第 52 卷 第 10 期 2003 年 10 月
1000-3290/2003/52(10)/2379-07

物 理 学 报
ACTA PHYSICA SINICA

Vol.52, No.10, October, 2003

非线性耦合标量场方程的新双周期解(Ⅱ)*

李德生[1)2)†] 张鸿庆[1)]

[1)] (大连理工大学应用数学系, 大连 116024)

[2)] (沈阳工业大学理学院, 沈阳 110023)

(2002 年 7 月 29 日收到; 2002 年 12 月 22 日收到修改稿)

基于具有双周期解的常微分方程,提出了一种构造非线性微分方程双周期解的新方法,在计算机符号软件帮助下方法可实现机械化. 应用此方法于非线性耦合标量场方程,得到了该方程的大量的新精确解.

关键词:非线性耦合标量场方程, 双周期解, 精确解

PACC:0340K, 0290

1. 引 言

本文继续研究非线性耦合标量场方程[1]

$$\sigma_{xx} = -\sigma + \sigma^3 + d\sigma^2\sigma, \tag{1}$$

$$\rho_{xx} = (f-d)\rho + \lambda\rho^3 + d\sigma^2\rho \tag{2}$$

的精确解问题. 对该方程的精确解的研究已有大量的结果[1—5]. 本文给出一个新的,可以求出方程多种双周期解(或雅可比椭圆函数解)的有效方法,该方法的一个优点是它避开了直接使用雅可比椭圆函数,从而,可在计算机符号软件的帮助下实现机械化求解. 利用该法再一次的讨论方程(1),(2)的新精确解. 我们就本文的有关问题进行一个简短的讨论,并给出一些结论和一个公开问题.

2. 构造非线性微分方程双周期解新方法

对于给定的非线性常微分方程

$$F(y, y_t, y_{tt}, \cdots) = 0, \tag{3}$$

通过使用新变量 $\omega = \omega(t)$,假设方程(3)具有如下形式的解

$$y(t) = y(\omega(t)) = A_0 + \sum_{i=1}^{n}[A_i\omega^i + B_i\omega^{-i}], \tag{4}$$

这里 $\omega = \omega(t)$ 满足

$$(\omega')^2 = f(\omega), \tag{5}$$

或它的等价形式

$$\omega'' = g(\omega), \tag{6}$$

这里 A_0, A_j, B_j $(j = 1, 2, \cdots, n)$ 为待定常数. 定义多项式次数函数为 $D(y(\omega)) = n$,我们有

$$D\left(y^p(\omega)\left(\frac{\mathrm{d}^s y(\omega)}{\mathrm{d}t^s}\right)^q\right) = np + q(n+s), \tag{7}$$

这样,可通过平衡最高阶线性项和非线性项来确定 n. 将(4)式代入到方程(3)中得到一关于 ω^i $(i = 0, \pm 1, \pm 2, \cdots, \pm n)$ 的多项式方程. 令它们的系数为 0,得到一关于未知数 A_j $(j = 0, 1, 2, \cdots, n)$ 和 B_j $(j = 1, 2, \cdots, n)$ 的代数方程组. 求解此代数方程组即可得到方程(3)的双周期解. 方程(5)和(6)在下面的求解过程中扮演着十分重要的角色,这里的函数 f, g 有着多种取法[6]. 下文中仅考虑其中的一种取法,即

$$f(\omega) = (1-\omega^2)(1-m^2\omega^2), \tag{8a}$$

$$g(\omega) = -(1+m^2)\omega + 2m^2\omega^3, \tag{8b}$$

这里 m 为雅可比椭圆函数的模数[6].

方程(5)或(6)的解为

$$\omega(t) = \mathrm{sn}(t, m), \tag{9}$$

且具有性质

$$\mathrm{ns}(t, m) = \frac{1}{\mathrm{sn}(t, m)} \tag{10}$$

注 1 容易看出当 $B_i = 0$ 或 $A_i = 0$ 时,文献[7—11]的方法是我们方法的特殊情形. 当 $A_iB_i \neq 0$ 时,我们的方法可以得到新的双周期解.

---

* 国家"973"项目(批准号:G1998030600)和国家自然科学基金(批准号:10072013)资助的课题.

† E-mail: deshengli-868@yahoo.com.cn

3. 非线性耦合标量场方程的新双周期解

对方程(1),(2)可假设它的解为

$$\sigma = A_0 + A_1\omega + B_1\omega^{-1},$$
$$\rho = C_0 + C_1\omega + D_1\omega^{-1},$$

其中

$$\frac{\mathrm{d}^2\omega}{\mathrm{d}x^2} = -(1+m^2)\omega + 2m^2\omega^3,$$
$$\left(\frac{\mathrm{d}\omega}{\mathrm{d}x}\right)^2 = (1-\omega^2)(1-m^2\omega^2).$$

通过简单的计算得到

$$\begin{aligned}
&-\sigma + \sigma^3 + d\rho^2\sigma - \sigma_{xx}\\
=& -A_0 + A_0^3 + 6A_0A_1B_1 + dA_0C_0^2\\
&+ 2dA_0C_1D_1 + 2dC_0D_1A_1 + 2dC_0C_1B_1\\
&+ [-A_1 + 3A_0^2A_1 + 3A_1B_1^2 + 2dC_0C_1A_0\\
&+ dA_1C_0^2 + dC_1^2B_1 + 2dC_1D_1A_1 + A_1(1+m^2)]\omega\\
&+ [-B_1 + 3A_0^2B_1 + 3A_1^2B_1 + 2dC_0A_0D_1\\
&+ dB_1C_0^2 + 2dC_1D_1B_1 + dD_1^2A_1 + B_1(1+m^2)]\omega^{-1}\\
&+ [3A_0A_1^2 + dC_1^2A_0 + 2dC_0C_1A_1]\omega^2\\
&+ [3B_1^2A_0 + dA_0D_1^2 + 2dC_0D_1B_1]\omega^{-2}\\
&+ [A_1^3 + dC_1^2A_1 - 2m^2A_1]\omega^3\\
&+ [B_1^3 + dD_1^2B_1 - 2B_1]\omega^{-3}\\
=& 0,
\end{aligned}\tag{11}$$

$$\begin{aligned}
&(f-d)\rho + \lambda\rho^3 + d\rho\sigma^2 - \rho_{xx}\\
=& (f-d)C_0 + \lambda C_0^3 + 6\lambda C_1C_0D_1\\
&+ dC_0A_0^2 + 2dA_1B_1C_0 + 2dA_0B_1C_1\\
&+ 2dA_0A_1D_1 + [(f-d)]C_1\\
&+ 3\lambda(C_0^2C_1 + C_1^2D_1) + 2dA_0A_1C_0\\
&+ 2dA_1B_1C_1 + dA_0^2C_1 + dA_1^2D_1\\
&+ C_1(1+m^2)]\omega + [(f-d)D_1\\
&+ 3\lambda(C_0^2D_1 + C_1D_1^2) + 2dA_0B_1C_0 + dC_1B_1^2\\
&+ dA_0^2D_1 + 2dA_1B_1D_1 + D_1(1+m^2)]\omega^{-1}\\
&+ [3\lambda C_0C_1^2 + dA_1^2C_0 + 2dA_0A_1C_1]\omega^2\\
&+ [3\lambda C_0D_1^2 + dC_0B_1^2 + 2dA_0B_1D_1]\omega^{-2}\\
&+ [\lambda C_1^3 + dC_1A_1^2 - 2m^2C_1]\omega^3\\
&+ [\lambda D_1^3 + dD_1B_1^2 - 2D_1]\omega^{-3}\\
=& 0,
\end{aligned}\tag{12}$$

令方程(11),(12)中的系数为零,则可得如下代数方程组:

$$\begin{aligned}
&-A_0 + A_0^3 + 6A_0A_1B_1 + dA_0C_0^2\\
&+ 2dA_0C_1D_1 + 2dC_0D_1A_1 + 2dC_0C_1B_1\\
=& 0,
\end{aligned}\tag{13a}$$

$$\begin{aligned}
&-A_1 + 3A_0^2A_1 + 3A_1B_1^2 + 2dC_0C_1A_0\\
&+ dA_1C_0^2 + dC_1^2B_1 + 2dC_1D_1A_1 + A_1(1+m^2)\\
=& 0,
\end{aligned}\tag{13b}$$

$$\begin{aligned}
&-B_1 + 3A_0^2B_1 + 3A_1^2B_1 + 2dC_0A_0D_1\\
&+ dB_1C_0^2 + 2dC_1D_1B_1 + dD_1^2A_1 1\\
&+ B_1(1+m^2)\\
=& 0
\end{aligned}\tag{13c}$$

$$3A_0A_1^2 + dC_1^2A_0 + 2dC_0C_1A_1 = 0,\tag{13d}$$

$$3B_1^2A_0 + dA_0D_1^2 + 2dC_0D_1B_1 = 0,\tag{13e}$$

$$A_1^3 + dC_1^2A_1 - 2m^2A_1 = 0,\tag{13f}$$

$$B_1^3 + dD_1^2B_1 - 2B_1 = 0,\tag{13g}$$

$$\begin{aligned}
&(f-d)C_0 + \lambda C_0^3 + 6\lambda C_1C_0D_1 + dC_0A_0^2\\
&+ 2dA_1B_1C_0 + 2dA_0B_1C_1 + 2dA_0A_1D_1\\
=& 0,
\end{aligned}\tag{13h}$$

$$\begin{aligned}
&(f-d)C_1 + 3\lambda(C_0^2C_1 + C_1^2D_1) + 2dA_0A_1C_0\\
&+ 2dA_1B_1C_1 + dA_0^2C_1 + dA_1^2D_1 + C_1(1+m^2)\\
=& 0,
\end{aligned}\tag{13i}$$

$$\begin{aligned}
&(f-d)D_1 + 3\lambda(C_0^2D_1 + C_1D_1^2) + 2dA_0B_1C_0\\
&+ dC_1B_1^2 + dA_0^2D_1 + 2dA_1B_1D_1\\
&+ D_1(1+m^2)\\
=& 0,
\end{aligned}\tag{13j}$$

$$3\lambda C_0C_1^2 + dA_1^2C_0 + 2dA_0A_1C_1 = 0,\tag{13k}$$

$$3\lambda C_0D_1^2 + dC_0B_1^2 + 2dA_0B_1D_1 = 0,\tag{13l}$$

$$\lambda C_1^3 + dC_1A_1^2 - 2m^2C_1 = 0,\tag{13m}$$

$$\lambda D_1^3 + dD_1B_1^2 - 2D_1 = 0.\tag{13n}$$

解此代数方程组,可得如下十六种解.

情形 1 当 $m = -6, f - d - 1 = 0$ 时,上述方程组有如下解:

$$\begin{aligned}
&A_0 = C_0 = 0,\\
&A_1 = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}},\\
&B_1 = \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}},\\
&C_1 = m\sqrt{\frac{2 - 2d}{\lambda - d^2}},\\
&D_1 = \sqrt{\frac{2 - 2d}{\lambda - d^2}},
\end{aligned}\tag{14}$$

$$A_0=C_0=0,\quad A_1=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{15}$$

$$A_0=C_0=0,\quad A_1=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=-m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=-\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{16}$$

$$A_0=C_0=0,\quad A_1=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=-m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=-\sqrt{\frac{2-2d}{\lambda-d^2}}.\tag{17}$$

情形 2　当 $m=6, d-f-1=0$ **时,上述方程组有如下解:**

$$A_0=C_0=0,\quad A_1=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=-m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{18}$$

$$A_0=C_0=0,\quad A_1=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=-\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{19}$$

$$A_0=C_0=0,\quad A_1=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=-m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{20}$$

$$A_0=C_0=0,\quad A_1=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=-\sqrt{\frac{2-2d}{\lambda-d^2}}.\tag{21}$$

情形 3　当 $m=\frac{6\lambda-8d+2d^2}{\lambda-d^2}$, $f-d+1+m^2+\frac{(8d\lambda-6\lambda-2d^2)m}{\lambda-d^2}=0$ **时,上述方程组有如下解:**

$$A_0=C_0=0,\quad A_1=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{22}$$

$$A_0=C_0=0,\quad A_1=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad B_1=-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}},\quad C_1=m\sqrt{\frac{2-2d}{\lambda-d^2}},\quad D_1=\sqrt{\frac{2-2d}{\lambda-d^2}},\tag{23}$$

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = -\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = -m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = -\sqrt{\frac{2 - 2d}{\lambda - d^2}},
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = -m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = -m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = -\sqrt{\frac{2 - 2d}{\lambda - d^2}}.
\end{aligned} \tag{25}
$$

情形 4 当 $m = \frac{8d - 6\lambda - 2d^2}{\lambda - d^2}$，$d - f - 1 - m^2 + \frac{(6\lambda - 8d\lambda + 2d^2)m}{\lambda - d^2} = 0$ 时，上述方程组有如下解：

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = -m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = -\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = -m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = \sqrt{\frac{2 - 2d}{\lambda - d^2}},
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = -m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = \sqrt{\frac{2 - 2d}{\lambda - d^2}},
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = -m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = -\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = -\sqrt{\frac{2 - 2d}{\lambda - d^2}},
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
& A_0 = C_0 = 0, \\
& A_1 = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& B_1 = \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}, \\
& C_1 = m\sqrt{\frac{2 - 2d}{\lambda - d^2}}, \\
& D_1 = -\sqrt{\frac{2 - 2d}{\lambda - d^2}}.
\end{aligned} \tag{29}
$$

显然，方程组还存在 $A_0 = C_0 = A_1 = C_1 = 0$ 而 $B_1 \neq 0$，$D_1 \neq 0$ 或 $A_0 = C_0 = B_1 = D_1 = 0$ 而 $A_1 \neq 0$，$C_1 \neq 0$ 及 $A_0 = A_1 = B_1 = 0$ 或 $C_0 = C_1 = D_1 = 0$ 等的解，限于篇幅，就不赘述了.

由此及(9)，(10)式得了方程(1)，(2)的如下十六种新的双周期解：

$$
\begin{aligned}
\sigma_1 &= -6\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6) \\
&\quad + \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{ns}(x, -6), \\
\rho_1 &= -6\sqrt{\frac{2 - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6) \\
&\quad + \sqrt{\frac{2 - 2d}{\lambda - d^2}}\mathrm{ns}(x, -6), \\
\sigma_2 &= 6\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6) \\
&\quad - \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{ns}(x, -6), \\
\rho_2 &= -6\sqrt{\frac{2 - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6) \\
&\quad + \sqrt{\frac{2 - 2d}{\lambda - d^2}}\mathrm{ns}(x, -6), \\
\sigma_3 &= 6\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6) \\
&\quad - \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\mathrm{ns}(x, -6), \\
\rho_3 &= 6\sqrt{\frac{2 - 2d}{\lambda - d^2}}\mathrm{sn}(x, -6)
\end{aligned}
$$

$$-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,-6),$$

$$\sigma_4=-6\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,-6)+\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,-6),$$

$$\rho_4=6\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,-6)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,-6),$$

这里要求 $d-f=-1$.

$$\sigma_5=6\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\rho_5=-6\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)+\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\sigma_6=6\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\rho_6=6\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\sigma_7=-6\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)+\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\rho_7=-6\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\sigma_8=-6\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)+\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

$$\rho_8=6\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,6)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,6),$$

这里要求 $d-f=1$.

$$\sigma_9=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)+\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\rho_9=m\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)+\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\sigma_{10}=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\rho_{10}=m\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)+\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\sigma_{11}=m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\rho_{11}=-m\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\sigma_{12}=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)+\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\rho_{12}=-m\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)-\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

这里要求 $m=\frac{6\lambda-8d+2d^2}{\lambda-d^2}$, $f-d+1+m^2+\frac{(8d\lambda-6\lambda-2d^2)m}{\lambda-d^2}=0$.

$$\sigma_{13}=-m\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)-\sqrt{\frac{2\lambda-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\rho_{13}=-m\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{sn}(x,m)+\sqrt{\frac{2-2d}{\lambda-d^2}}\,\mathrm{ns}(x,m),$$

$$\sigma_{14} = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) + \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

$$\rho_{14} = -m\sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) + \sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

$$\sigma_{15} = -m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) - \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

$$\rho_{15} = m\sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) - \sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

$$\sigma_{16} = m\sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) + \sqrt{\frac{2\lambda - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

$$\rho_{16} = m\sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{sn}(x, m) - \sqrt{\frac{2 - 2d}{\lambda - d^2}}\,\mathrm{ns}(x, m),$$

这里要求 $m = \frac{8d - 6\lambda - 2d^2}{\lambda - d^2}$，$d - f - 1 - m^2 + \frac{(6\lambda - 8d\lambda + 2d^2)m}{\lambda - d^2} = 0$.

至此，我们已求得了方程(1)，(2)的十六组双周期解.此外，在退化的情形下，还可得到八组孤波解及八组三角函数解，将所得的结果与提出偏微分方程的雅可比椭圆函数求解法的文献[7—11]相比较，我们不仅可以给出单个雅可比椭圆函数所表达的解，而且还可以给出它们的组合解.此外，这里使用的方法具有一个显著的优点：在整个求解的过程中，雅可比椭圆函数的出现是间接的，这避开了计算机的数学软件中没有雅可比椭圆函数这一缺点.从而，该方法也可以在计算机的符号计算软件的帮助下，实现一类偏微分方程的雅可比椭圆函数求解的数学机械化.

4. 有关问题的讨论及结论

本文基于文献[6]的一个微分方程，提出了一个构造微分方程双周期解的新方法.而具有此类性质的微分方程有许多种，就文献[6]中给出的结果来看，就有 12 种，我们可选取 6 种来构造相应的算法.这样，可以直接算出，利用我们的算法，可给出非线性耦合标量场方程的解大约至少有 $16\times3\times6$(双周期解) $+8\times3\times6$(孤波解) $+8\times3\times6$(三角函数解) $=576$ 个.由此可见此方法是一个十分有效的方法，同时也证明非线性耦合标量场方程具有丰富的解的结构.此外，该方程的解的结构自然的提示我们猜测，它不仅具有上述形式的由两个不同的函数的线性组合解，是否也应具有更多的不同的函数的线性组合解呢，那么，该如何证实这一问题呢，其算法又是什么样的呢？

总之，本文提出了一个构造微分方程双周期解的新方法，并首次利用该方法于非线性耦合标量场方程，得到了该方程的大量的新双周期解和精确解.显然此方法也适用于非线性偏微分方程的行波解的计算.

[1] Rajaraman R. 1979 *Phys. Rev. Lett.* **42** 200
[2] Wang X Y *et al* 1993 *Phys. Lett.* A **173** 30
[3] Fan E G *et al* 1998 *Acta. Phys. Sin.* **47** 1064 (in Chinese)[范恩贵等 1998 物理学报 **47** 1064]
[4] Cao D B 2002 *Phys. Lett.* A **296** 27
[5] Li D S *et al* 2003 *Acta. Phys. Sin.* **52** 2373(in Chinese)[李德生等 2003 物理学报 **52** 2373]
[6] Liu S K，Liu S D 2000 *Nonlinear Equations in Physics* (Beijing: Peking University Press) (in Chinese)[刘式适，刘式达 2000 物理学中的非线性方程，北京大学出版社].
[7] Liu S K *et al* 2001 *Acta. Phys. Sin.* **50** 2068 (in Chinese)[刘式适等 2001 物理学报 **50** 2068]
[8] Liu S K *et al* 2002 *Acta. Phys. Sin.* **50** 10(in Chinese)[刘式适等 2002 物理学报 **51** 10]
[9] Liu S K *et al* 2002 *Acta. Phys. Sin.* **51** 718 (in Chinese)[刘式适等 2002 物理学报 **51** 718]
[10] Liu S K *et al* 2001 *Phys. Lett.* A **289** 69
[11] Fu Z *et al* 2001 *Phys. Lett.* A **290** 72

The new doubly-periodic solutions for nonlinear coupled scalar field equations(Ⅱ)*

Li De-Sheng[1)2)] Zhang Hong-Qing[1)]
[1)] (*Department of Applied Mathematics*, *Dalian University of Technology*, *Dalian* 116024, *China*)
[2)] (*Science School of Shenyang University of Technology*, *Shenyang* 110023, *China*)
(Received 29 July 2002; revised manuscript received 22 December 2002)

Abstract

Based on an ordinary differential equation which possesses doubly-periodic solutions, a new method for constructing doubly-periodic solutions for differential equations and its algorithm are proposed. This method can be carried out in computer by the aid of symbolic computation. A lot of new exact solutions of the nonlinear coupled scalar field equations are found by using this method.

Keywords: nonlinear coupled scalar field equations, doubly-periodic solutions, exact solutions
PACC: 0340K, 0290

* Project supported by the National Key Basic Research Special Foundation of China (Grant No. G1998030600) and the National Natural Science Foundation of China (Grant No. 10072013).