1) - 2Q^2}. \tag{27}$$

4. 讨 论

1. 容易验证(18)式描述的视界方程与用零曲面方程得到的视界方程是完全一致的.

2. 当 $Q = 0$ 时,(18)式和(19)式可以分别化为

$$r_H = \frac{2M}{1 - 2\dot{r}_H e^{-\psi}}, \tag{28}$$

$$\kappa = \frac{1 - 2M' - 2\dot{r}_H e^{-\psi}(1 - r_H \psi')}{4M + 2r_H(e^{-\psi} - 1)}, \tag{29}$$

与一般球对称蒸发黑洞的结论[6]一致.

3. 当 $\psi = 0$,$M = M(v)$ 时,(18)式和(19)式可以分别化为

$$r_H = \frac{M + \sqrt{M^2 - Q^2(1 - 2\dot{r}_H)}}{1 - 2\dot{r}_H}, \tag{30}$$

$$\kappa = \frac{r_H - M - 2r_H \dot{r}_H}{2Mr_H - Q^2}, \tag{31}$$

与球对称带电黑洞的结论[15]一致.

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Temperature of a general spherically symmetric and charged evaporating black hole*

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Abstract

Starting from the Klein-Gordon equation near the event horizon, we exactly determine the Hawking temperature and radiation spectrum of a general spherically symmetric and charged evaporating black hole. The equation of the location of the event horizon is also obtained, which is in accord with the result from the equation of null surface.

Keywords: evaporating black hole, event horizon, radiation

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