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# 电磁直线加速动态黑洞时空中 Dirac 粒子的 Hawking 辐射*

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研究了作直线加速运动的电磁黑洞视界面上 Dirac 粒子的 Hawking 辐射.首先,构造对称化零标架,计算旋系数,导出 Dirac 方程,并对其进行化简.然后,通过引入广义乌龟坐标,在视界面上将 Dirac 方程退耦.利用 Damour-Ruffini 方法,求出了温度以及热谱公式,并对所得结果进行了讨论.

关键词:加速动态黑洞,Dirac 粒子,Dirac 方程,Hawking 辐射

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## 1. 引　　言

近几年来,人们对于各类黑洞视界面上标量粒子的 Hawking 辐射进行了深入的研究[1—9].Dirac 粒子的 Hawking 辐射问题也研究了球对称黑洞以及稳态 Kerr 黑洞等几种情况[10—14].但对于动态非球对称黑洞的 Dirac 粒子 Hawking 辐射的研究则遇到了极大的困难.原因在于在非球对称背景时空中,Dirac 方程不易退耦和分离变量,因而不能严格求解.本文引入较为对称的零标架,并通过适当变换化简 Dirac 方程,然后引入广义乌龟坐标,在视界面上进一步化简,较好地解决了退耦问题.利用 Damour-Ruffini 方法,求出了视界面上的温度表达式以及热谱公式.

## 2. 零标架与 Dirac 方程

带有电荷磁荷、作任意加速运动的黑洞背景时空线元为[15]

$$\mathrm{d}s^2 = g_{uu}\mathrm{d}u^2 + 2g_{ur}\mathrm{d}u\mathrm{d}r + 2g_{u\theta}\mathrm{d}u\mathrm{d}\theta + 2g_{u\varphi}\mathrm{d}u\mathrm{d}\varphi + g_{\theta\theta}\mathrm{d}\theta^2 + g_{\varphi\varphi}\mathrm{d}\varphi^2, \quad (1)$$

式中

$$\begin{aligned} g_{uu} &= 1 - 2Mr^{-1} + (e^2+q^2)r^{-2} - 2ar\cos\theta \\ &\quad - 4a(e^2+q^2)\cos\theta r^{-1} - r^2(b\sin\varphi + c\cos\varphi \\ &\quad - a\sin\theta)^2 - r^2(b\cos\varphi - c\sin\varphi)^2\cos^2\theta - \frac{1}{3}\lambda r^2, \\ g_{ur} &= 1, \\ g_{u\theta} &= r^2(b\sin\varphi + c\cos\varphi - a\sin\theta), \\ g_{u\varphi} &= r^2\sin\theta\cos\theta(b\cos\varphi - c\sin\varphi), \\ g_{\theta\theta} &= -r^2, \\ g_{\varphi\varphi} &= -r^2\sin^2\theta, \end{aligned}$$

其中 $a, b, c, e, q$ 和 $M$ 为推迟 Eddington-Finkelstein 坐标 $u$ 的任意函数.$a$ 为加速度的大小,$b$ 和 $c$ 为描述加速度方向变化的速率.$M$,$e$ 和 $q$ 分别为引力源的质量、电荷和磁荷.对于直线加速的情况,我们有 $b = c = 0$.度规简化为

$$\begin{aligned} \mathrm{d}s^2 &= \Big(1 - 2ar\cos\theta - a^2r^2\sin^2\theta - 2Mr^{-1} \\ &\quad + \frac{e^2+q^2}{r^2} - \frac{4a(e^2+q^2)}{r}\cos\theta - \frac{1}{3}\lambda r^2\Big)\mathrm{d}u^2 \\ &\quad + 2\mathrm{d}u\mathrm{d}r - 2ar^2\sin\theta\mathrm{d}u\mathrm{d}\theta - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2. \end{aligned} \quad (2)$$

用超前 Eddington-Finkelstein 坐标 $v$ 替换推迟坐标 $u$,度规变为

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$$ds^2=\Big(1-2ar\cos\theta-a^2r^2\sin^2\theta-2Mr^{-1}+\frac{e^2+q^2}{r^2}-\frac{4a(e^2+q^2)}{r}\cos\theta-\frac{1}{3}\lambda r^2\Big)dv^2-2dvdr+2ar^2\sin\theta dvd\theta-r^2d\theta^2-r^2\sin^2\theta d\varphi^2. \tag{3}$$

我们首先构造对称化的零标架

$$\begin{aligned}
l^\mu&=\frac{1}{\sqrt2}[0,1,0,0],\\
n^\mu&=\sqrt2\Big[-1,-\frac12\Sigma,-a\sin\theta,0\Big],\\
m^\mu&=\frac{1}{\sqrt2}\Big[0,0,\frac1r,\frac{i}{r\sin\theta}\Big],\\
\bar m^\mu&=\frac{1}{\sqrt2}\Big[0,0,\frac1r,\frac{-i}{r\sin\theta}\Big],
\end{aligned}\tag{4}$$

其中

$$\Sigma=1-2ar\cos\theta-2Mr^{-1}+\frac{e^2+q^2}{r^2}-\frac{4a(e^2+q^2)}{r}\cos\theta-\frac13\lambda r^2. \tag{5}$$

不为零的旋系数为

$$\begin{aligned}
\rho&=-\frac{1}{\sqrt2 r},\\
\alpha&=-\frac{\cot\theta}{2\sqrt2 r},\\
\beta&=\frac{\cot\theta}{2\sqrt2 r},\\
\mu&=\sqrt2\Big(-\frac{1}{2r}\Sigma-a\cos\theta\Big),\\
\gamma&=\frac{\sqrt2}{4}\frac{\partial\Sigma}{\partial r},\\
\nu&=\sqrt2 a\Big[1+\frac{2(e^2+q^2)}{r^2}\Big]\sin\theta.
\end{aligned}\tag{6}$$

4 个方向导数为

$$\begin{aligned}
D&=l^\mu\frac{\partial}{\partial x^\mu}=\frac{1}{\sqrt2}\frac{\partial}{\partial r},\\
\Delta&=n^\mu\frac{\partial}{\partial x^\mu}=\sqrt2\Big(-\frac{\partial}{\partial v}-\frac12\Sigma\frac{\partial}{\partial r}-a\sin\theta\frac{\partial}{\partial\theta}\Big),\\
\delta&=m^\mu\frac{\partial}{\partial x^\mu}=\frac{1}{\sqrt2 r}\Big(\frac{\partial}{\partial\theta}+\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\Big),\\
\bar\delta&=\bar m^\mu\frac{\partial}{\partial x^\mu}=\frac{1}{\sqrt2 r}\Big(\frac{\partial}{\partial\theta}-\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\Big).
\end{aligned}\tag{7}$$

弯曲时空中的 Dirac 方程为

$$\Big(\frac{\partial}{\partial r}+\frac1r\Big)F_1+\Big(\frac1r\frac{\partial}{\partial\theta}-\frac{i}{r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cot\theta}{2r}\Big)F_2=i\sqrt2\mu_0G_1, \tag{8a}$$

$$\Big(-2\frac{\partial}{\partial v}-\Sigma\frac{\partial}{\partial r}-2a\sin\theta\frac{\partial}{\partial\theta}+\Big[-\frac1r\Sigma-2a\cos\theta-\frac12\frac{\partial\Sigma}{\partial r}\Big]\Big)F_2+\Big(\frac1r\frac{\partial}{\partial\theta}+\frac{i}{r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cot\theta}{2r}\Big)F_1=i\sqrt2\mu_0G_2 \tag{8b}$$

$$\Big(\frac{\partial}{\partial r}+\frac1r\Big)G_2-\Big(\frac1r\frac{\partial}{\partial\theta}+\frac{i}{r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cot\theta}{2r}\Big)G_1=i\sqrt2\mu_0F_2, \tag{8c}$$

$$\Big(-2\frac{\partial}{\partial v}-\Sigma\frac{\partial}{\partial r}-2a\sin\theta\frac{\partial}{\partial\theta}+\Big[-\frac1r\Sigma-2a\cos\theta-\frac12\frac{\partial\Sigma}{\partial r}\Big]\Big)G_1-\Big(\frac1r\frac{\partial}{\partial\theta}-\frac{i}{r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cot\theta}{2r}\Big)G_2=i\sqrt2\mu_0F_1, \tag{8d}$$

式中 $F_1,F_2,G_1,G_2$ 为波函数的 4 个分量,$\mu_0$ 是 Dirac 粒子的静止质量.

## 3. Hawking 温度与波谱公式

考虑到时空的对称性,令

$$\begin{aligned}
F_1&=\frac{1}{r\sqrt{\sin\theta}}f_1e^{im\varphi},\\
F_2&=\frac{1}{r\sqrt{\sin\theta}}f_2e^{im\varphi},\\
G_1&=\frac{1}{r\sqrt{\sin\theta}}g_1e^{im\varphi},\\
G_2&=\frac{1}{r\sqrt{\sin\theta}}g_2e^{im\varphi},
\end{aligned}\tag{9}$$

式中 $m$ 为磁量子数.将(9)式代入(8)式,得

$$\frac{\partial f_1}{\partial r}+\frac1r\frac{\partial f_2}{\partial\theta}+\frac{m}{r\sin\theta}f_2=i\sqrt2\mu_0g_1, \tag{10a}$$

$$-r\Sigma\frac{\partial f_2}{\partial r}-2ar\sin\theta\frac{\partial f_2}{\partial\theta}-2r\frac{\partial f_2}{\partial v}+\frac{\partial f_1}{\partial\theta}-\frac{m}{\sin\theta}f_1-\Big(ar\cos\theta+\frac12r\frac{\partial\Sigma}{\partial r}\Big)f_2=i\sqrt2r\mu_0g_2, \tag{10b}$$

$$\frac{\partial g_2}{\partial r}-\frac1r\frac{\partial g_1}{\partial\theta}+\frac{m}{r\sin\theta}g_1=i\sqrt2\mu_0f_2, \tag{10c}$$

$$-r\Sigma\frac{\partial g_1}{\partial r}-2ar\sin\theta\frac{\partial g_1}{\partial\theta}-2r\frac{\partial g_1}{\partial v}-\frac{\partial g_2}{\partial\theta}-\Big(ar\cos\theta+\frac12r\frac{\partial\Sigma}{\partial r}\Big)g_1-\frac{m}{\sin\theta}g_2=i\sqrt2r\mu_0f_1. \tag{10d}$$

对方程(10a)和(10c)求 $\theta$ 的偏导数,对方程(10b)和

(10d)求 $r$ 的偏导数,得

$$\frac{\partial^2 f_1}{\partial\theta\partial r}+\frac{1}{r}\frac{\partial^2 f_2}{\partial\theta^2}-\frac{m\cot\theta}{r\sin\theta}f_2+\frac{m}{r\sin\theta}\frac{\partial f_2}{\partial\theta}=\mathrm{i}\sqrt{2}\mu_0\frac{\partial g_1}{\partial\theta}, \tag{11a}$$

$$\frac{\partial^2 g_2}{\partial\theta\partial r}-\frac{1}{r}\frac{\partial^2 g_1}{\partial\theta^2}-\frac{m\cot\theta}{r\sin\theta}g_1+\frac{m}{r\sin\theta}\frac{\partial g_1}{\partial\theta}=\mathrm{i}\sqrt{2}\mu_0\frac{\partial f_2}{\partial\theta}, \tag{11b}$$

$$r\Sigma\frac{\partial^2 f_2}{\partial r^2}+2ar\sin\theta\frac{\partial^2 f_2}{\partial r\partial\theta}+2r\frac{\partial^2 f_2}{\partial r\partial v}+\left(\Sigma+ar\cos\theta+\frac{3}{2}r\frac{\partial\Sigma}{\partial r}\right)\frac{\partial f_2}{\partial r}+2a\sin\theta\frac{\partial f_2}{\partial\theta}+2\frac{\partial f_2}{\partial v}+\left(a\cos\theta+\frac{1}{2}\frac{\partial\Sigma}{\partial r}+\frac{1}{2}r\frac{\partial^2\Sigma}{\partial r^2}\right)f_2-\frac{\partial^2 f_1}{\partial r\partial\theta}+\frac{m}{\sin\theta}\frac{\partial f_1}{\partial r}+\mathrm{i}\sqrt{2}\left(\mu_0 g_2+r\mu_0\frac{\partial g_2}{\partial r}\right)=0, \tag{12a}$$

$$r\Sigma\frac{\partial^2 g_1}{\partial r^2}+2ar\sin\theta\frac{\partial^2 g_1}{\partial r\partial\theta}+2r\frac{\partial^2 g_1}{\partial r\partial v}+\left(\Sigma+ar\cos\theta+\frac{3}{2}r\frac{\partial\Sigma}{\partial r}\right)\frac{\partial g_1}{\partial r}+2a\sin\theta\frac{\partial g_1}{\partial\theta}+2\frac{\partial g_1}{\partial v}+\left(a\cos\theta+\frac{1}{2}\frac{\partial\Sigma}{\partial r}+\frac{1}{2}r\frac{\partial^2\Sigma}{\partial r^2}\right)g_1+\frac{\partial^2 g_2}{\partial r\partial\theta}+\frac{m}{\sin\theta}\frac{\partial g_2}{\partial r}+\mathrm{i}\sqrt{2}\left(\mu_0 f_1+r\mu_0\frac{\partial f_1}{\partial r}\right)=0, \tag{12b}$$

将方程(10)和(11)代入(12)式,得

$$2\frac{\partial^2 f_2}{\partial v\partial r}+\Sigma\frac{\partial^2 f_2}{\partial r^2}+2a\sin\theta\frac{\partial^2 f_2}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial^2 f_2}{\partial\theta^2}+\frac{2}{r}\frac{\partial f_2}{\partial v}+\frac{1}{r}\left(\Sigma+ar\cos\theta+\frac{3}{2}r\frac{\partial\Sigma}{\partial r}\right)\frac{\partial f_2}{\partial r}+\frac{2a\sin\theta}{r}\frac{\partial f_2}{\partial\theta}+\frac{1}{r}\left(a\cos\theta+\frac{1}{2}\frac{\partial\Sigma}{\partial r}+\frac{1}{2}r\frac{\partial^2\Sigma}{\partial r^2}-\frac{m\cot\theta}{r\sin\theta}-\frac{m^2}{r\sin^2\theta}-2\mu_0^2\right)f_2+\frac{\mathrm{i}\sqrt{2}\mu_0}{r}g_2=0, \tag{13a}$$

$$2\frac{\partial^2 g_1}{\partial v\partial r}+\Sigma\frac{\partial^2 g_1}{\partial r^2}+2a\sin\theta\frac{\partial^2 g_1}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial^2 g_1}{\partial\theta^2}+\frac{2}{r}\frac{\partial g_1}{\partial v}+\frac{1}{r}\left(\Sigma+ar\cos\theta+\frac{3}{2}r\frac{\partial\Sigma}{\partial r}\right)\frac{\partial g_1}{\partial r}+\frac{2a\sin\theta}{r}\frac{\partial g_1}{\partial\theta}+\frac{1}{r}\left(a\cos\theta+\frac{1}{2}\frac{\partial\Sigma}{\partial r}+\frac{1}{2}r\frac{\partial^2\Sigma}{\partial r^2}+\frac{m\cot\theta}{r\sin\theta}-\frac{m^2}{r\sin^2\theta}-2\mu_0^2\right)g_1+\frac{\mathrm{i}\sqrt{2}\mu_0}{r}f_1=0. \tag{13b}$$

引入广义乌龟坐标 $r_*$,$v_*$,和 $\theta_*$[16]

$$r_*=r+\frac{1}{2\kappa(v_0,\theta_0)}\ln[r-r_H(v,\theta)],$$
$$v_*=v-v_0,$$
$$\theta_*=\theta-\theta_0, \tag{14}$$

式中 $\kappa$ 是一可调节的参数,$v_0$ 和 $\theta_0$ 是两个任意的不变参量,它们在乌龟坐标变换下为常数.

由(14)式我们可以得出

$$\mathrm{d}r_*=\left[1+\frac{1}{2\kappa(r-r_H)}\right]\mathrm{d}r-\frac{\dot r_H}{2\kappa(r-r_H)}\mathrm{d}v-\frac{r'_H}{2\kappa(r-r_H)}\mathrm{d}\theta,$$
$$\mathrm{d}v_*=\mathrm{d}v,$$
$$\mathrm{d}\theta_*=\mathrm{d}\theta,$$
$$\frac{\partial}{\partial r}=\left[1+\frac{1}{2\kappa(r-r_H)}\right]\frac{\partial}{\partial r_*},$$
$$\frac{\partial}{\partial v}=\frac{\partial}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},$$
$$\frac{\partial}{\partial\theta}=\frac{\partial}{\partial\theta_*}-\frac{r'_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*}, \tag{15}$$

式中 $\dot r_H=\frac{\partial r_H}{\partial v}$,$r'_H=\frac{\partial r_H}{\partial\theta}$.

将(15)式代入(13)式,当 $r\to r_H(v_0,\theta_0)$,$v\to v_0$,$\theta\to\theta_0$,方程(13)可以简化为

$$\alpha\frac{\partial^2 f_2}{\partial r_*^2}+2\frac{\partial^2 f_2}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 f_2}{\partial r_*\partial\theta_*}-G_0\frac{\partial f_2}{\partial r_*}=0, \tag{16a}$$

$$\alpha\frac{\partial^2 g_1}{\partial r_*^2}+2\frac{\partial^2 g_1}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 g_1}{\partial r_*\partial\theta_*}-G_0\frac{\partial g_1}{\partial r_*}=0, \tag{16b}$$

式中

$$\alpha=\lim_{r\to r_H,v\to v_0,\theta\to\theta_0}\frac{\{-2\dot r_H+\Sigma[2\kappa(r-r_H)+1]-(2a\sin\theta)r'_H\}r^2[2\kappa(r-r_H)+1]+(r'_H)^2}{2\kappa(r-r_H)[2\kappa(r-r_H)+1]r^2}, \tag{17}$$

$$\Omega=\left(a\sin\theta-\frac{r'_H}{r_H^2}\right)_{v\to v_0,\theta\to\theta_0}, \tag{18}$$

$$G_0=-\frac{1}{r_H^2}\left(M-\frac{e^2+q^2}{r_H}+2a(e^2+q^2)\cos\theta\right.$$

$$-\frac{1}{3}\lambda r_H^3-r_H''+\frac{(r_H')^2}{r_H}\Big)_{v\to v_0,\theta\to\theta_0}. \tag{19}$$

将(15)式代入(10a)和(10c)式，得

$$\frac{\partial f_2}{\partial r_*}=\frac{r[2\kappa(r-r_H)+1]}{r_H'}\frac{\partial f_1}{\partial r_*}+\frac{2\kappa(r-r_H)}{r_H'}\frac{\partial f_2}{\partial\theta_*}+\frac{m}{r\sin\theta}\frac{2r\kappa(r-r_H)}{r_H'}f_2-\mathrm{i}\sqrt{2}\mu_0 g_1\frac{2r\kappa(r-r_H)}{r_H'}, \tag{20a}$$

$$\frac{\partial g_1}{\partial r_*}=-\frac{r[2\kappa(r-r_H)+1]}{r_H'}\frac{\partial g_2}{\partial r_*}+\frac{2\kappa(r-r_H)}{r_H'}\frac{\partial g_1}{\partial\theta_*}-\frac{m}{r\sin\theta}\frac{2r\kappa(r-r_H)}{r_H'}g_1+\mathrm{i}\sqrt{2}\mu_0 f_2\frac{2r\kappa(r-r_H)}{r_H'}. \tag{20b}$$

继续对(20)式求 $r_*$，$\theta_*$ 和 $v_*$ 的偏导数，求得 $\frac{\partial^2 f_2}{\partial r_*^2}$，$\frac{\partial^2 f_2}{\partial\theta_*\partial r_*}$，$\frac{\partial^2 f_2}{\partial v_*\partial r_*}$，$\frac{\partial^2 g_1}{\partial r_*^2}$，$\frac{\partial^2 g_1}{\partial\theta_*\partial r_*}$，$\frac{\partial^2 g_1}{\partial v_*\partial r_*}$ 的表达式. 然后将其代入(16)式，当 $r\to r_H(v_0,\theta_0)$，$v\to v_0$ 和 $\theta\to\theta_0$ 时，得分量 $f_1$，$g_2$ 在视界附近满足的方程为

$$\alpha\frac{\partial^2 f_1}{\partial r_*^2}+2\frac{\partial^2 f_1}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 f_1}{\partial r_*\partial\theta_*}-G_0'\frac{\partial f_1}{\partial r_*}=0, \tag{21a}$$

$$\alpha\frac{\partial^2 g_2}{\partial r_*^2}+2\frac{\partial^2 g_2}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 g_2}{\partial r_*\partial\theta_*}-G_0'\frac{\partial g_2}{\partial r_*}=0, \tag{21b}$$

式中

$$G_0'=G_0-\frac{2}{r_H r_H'}(\dot r_H r_H'-r_H\dot r_H')-\frac{2\Omega}{r_H r_H'}[(r_H')^2-r_H r_H''].\tag{22}$$

而

$$r_H''=\frac{\partial^2 r_H}{\partial\theta^2},\qquad \dot r'=\frac{\partial^2 r_H}{\partial v\partial\theta}.$$

适当选择参量 $\kappa$，使得

$$\kappa=\frac{1}{2r_H}\frac{M/r_H^2-a\cos\theta-(e^2+q^2)/r_H^3+2a(e^2+q^2)\cos\theta/r_H^2-\frac{1}{3}\lambda r_H-(r_H')^2/r_H^3}{M/r_H^2+a\cos\theta-(e^2+q^2)/r_H^3+2a(e^2+q^2)\cos\theta/r_H^2+\frac{1}{6}\lambda r_H+(r_H')^2/2r_H^3}\Bigg|_{v\to v_0,\ \theta\to\theta_0}, \tag{23}$$

则 $\alpha=1$. 方程(16)和(21)可以简化为

$$\frac{\partial^2 f_1}{\partial r_*^2}+2\frac{\partial^2 f_1}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 f_1}{\partial r_*\partial\theta_*}-G_0'\frac{\partial f_1}{\partial r_*}=0, \tag{24a}$$

$$\frac{\partial^2 f_2}{\partial r_*^2}+2\frac{\partial^2 f_2}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 f_2}{\partial r_*\partial\theta_*}-G_0\frac{\partial f_2}{\partial r_*}=0, \tag{24b}$$

$$\frac{\partial^2 g_1}{\partial r_*^2}+2\frac{\partial^2 g_1}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 g_1}{\partial r_*\partial\theta_*}-G_0\frac{\partial g_1}{\partial r_*}=0, \tag{24c}$$

$$\frac{\partial^2 g_2}{\partial r_*^2}+2\frac{\partial^2 g_2}{\partial v_*\partial r_*}+2\Omega\frac{\partial^2 g_2}{\partial r_*\partial\theta_*}-G_0'\frac{\partial g_2}{\partial r_*}=0. \tag{24d}$$

从(24)式可以看出，Dirac 方程对应波函数的 4 个分量除了对 $r_*$ 的一阶偏导数的系数不同外，其他系数相同. 我们用第一个分量进行讨论.

对方程(24a)进行分离变量，

$$f_1=\phi_1(r_*)\mathrm{e}^{-\mathrm{i}\omega v_*+\mathrm{i}k_\theta\theta_*}. \tag{25}$$

容易证明，方程(24a)解得的径向分量分别为

$$(\phi_1)_{\text{in}}=\mathrm{e}^{-\mathrm{i}\omega v_*},$$
$$(\phi_1)_{\text{out}}=\mathrm{e}^{-\mathrm{i}\omega v_*+G_o'r_*+\mathrm{i}(2\omega-2\Omega k_\theta)r_*}, \tag{26}$$

式中 $(\phi_1)_{\text{in}}$ 表示入射波，而 $(\phi_1)_{\text{out}}$ 表示出射波. 在视界 $r_H$ 附近，$r_*=\frac{1}{2\kappa}\ln(r-r_H)$，$(\phi_1)_{\text{out}}$ 可以写成如下形式：

$$(\phi_1)_{\text{out}}=\mathrm{e}^{-\mathrm{i}\omega v_*}(r-r_H)^{G_0'/2\kappa}(r-r_H)^{\mathrm{i}(\omega-\Omega k_\theta)/\kappa}. \tag{27}$$

该出射波函数在视界面上奇异，只能描述视界外的出射粒子. 为此，我们把 $(\phi_1)_{\text{out}}$ 解析延拓到视界内. 以奇点 $r=r_H$ 为圆心，以 $|r-r_H|$ 为半径，沿下半复平面作解析延拓，转动 $(-\pi)$ 角[16]，此时

$$(r-r_H)\to|r-r_H|\mathrm{e}^{-\mathrm{i}\pi}=(r-r_H)\mathrm{e}^{-\mathrm{i}\pi}. \tag{28}$$

于是得到视界内的出射波为

$$(\phi_1)_{\text{out}}\to(\phi_1')_{\text{out}}=\mathrm{e}^{-\mathrm{i}\omega v_*+G_o'r_*+\mathrm{i}(2\omega-2\Omega k_\theta)r_*}\mathrm{e}^{-\mathrm{i}\pi G_o'/2\kappa}\mathrm{e}^{\pi(\omega-\Omega k_\theta)/\kappa}. \tag{29}$$

出射波在视界面上的相对散射概率为

$$\left|\frac{(\phi_1)_{\rm out}}{(\phi_1')_{\rm out}}\right|^2 = e^{-2\pi(\omega-\Omega k_\theta)/\kappa}. \tag{30}$$

其 Hawking 辐射谱为

$$N_\omega = (e^{\frac{\omega-\Omega k_\theta}{T}}+1)^{-1}, \tag{31}$$

式中

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi}\frac{1}{2r_H}\frac{M/r_H^2 - a\cos\theta - (e^2+q^2)/r_H^3 + 2a(e^2+q^2)\cos\theta/r_H^2 - \frac{1}{3}\lambda r_H - (r_H')^2/r_H^3}{M/r_H^2 + a\cos\theta - (e^2+q^2)/r_H^3 + 2a(e^2+q^2)\cos\theta/r_H^2 + \frac{1}{6}\lambda r_H + (r_H')^2/2r_H^3}\Bigg|_{v\to v_0,\theta\to\theta_0} \tag{32}$$

为 Hawking 温度.

## 4. 讨 论

1) 从(24)式及其后的讨论可以看出,对应波函数的 4 个分量 $f_1$, $f_2$, $g_1$, $g_2$ 满足类似的波动方程,4 个分量都有辐射,且具有相同的波谱公式.对于无穷远处的观测者而言,既可观测到自旋向上的辐射粒子,又可观测到自旋向下的辐射粒子.其温度表达式(32)在令 $e=q=\lambda=0$ 的情况下,退化为直线加速动态黑洞背景下的热辐射温度公式[16].

2) 在波谱公式中出现了物理意义不明的两个量 $k_\theta$ 和 $\Omega$. $k_\theta$ 的物理意义从(25)式可以看出其对应为沿 $\theta$ 方向波矢.为了弄清楚 $\Omega$ 的物理意义,我们可以用文献[17,18]中提供的方法进行讨论.将度规(3)式写成如下一般形式:

$$ds^2 = g_{00}dv^2 + 2g_{01}dvdr + 2g_{02}dvd\theta + g_{22}d\theta^2 + g_{33}d\varphi^2. \tag{33}$$

引入坐标变换[19]

$$R = r - r_H,$$
$$dR = dr - \dot r_H dv - r_H' d\theta. \tag{34}$$

度规(33)式变为

$$ds^2 = (g_{00}+2\dot r_H g_{01})dv^2 + 2g_{01}dvdR + 2(r_H'g_{01}+g_{02})dvd\theta + g_{22}d\theta^2 + g_{33}d\varphi^2. \tag{35}$$

(35)式可以写成

$$ds^2 = \left(g_{00}+2\dot r_H g_{01} - \frac{(r_H'g_{01}+g_{02})^2}{g_{22}}\right)dv^2 + 2g_{01}dvdR + g_{22}(-\Omega dv + d\theta)^2 + g_{33}d\varphi^2, \tag{36}$$

其中

$$\Omega = -\frac{r_H'g_{01}+g_{02}}{g_{22}} = a\sin\theta - \frac{r_H'}{r^2} \tag{37}$$

为类似于旋转黑洞而定义的拖曳角速度[16].如果令

$$\frac{d\theta}{dv} = \Omega, \tag{38}$$

则(37)式将变为拖曳系中的线元

$$ds^2 = \left(g_{00}+2\dot r_H g_{01} - \frac{(r_H'g_{01}+g_{02})^2}{g_{22}}\right)dv^2 + 2g_{01}dvdR + g_{33}d\varphi^2. \tag{39}$$

因此,在引入坐标变换(34)式后,$\Omega$ 为类似于旋转黑洞而定义的拖曳角速度.

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# Hawking radiation of Dirac particles in a nonuniformly rectilinearly accelerating black hole with electric and magnetic charge *

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Abstract

The Hawking radiation of Dirac particles on event horizon of nonuniformly rectilinearly accelerating black hole with electric and magnetic charge is studied in this paper. First, we construct the symmetrized null tetrad from which the spin coefficients and Dirac equation are derived. Next, by proposing a generalized tortoise coordinate transformation, the Dirac equation is decoupled successfully near the event horizon surface. Finally, following the method of Damour and Ruffini, the temperature on the horizon surface and the thermal spectrum formula of Dirac particles are obtained. The result is also discussed.

**Keywords**: accelerating non-stationary black hole, Dirac particles, Dirac equation, Hawking radiation

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