第 52 卷 第 5 期 2003 年 5 月
1000-3290/2003/52(05)/1051-06

物 理 学 报
ACTA PHYSICA SINICA

Vol.52, No.5, May, 2003

非完整系统 Hamilton 正则方程的形式不变性*

乔永芬[1)] 张耀良[2)] 韩广才[2)]

1)(东北农业大学工程学院, 哈尔滨 150030)

2)(哈尔滨工程大学二系, 哈尔滨 150001)

(2002 年 7 月 9 日收到; 2002 年 8 月 26 日收到修改稿)

研究非完整系统 Hamilton 正则方程的形式不变性, 给出形式不变性的定义和判据, 建立形式不变性和系统守恒量之间的关系, 并举例说明结果的应用.

关键词: 非完整系统, Hamilton 正则方程, 形式不变性, 守恒量

PACC: 0320, 0200

1. 引 言

动力学系统的守恒量不仅具有数学重要性, 而且表现为深刻的物理规律, 它已成为近代分析力学的一个重要研究方向. 寻求守恒量的主要方法有 Noether 理论[1] 和 Lie 对称性[2], 近年来对这类方法的研究已取得重要进展[3—7].

形式不变性是指运动微分方程中出现的动力学函数在经无限小变换后仍满足原来的方程. 文献[8—14]中分别研究了 Lagrange 方程、Appell 方程、Gibbs-Appel 方程、Nielsen 方程和 Chaplygin 方程的形式不变性.

本文研究非完整系统 Hamilton 正则方程在无限小变换下的形式不变性定义和判据, 并建立形式不变性和守恒量之间的关系, 举例说明结果的应用.

2. 相应完整系统的 Hamilton 正则方程

设力学系统的位形由 n 个广义坐标 $q_1, q_1, \cdots, q_n$ 确定, 系统的运动受有 g 个理想、双面 Chetaev 型非完整约束

$$f_\beta(q_s, \dot{q}_s, t) = 0$$
$$(\beta = 1,2,\cdots,g;\ s = 1,2,\cdots,n). \tag{1}$$

虚位移方程为

$$\frac{\partial f_\beta}{\partial \dot{q}_s}\delta q_s = 0$$
$$(\beta = 1,2,\cdots,g;\ s = 1,2,\cdots,n). \tag{2}$$

系统的 Routh 方程可表为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = Q_s + \lambda_\beta \frac{\partial f_\beta}{\partial \dot{q}_s}$$
$$(s = 1,2,\cdots,n;\ \beta = 1,2,\cdots,g). \tag{3}$$

在运动微分方程积分之前, 可由方程(1), (3)求出 λ_β 为 $t, q, \dot{q}$ 的函数, 这样方程(3)可写为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = Q_s + \Lambda_s$$
$$(s = 1,2,\cdots,n). \tag{4}$$

其中 $\Lambda_s = \lambda_\beta \dfrac{\partial f_\beta}{\partial \dot{q}_s}$, 它是广义非完整约束力, 已表为 $t, q, \dot{q}$ 的函数. 方程(4)称为与非完整系统(1), (3)相应的完整系统的运动微分方程. 只要对初始条件施加非完整约束(1)的限制条件, 则非完整系统(1), (3)的积分可通过相应的完整系统(4)求得.

引入广义动量和 Hamilton 函数

$$p_s = \frac{\partial L}{\partial \dot{q}_s} \qquad (s = 1,2,\cdots,n), \tag{5}$$

$$H = p_s\dot{q}_s - L \qquad (s = 1,2,\cdots,n). \tag{6}$$

于是得到相应完整系统的 Hamilton 正则方程

$$\dot{p}_s = -\frac{\partial H}{\partial q_s} + \tilde{Q}_s + \tilde{\Lambda}_s, \tag{7}$$

---

* 国家自然科学基金(批准号 19872022)资助的课题.

$$\dot{q}_s = \frac{\partial H}{\partial p_s} \quad (s = 1,2,\cdots,n). \tag{8}$$

这里记号 ~ 表示 $\dot{q}$ 用 t,q,p 代换所得的表达式.

3. 相应完整系统 Hamilton 正则方程的形式不变性

取时间、广义坐标和广义动量的无限小变换

$$t^* = t + \Delta t,\ q_s^*(t^*) = q_s(t) + \Delta q_s,$$
$$p_s^*(t^*) = p_s(t) + \Delta p_s \quad (s = 1,2,\cdots,n). \tag{9}$$

或其展开式为

$$t^* = t + \varepsilon R(t,q,p),$$
$$q_s^*(t^*) = q_s(t) + \varepsilon G_s(t,q,p),$$
$$p_s^*(t^*) = p_s(t) + \varepsilon \eta_s(t,q,p), \tag{10}$$

式中 ε 为无限小参数,R,G_s 和 η_s 为无限小生成元.

假设在无限小变换(10)下,Hamilton 函数 $H(t,q,p)$ 成为 $H^* = H(t^*,q^*,p^*)$;广义非势力 $\tilde{Q}_s$ 成为 $\tilde{Q}_s^* = \tilde{Q}_s(t^*,q^*,p^*)$;广义约束反力 $\tilde{\Lambda}_s$ 成为 $\tilde{\Lambda}_s^* = \tilde{\Lambda}_s(t^*,q^*,p^*)$.

3.1. 定 义

在无限小变换(10)下,如果正则方程(7),(8)的形式保持不变,即

$$\dot{p}_s^* = -\frac{\partial H^*}{\partial q_s} + \tilde{Q}_s^* + \tilde{\Lambda}_s^*,$$
$$\dot{q}_s^* = \frac{\partial H^*}{\partial p_s} \quad (s = 1,2,\cdots,n) \tag{11}$$

成立,则称这种不变性为相应完整系统的 Hamilton 正则方程(7),(8)的形式不变性.

展开 H^*,$\tilde{Q}_s^*$,$\tilde{\Lambda}_s^*$,$\dot{p}_s^*$ 和 $\dot{q}_s^*$,有

$$H^* = H(t^*,q^*,p^*) = H(t,q,p) + \varepsilon\left[\frac{\partial H}{\partial t}R + \frac{\partial H}{\partial q_s}G_s + \frac{\partial H}{\partial p_s}\eta_s\right] + O(\varepsilon^2),$$

$$\tilde{Q}_s^* = \tilde{Q}_s(t^*,q^*,p^*) = \tilde{Q}_s(t,q,p) + \varepsilon\left[\frac{\partial \tilde{Q}_s}{\partial t}R + \frac{\partial \tilde{Q}_s}{\partial q_k}G_k + \frac{\partial \tilde{Q}_s}{\partial p_k}\eta_k\right] + O(\varepsilon^2),$$

$$\tilde{\Lambda}_s^* = \tilde{\Lambda}_s(t^*,q^*,p^*) = \tilde{\Lambda}_s(t,q,p) + \varepsilon\left[\frac{\partial \tilde{\Lambda}_s}{\partial t}R + \frac{\partial \tilde{\Lambda}_s}{\partial q_k}G_k + \frac{\partial \tilde{\Lambda}_s}{\partial p_k}\eta_k\right] + O(\varepsilon^2),$$

$$\dot{q}_s^* = \dot{q}_s + \varepsilon\dot{G}_s,\ \dot{p}_s^* = \dot{p}_s + \varepsilon\dot{\eta}_s$$
$$(s,k = 1,2,\cdots,n). \tag{12}$$

由于

$$\Delta\dot{q}_k = \varepsilon(\dot{G}_k - \dot{q}_k\dot{R}), \tag{13}$$
$$\Delta p_s = \varepsilon\eta_s,$$
$$\Delta p_s = \frac{\partial p_s}{\partial t}\Delta t + \frac{\partial p_s}{\partial q_k}\Delta q_k + \frac{\partial p_s}{\partial \dot{q}_k}\Delta\dot{q}_k = \varepsilon\left[\frac{\partial p_s}{\partial t}R + \frac{\partial p_s}{\partial q_k}G_k + \frac{\partial p_s}{\partial \dot{q}_k}(\dot{G}_k - \dot{q}_k\dot{R})\right],$$

故有

$$\eta_s = \frac{\partial p_s}{\partial t}R + \frac{\partial p_s}{\partial q_k}G_k + \frac{\partial p_s}{\partial \dot{q}_k}(\dot{G}_k - \dot{q}_k\dot{R})$$
$$(s,k = 1,2,\cdots,n). \tag{14}$$

将(12)式代入方程(11),舍去 ε^2 及更高阶小项,并注意(13),(14)式,得到

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial p_s}{\partial t}R + \frac{\partial p_s}{\partial q_k}G_k + \frac{\partial p_s}{\partial \dot{q}_k}(\dot{G}_k - \dot{q}_k\dot{R})\right] + \frac{\partial}{\partial q_s}\left\{\frac{\partial H}{\partial t}R + \frac{\partial H}{\partial q_k}G_k + \frac{\partial H}{\partial p_k}\left[\frac{\partial p_k}{\partial t}R + \frac{\partial p_k}{\partial q_j}G_j + \frac{\partial p_k}{\partial \dot{q}_j}(\dot{G}_j - \dot{q}_j\dot{R})\right]\right\}$$
$$- \left(\frac{\partial \tilde{Q}_s}{\partial t} + \frac{\partial \tilde{\Lambda}_s}{\partial t}\right)R - \left(\frac{\partial \tilde{Q}_s}{\partial q_k} + \frac{\partial \tilde{\Lambda}_s}{\partial q_k}\right)G_k - \left(\frac{\partial \tilde{Q}_s}{\partial p_k} + \frac{\partial \tilde{\Lambda}_s}{\partial p_k}\right)\left[\frac{\partial p_k}{\partial t}R + \frac{\partial p_k}{\partial q_j}G_j + \frac{\partial p_k}{\partial \dot{q}_j}(\dot{G}_j - \dot{q}_j\dot{R})\right] = 0$$
$$(s,k,j = 1,2,\cdots,n), \tag{15}$$

$$\dot{G}_s - \frac{\partial}{\partial p_s}\left\{\frac{\partial H}{\partial t}R + \frac{\partial H}{\partial q_k}G_k + \frac{\partial H}{\partial p_k}\left[\frac{\partial p_k}{\partial t}R + \frac{\partial p_k}{\partial q_j}G_j + \frac{\partial p_k}{\partial \dot{q}_j}(\dot{G}_j - \dot{q}_j\dot{R})\right]\right\} = 0$$
$$(s,k,j = 1,2,\cdots,n). \tag{16}$$

于是有下面的判据.

3.2. 判 据

如果无限小生成元 R,G_s 满足方程(15)和(16),则相应完整系统(7),(8)是形式不变的.

一般说来,形式不变性不一定总导致守恒量,下面的命题给出形式不变性导致守恒量的条件.

3.3. 命 题

如果无限小生成元 R,G_s 使方程(7),(8)是形

式不变的,而且又若 G_s 是方程(7)的积分因子,即 G_s 满足必要条件[15,16]

$$-R\frac{\partial H}{\partial t}+(\tilde{Q}_s+\tilde{\Lambda}_s)\left(G_s-R\frac{\partial H}{\partial p_s}\right)+p_s\dot{G}_s-\frac{\partial H}{\partial q_s}G_s-H\dot{R}-\dot{B}+I=0, \quad (17)$$

其中

$$I=\mu_s\left(\dot{p}_s+\frac{\partial H}{\partial q_s}-\tilde{Q}_s-\tilde{\Lambda}_s\right),$$

则形式不变性将导致相应完整系统(7),(8)具有如下的守恒量:

$$D=p_sG_s-HR-B \quad (s=1,2,\cdots,n). \quad (18)$$

由方程(15),(16)和(17)可以看出,形式不变性与积分因子的必要条件一般说是不同的.

4. 非完整系统 Hamilton 正则方程的形式不变性

根据文献[17],由方程(1)和(3)表示的非完整系统,其 Hamilton 正则方程为

$$\dot{p}_s=-\frac{\partial H}{\partial q_s}+\tilde{Q}_s+\lambda_\beta\frac{\partial f_\beta}{\partial \dot{q}_s},$$

$$\dot{q}_s=\frac{\partial H}{\partial p_s}$$

$$(\beta=1,2,\cdots,g;s=1,2,\cdots,n). \quad (19)$$

约束方程(1)和乘子 λ_β 可写为

$$\tilde{f}_\beta(t,q_s,p_s)=0$$
$$(\beta=1,2,\cdots,g;s=1,2,\cdots,n), \quad (20)$$

$$\tilde{\lambda}_\beta=\tilde{\lambda}_\beta(t,q_s,p_s)$$
$$(\beta=1,2,\cdots,g;s=1,2,\cdots,n). \quad (21)$$

假设在无限小变换(10)下,$H,\tilde{Q}_s,\tilde{\lambda}_\beta,\tilde{f}_\beta$ 和 $\tilde{\Lambda}_s$ 成为

$$H^*=H(t^*,q^*,p^*),$$
$$\tilde{Q}_s^*=\tilde{Q}_s(t^*,q^*,p^*),$$
$$\tilde{\lambda}_\beta^*=\tilde{\lambda}_\beta(t^*,q^*,p^*),$$
$$\tilde{f}_\beta^*=\tilde{f}_\beta(t^*,q^*,p^*),$$
$$\tilde{\Lambda}_s^*=\tilde{\Lambda}_s(t^*,q^*,p^*).$$

4.1. 定 义

在无限小变换(10)下,如果约束方程(20)和相应完整系统正则方程(7),(8)的形式保持不变,即

$$\tilde{f}_\beta^*=\tilde{f}_\beta(t^*,q^*,p^*)=0$$
$$(\beta=1,2,\cdots,g), \quad (22)$$

$$\dot{p}_s^*=-\frac{\partial H^*}{\partial q_s}+\tilde{Q}_s^*+\tilde{\Lambda}_s^*$$
$$(s=1,2,\cdots,n), \quad (23)$$

$$\dot{q}_s^*=\frac{\partial H^*}{\partial p_s} \quad (24)$$

成立,则称这种不变性为非完整系统正则方程(19)的形式不变性.

展开 $H^*,\tilde{Q}_s^*,\tilde{\Lambda}_s^*$ 和 $\tilde{f}_\beta^*$,有

$$H^*=H(t^*,q^*,p^*)=H(t,q,p)+\varepsilon\left[\frac{\partial H}{\partial t}R+\frac{\partial H}{\partial q_s}G_s+\frac{\partial H}{\partial p_s}\eta_s\right]+O(\varepsilon^2),$$

$$\tilde{Q}_s^*=\tilde{Q}_s(t^*,q^*,p^*)=\tilde{Q}_s(t,q,p)+\varepsilon\left[\frac{\partial\tilde{Q}_s}{\partial t}R+\frac{\partial\tilde{Q}_s}{\partial q_k}G_k+\frac{\partial\tilde{Q}_s}{\partial p_k}\eta_k\right]+O(\varepsilon^2),$$

$$\tilde{\Lambda}_s^*=\tilde{\Lambda}_s(t^*,q^*,p^*)=\tilde{\Lambda}_s(t,q,p)+\varepsilon\left[\frac{\partial\tilde{\Lambda}_s}{\partial t}R+\frac{\partial\tilde{\Lambda}_s}{\partial q_k}G_k+\frac{\partial\tilde{\Lambda}_s}{\partial p_k}\eta_k\right]+O(\varepsilon^2), \quad (25)$$

$$\tilde{f}_\beta^*=\tilde{f}_\beta(t^*,q^*,p^*)=\tilde{f}_\beta(t,q,p)+\varepsilon\left[\frac{\partial\tilde{f}_\beta}{\partial t}R+\frac{\partial\tilde{f}_\beta}{\partial q_k}G_k+\frac{\partial\tilde{f}_\beta}{\partial p_k}\eta_k\right]+O(\varepsilon^2). \quad (26)$$

又

$$\dot{q}_s^*=\dot{q}_s+\varepsilon\dot{G}_s,\ \dot{p}_s^*=\dot{p}_s+\varepsilon\dot{\eta}_s. \quad (27)$$

现将(26)式代入方程(20);(25)及(27)式代入方程(23),(24),通过应用方程(20),(7)和(8),舍去 ε^2 及更高阶小项,并注意(13),(14)式,得到

$$\frac{\partial\tilde{f}_\beta}{\partial t}R+\frac{\partial\tilde{f}_\beta}{\partial q_k}G_k+\frac{\partial\tilde{f}_\beta}{\partial p_k}\left[\frac{\partial p_k}{\partial t}R+\frac{\partial p_k}{\partial q_j}G_j+\frac{\partial p_k}{\partial\dot{q}_j}(\dot{G}_j-\dot{q}_j\dot{R})\right]=0, \quad (28)$$

及方程(15)和(16).

于是有下面的判据.

4.2. 判 据

对于由方程(19)和(20)表示的非完整系统,如果无限小生成元 R,G_s 满足(15),(16)和(28)式,则在无限小变换(10)下,该系统是形式不变的.

对于非完整系统,形式不变性不一定导致守恒量.下面的命题给出形式不变性导致守恒量的条件.

4.3. 命 题

在无限小变换(10)下,如果由方程(19)和(20)给出的非完整系统是形式不变的,并且,又若 G_s 是方程(7)的积分因子[15,16],即 G_s 满足必要条件(17),则形式不变性将导致非完整系统具有守恒量

$$D = p_s G_s - HR - B \qquad (s = 1,2,\cdots,n). \tag{29}$$

5. Killing 方程

利用积分因子理论寻求动力学系统的守恒量,关键在于找到函数 $R = R(t,q,p)$,$B = B(t,q,p)$,$G_s = G_s(t,q,p)$.通常将必要条件(17)展开,并分解对 R,B 和 G_s 的一阶偏微分方程,这类偏微分方程称为 Killing 方程,解 Killing 方程便有可能找到这类函数.根据文献[13],将必要条件(17)展开,稍加变换,得

$$\begin{aligned}
&-R\frac{\partial H}{\partial t} + (\tilde{Q}_s + \tilde{\Lambda}_s)\left(G_s - R\frac{\partial H}{\partial p_s}\right) + p_s\left(\frac{\partial G_s}{\partial t}\right.\\
&\left.+\frac{\partial G_s}{\partial q_k}\frac{\partial H}{\partial p_k}\right) - \frac{\partial H}{\partial q_s}G_s - H\left(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial q_k}\frac{\partial H}{\partial p_k}\right)\\
&-\frac{\partial B}{\partial t} - \frac{\partial B}{\partial q_k}\frac{\partial H}{\partial p_k} + \left(\tilde{Q}_s + \tilde{\Lambda}_s - \frac{\partial H}{\partial q_k}\right)\\
&\times\left(p_s\frac{\partial G_s}{\partial p_k} - H\frac{\partial R}{\partial p_k} - \frac{\partial B}{\partial p_k}\right) = 0\\
&\qquad (s,k = 1,2,\cdots,n).
\end{aligned} \tag{30}$$

这是线性偏微分方程,在一般情况下,$(n+2)$个函数 R,B 和 G_s 中任意一个函数被认为是未知的.

6. 举 例

Appell-Hamel 例

已知力学系统的 Lagrange 函数为

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) - mgq_3, \tag{31}$$

非完整约束

$$f = \dot{q}_1^2 + \dot{q}_2^2 - \dot{q}_3^2 = 0. \tag{32}$$

试求系统的守恒量.

第一步 建立相应完整系统的 Hamilton 正则方程.

方程(3)给出

$$m\ddot{q}_1 = 2\lambda\dot{q}_1,\quad m\ddot{q}_2 = 2\lambda\dot{q}_2,$$
$$m\ddot{q}_3 + mg = -2\lambda\dot{q}_3. \tag{33}$$

从方程(32),(33)解得

$$\lambda = -\frac{mg\dot{q}_3}{2(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2)} = -\frac{mg}{4\dot{q}_3}, \tag{34}$$

于是,有

$$m\ddot{q}_1 = -\frac{mg\dot{q}_1}{2\dot{q}_3},\quad m\ddot{q}_2 = -\frac{mg\dot{q}_2}{2\dot{q}_3},$$
$$m\ddot{q}_3 + mg = \frac{1}{2}mg. \tag{35}$$

对照方程(4),得

$$Q_1 + \Lambda_1 = -\frac{mg\dot{q}_1}{2\dot{q}_3},$$
$$Q_2 + \Lambda_2 = -\frac{mg\dot{q}_2}{2\dot{q}_3},$$
$$Q_3 + \Lambda_3 = \frac{1}{2}mg. \tag{36}$$

广义动量为

$$p_1 = m\dot{q}_1,\quad p_2 = m\dot{q}_2,\quad p_3 = m\dot{q}_3,$$
$$\dot{q}_1 = \frac{p_1}{m},\ \dot{q}_2 = \frac{p_2}{m},\ \dot{q}_3 = \frac{p_3}{m}. \tag{37}$$

于是

$$\tilde{Q}_1 + \tilde{\Lambda}_1 = -\frac{mgp_1}{2p_3},$$
$$\tilde{Q}_2 + \tilde{\Lambda}_2 = -\frac{mgp_2}{2p_3},$$
$$\tilde{Q}_3 + \tilde{\Lambda}_3 = \frac{1}{2}mg. \tag{38}$$

引入 Hamilton 函数

$$H = \frac{p_1^2 + p_2^2 + p_3^2}{2m} + mgq_3. \tag{39}$$

由此可得

$$\dot{p}_1 = -\frac{\partial H}{\partial q_1} + \tilde{Q}_1 + \tilde{\Lambda}_1 = -\frac{mgp_1}{2p_3},$$
$$\dot{p}_2 = -\frac{\partial H}{\partial q_2} + \tilde{Q}_2 + \tilde{\Lambda}_2 = -\frac{mgp_2}{2p_3},$$
$$\dot{p}_3 = -\frac{\partial H}{\partial q_3} + \tilde{Q}_3 + \tilde{\Lambda}_3 = -\frac{1}{2}mg. \tag{40}$$

第二步 研究形式不变性.

取
$$R = 1,\ G_1 = G_2 = G_3 = 0, \tag{41}$$
$$R = 0,\ G_1 = G_2 = 0,\ G_3 = 1. \tag{42}$$

则(41)式和(42)式分别满足(15),(16)式,且对应系统的形式不变性.

第三步　求系统的守恒量.

由 Killing 方程(30),得

$$\left(-\frac{mgp_1}{2p_3}\right)\left(G_1-R\frac{p_1}{m}\right)+\left(-\frac{mgp_2}{2p_3}\right)\times\left(G_2-R\frac{p_2}{m}\right)+\frac{mg}{2}\left(G_3-R\frac{p_3}{m}\right)$$
$$+p_1\left(\frac{\partial G_1}{\partial t}+\frac{\partial G_1}{\partial q_1}\frac{p_1}{m}+\frac{\partial G_1}{\partial q_2}\frac{p_2}{m}+\frac{\partial G_1}{\partial q_3}\frac{p_3}{m}\right)$$
$$+p_2\left(\frac{\partial G_2}{\partial t}+\frac{\partial G_2}{\partial q_1}\frac{p_1}{m}+\frac{\partial G_2}{\partial q_2}\frac{p_2}{m}+\frac{\partial G_2}{\partial q_3}\frac{p_3}{m}\right)$$
$$+p_3\left(\frac{\partial G_3}{\partial t}+\frac{\partial G_3}{\partial q_1}\frac{p_1}{m}+\frac{\partial G_3}{\partial q_2}\frac{p_2}{m}+\frac{\partial G_3}{\partial q_3}\frac{p_3}{m}\right)$$
$$-\frac{\partial H}{\partial q_1}G_1-\frac{\partial H}{\partial q_2}G_2-\frac{\partial H}{\partial q_3}G_3-H\left(\frac{\partial R}{\partial t}+\frac{\partial R}{\partial q_1}\frac{p_1}{m}+\frac{\partial R}{\partial q_2}\frac{p_2}{m}+\frac{\partial R}{\partial q_3}\frac{p_3}{m}\right)-\frac{\partial B}{\partial t}-\frac{\partial B}{\partial q_1}\frac{p_1}{m}-\frac{\partial B}{\partial q_2}\frac{p_2}{m}$$
$$-\frac{\partial B}{\partial q_3}\frac{p_3}{m}+\left(-\frac{mg}{2}\frac{p_1}{p_3}\right)\left(p_1\frac{\partial G_1}{\partial p_1}+p_2\frac{\partial G_2}{\partial p_1}+p_3\frac{\partial G_3}{\partial p_1}-H\frac{\partial R}{\partial p_1}-\frac{\partial B}{\partial p_1}\right)+\left(-\frac{mg}{2}\frac{p_2}{p_3}\right)$$
$$\times\left(p_1\frac{\partial G_1}{\partial p_2}+p_2\frac{\partial G_2}{\partial p_2}+p_3\frac{\partial G_3}{\partial p_2}-H\frac{\partial R}{\partial p_2}-\frac{\partial B}{\partial p_2}\right)+\left(\frac{mg}{2}-mg\right)\left(p_1\frac{\partial G_1}{\partial p_3}+p_2\frac{\partial G_2}{\partial p_3}\right.$$
$$\left.+p_3\frac{\partial G_3}{\partial p_3}-H\frac{\partial R}{\partial p_3}-\frac{\partial B}{\partial p_3}\right)=0. \tag{43}$$

现将(41)式和(42)式分别代入方程(43),相应的得到

$$B_1=0, \tag{44}$$

$$B_2=-\frac{1}{2}mgt. \tag{45}$$

由于(41),(44)式和(42),(45)式均满足必要条件(17),于是守恒量(18)式给出

$$D_1=-\frac{1}{2}m(\dot q_1^{\,2}+\dot q_2^{\,2}+\dot q_3^{\,2})-mgq_3=\mathrm{const}, \tag{46}$$

$$D_2=m\dot q_3+\frac{1}{2}mgt=\mathrm{const}. \tag{47}$$

[1] Noether A E 1918 *Nachr Akad Wiss Göttingen Math. Phys*. KI. II 235

[2] Lutzky M 1979 *Phys A*:*Math Gen*. **12** 973

[3] Li Z P 1981 *Acta Phys. Sin*. **30** 1659(in Chinese)[李子平 1981 物理学报 **30** 1659]

[4] Liu D 1989 *Acta Mech. Sin*. **21** 75(in Chinese)[刘　端 1989 力学学报 **21** 75]

[5] Mei F X 1999 *Applications of Lie groups and Lie algebras to constrained mechanical systems* (Beijing:Science Press) p90 [梅凤翔 1999 李群和李代数对约束力学系统的应用(北京:科学出版社)第 90 页]

[6] Qiao Y F and Zhao S H 2001 *Acta Phys. Sin*. **50** 1(in Chinese)[乔永芬、赵淑红 2001 物理学报 **50** 1]

[7] Zhang R C and Mei F X 2000 *Chin. Phys*. **9** 801

[8] Mei F X 2000 *Journal of Beijing Institute of Technology* **9** 120(in Chinese)[梅凤翔 2000 北京理工大学学报 **9** 120]

[9] Mei F X 2001 *Chin. Phys*. **10** 177

[10] Li R J,Qiao Y F and Meng J 2002 *Acta Phys. Sin*. **51** 1 (in Chinese)[李仁杰、乔永芬、孟　军 2002 物理学报 **51** 1]

[11] Wang S Y and Mei F X 2001 *Chin. Phys*. **10** 373

[12] Ge W K 2002 *Acta Phys. Sin*. **51** 939(in Chinese)[葛伟宽 2002 物理学报 **51** 939]

[13] Wang S Y and Mei F X 2002 *Chin. Phys*. **11** 5

[14] Fang J H,Xue Q Z and Zhao S Q 2002 *Acta Phys. Sin*. **51** 2183(in Chinese)[方建会、薛庆忠、赵嵩卿 2002 物理学报 **51** 2183]

[15] Djukic'Dj S and Tytti Sutela 1984 *Int. J. Nonlinear Mechanics* **19** 331

[16] Qiao Y F,Zhang Y L and Zhao S H 2002 *Acta Phys. Sin*. **51** 1661 (in Chinese)[乔永芬、张耀良、赵淑红 2002 物理学报 **51** 1661]

[17] Mei F X 1985 *Foundations of mechanics of nonholonomic systems* (Beijing:Beijing Institute of Technology Press) p334 (in Chinese)[梅凤翔 1985 非完整系统力学基础(北京工业学院出版社)第 334 页]

Form invariance of Hamilton 's canonical equations of a nonholonomic mechanical system*

Qiao Yong-Fen[1)] Zhang Yao-Liang[2)] Han Guang-Cai[2)]

[1)] (*Engineering College of Northeast Agriculture University*, *Harbin* 150001, *China*)

[2)] (*Department of Civil Engineering.*, *Harbin Engineering University*, *Harbin* 150001, *China*)

(Received 9 July 2002; revised manuscript received 26 August 2002)

Abstract

In this paper, a form invariance of Hamilton 's canonical equations of a nonholonomic mechanical system is studied. First, the definition and criterion of the form invariance in the system under infinitesimal transformations of groups are given. Next, the relation between the form invariance and the conserved quantity of the system is established. Finally, an example is given to illustrate the application of the result.

Keywords: nonholonomic system, Hamilton 's canonical equation, form invariance, conserved quantity

PACC: 0320, 0200

* Project supported by the National Natural Science Foundation of China (Grant No. 19872022).