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Hybrid-Lattice 系统和 Ablowitz-Ladik-Lattice 系统的新解探索*

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对 tanh 函数方法进行了对称延拓,并拓广了它的应用范围,将其应用于非线性离散系统的求解.研究了 Hybrid-Lattice 系统和 Ablowitz-Ladik-Lattice 系统.得到了方程的孤波解和周期波解.

关键词:改进的 tanh 函数方法,离散系统,孤波解,周期波解

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1. 引　言

寻找非线性物理模型的精确解长期以来一直是数学家和物理学家研究的重要课题.众所周知,每一个非线性偏微分方程存在无穷多解,为了寻找其精确解,在实践中人们建立起了许多行之有效的求解方法,如齐次平衡法、Darboux 变换、标准的 Painlevé 截断法、相似约化法、分离变量法、Hirota 变换、tanh 函数法等[1—8].但这些方法大多用于连续非线性系统的求解,而较少地用于求解离散非线性系统问题[12—22].而由于很多物理、化学、生物、经济问题的数学模型本身是离散的,再加上对微分方程进行数值计算时首先要将它离散化.因此离散方程的求解,特别是非线性离散系统的精确解的获得就有十分重要的理论应用价值.tanh 函数方法是一种十分有效的寻找方程孤波解的方法,被广泛应用于非线性连续问题的求解,它的主要思想是将非线性演化方程转化为一个非线性代数方程组,然后利用吴消元法求解此方程组,从而获得非线性演化方程的精确解.最近 Baldwin 等提出用该方法求非线性离散系统得到了类冲击波解[9].本文改进 tanh 函数方法,将求和范围 j 从原来的 0 到 m 延拓至 $-m$ 至 m,从而可得到更多的孤波解,此外由 tanh 函数和 tan 函数的性质可知,若将 tanh 函数换成 tan 函数,也可得到系统的周期波解.为了说明以上观点,本文用以上方法求解典型的非线性离散的 Hybrid-Lattice 系统和 Ablowitz-Ladik-Lattice 系统[10,11].

2. 改进的 tanh 方法

为方便对改进的 tanh 法的论述,以(2+1)维非线性差分微分方程为例.设非线性差分微分方程(DDE)为

$$\begin{aligned}\Delta(&u_{n+p_1}(x,t),u_{n+p_2}(x,t),\cdots,u_{n+p_k}(x,t),\\&u'_{n+p_1}(x,t),u'_{n+p_2}(x,t),\cdots,u'_{n+p_k}(x,t),\cdots,\\&u^{(r)}_{n+p_1}(x,t),u^{(r)}_{n+p_2}(x,t),\cdots,u^{(r)}_{n+p_k}(x,t))=0,\end{aligned}\quad(1)$$

式中 $u(n)$是连续变量 x,t 和离散变量 n 的函数,p_k 是离散变量 n 的分量.

首先对方程(1)进行行波约化,令

$$u_n(x,t,n)=u_n(\xi),$$
$$\xi=d_1n+c_1x+c_2t+\xi_0,\quad(2)$$

式中 d_1,c_1,c_2,ξ_0 是常数.引入变量 $T_n=\tanh(\xi)$,利用双曲正切函数的性质

$$\frac{\mathrm{d}}{\mathrm{d}\xi}=(1-T_n^2)\frac{\mathrm{d}}{\mathrm{d}T_n}$$

和

$$\tanh(x+y)=\frac{\tanh(x)+\tanh(y)}{1+\tanh(x)\tanh(y)}.\quad(3)$$

有

$$T_{n+p_s}=\frac{T_n+\tanh(\phi_s)}{1+T_n\tanh(\phi_s)},\quad(4)$$

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式中 $\phi_s = p_{s1}d_1 + p_{s2}d_2 + \cdots + p_{sQ}d_Q$，$d_1, d_2, \cdots, d_Q$ 是常数. 因而可将(1)式化成变量为 T_n 的方程.

$$\Delta(u_{n+p_1}(T_n), u_{n+p_2}(T_n), \cdots, u_{n+p_k}(T_n), u'_{n+p_1}(T_n), u'_{n+p_2}(T_n), \cdots, u'_{n+p_k}(T_n), \cdots, u^{(r)}_{n+p_1}(T_n), u^{(r)}_{n+p_2}(T_n), \cdots, u^{(r)}_{n+p_k}(T_n)) = 0. \quad (5)$$

其次，假定方程(5)的形式解为 T_n 的 m 阶多项式并表示为

$$u_n(T_n) = \sum_{j=-m}^{m} A_j T_n^j,$$

$$u_{n+p_s}(T_n) = \sum_{j=-m}^{m} A_j \left(\frac{T_n + \tanh(\phi_s)}{1 + T_n \tanh\phi_s}\right)^j, \quad (6)$$

其中系数 $A_j (j = 1, 2, \cdots, m)$ 是待定参数. 利用齐次平衡原则确定 m，然后将(6)式代入方程(5)，合并 T_n 的同次幂系数并取为零，得到参数 A_j 及 $c_1, c_2, \tanh(d_1), \tanh(d_2), \cdots, \tanh(d_Q)$ 的非线性代数方程组.

最后通过联立求解上述代数方程组，可求得相关参数并给出非线性差分微分方程的孤波解.

依照同样的思路，如果将 tanh 函数换成 tan 函数，引入变量 $T_n = \tan(\xi)$，利用正切函数的性质

$$\frac{\mathrm{d}}{\mathrm{d}\xi} = (1 + T_n^2)\frac{\mathrm{d}}{\mathrm{d}T_n}$$

和

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}. \quad (7)$$

有

$$T_{n+p_s} = \frac{T_n + \tan(\phi_s)}{1 - T_n \tan(\phi_s)}. \quad (8)$$

同样可将(1)式化成(5)式的形式. 然后进行 tan 函数的级数展开，和 tanh 同样过程不难得到非线性离散系统的三角函数周期波解.

3. (2+1)维 Hybrid-Lattice 系统的孤波解和周期波解

作为一种典型的非线性差分微分方程，离散型(2+1)维 Hybrid-Lattice 方程可写为

$$\dot{u}_n(t) = (1 + \alpha u_n + \beta u_n^2)(u_{n-1} - u_{n+1}), \quad (9)$$

式中 α, β 是常数.

3.1. (2+1)维 Hybrid-Lattice 系统的孤波解

用 tanh 函数对方程(9)进行行波约化和参量变换可得

$$c_2(1 - T_n^2)u'_n - (1 + \alpha u_n + \beta u_n^2) \times (u_{n-1} - u_{n+1}) = 0, \quad (10)$$

式中所有变量都是 $T_n = \tanh(\xi)$ 的函数. 根据齐次平衡原则进行领头项分析可知，$m = 1$. 设方程的形式解为

$$u_n(T_n) = a_0 + a_1 T_n + a_2 T^{-1},$$

$$u_{n+p_s}(T_n) = a_0 + a_1 \frac{T_n + \tanh(\phi_s)}{1 + T_n \tanh(\phi_s)} + a_2\left(\frac{T_n + \tanh(\phi_s)}{1 + T_n \tanh(\phi_s)}\right)^{-1}, \quad (11)$$

式中 a_0, a_1, a_2 是待定常数.

将(11)式代入方程(10)，合并 T_n 的同次幂项系数并取为零可得方程组

$$2\beta a_1^2 a_2 \tanh(d_1)^3 + 2\tanh(d_1)\beta a_1^3 - c_2 a_1 \tanh(d_1)^2 = 0,$$

$$2\tanh(d_1)\alpha a_1^2 + 2\alpha a_2 a_1 \tanh(d_1)^3 + 4\beta a_0 a_2 a_1 \tanh(d_1)^3 + 4\tanh(d_1)\beta a_0 a_1^2 = 0,$$

$$2a_2\tanh(d_1)^3 + c_2 a_1 \tanh(d_1)^4 + 2\beta a_0^2 a_2 \tanh(d_1)^3 + 4\beta a_2^2 a_1 \tanh(d_1)^3 - 2\tanh(d_1)\beta a_1^3 + c_2 a_2 \tanh(d_1)^2 - 2\beta a_1^2 a_2 \tanh(d_1)^3 + 2\alpha a_0 a_2 \tanh(d_1)^3 + 2\tanh(d_1)\beta a_1^2 a_2 + c_2 a_1 \tanh(d_1)^2 + c_2 a_1 + 2\tanh(d_1)\alpha a_0 a_1 + 2a_1\tanh(d_1) - 2\beta a_1^3 \tanh(d_1)^3 + 2\tanh(d_1)\beta a_0^2 a_1 = 0,$$

$$4\beta a_0 a_2^2 \tanh(d_1)^3 + 2\alpha a_2^2 \tanh(d_1)^3 - 4\beta a_0 a_2 a_1 \tanh(d_1)^3 - 4\tanh(d_1)\beta a_0 a_1^2 - 2\alpha a_2 a_1 \tanh(d_1)^3 - 4\beta a_0 a_1^2 \tanh(d_1)^3 - 2\tanh(d_1)\alpha a_1^2 - 2\alpha a_1^2 \tanh(d_1)^3 = 0,$$

$$-4\beta a_2^2 a_1 \tanh(d_1)^3 - 2\alpha a_0 a_2 \tanh(d_1)^3 - 2a_2\tanh(d_1)^3 - 2\tanh(d_1)\alpha a_0 a_1 - c_2 a_2 \tanh(d_1)^4 - 2\alpha a_0 a_1 \tanh(d_1)^3 - 2a_1\tanh(d_1) - 2a_2\tanh(d_1) - c_2 a_2 \tanh(d_1)^2 + 2\beta a_1^3 \tanh(d_1)^3 - 2\tanh(d_1)\alpha a_0 a_2 - 2\tanh(d_1)\beta a_1^2 a_2 - c_2 a_2 - c_2 a_1 \tanh(d_1)^2 - 4\beta a_1^2 a_2 \tanh(d_1)^3 + 2\beta a_2^3 \tanh(d_1)^3 - 2\tanh(d_1)\beta a_1 a_2^2 - 2\tanh(d_1)\beta a_0^2 a_2 - 2\beta a_0^2 a_1 \tanh(d_1)^3 - 2a_1\tanh(d_1)^3 - c_2 a_1 \tanh(d_1)^4 - 2\beta a_0^2 a_2 \tanh(d_1)^3 - c_2 a_1 - 2\tanh(d_1)\beta a_0^2 a_1 = 0,$$

$$2\alpha a_1^2 \tanh(d_1)^3 - 4\beta a_0 a_2 a_1 \tanh(d_1)^3$$

$$
\begin{aligned}
&-2\alpha a_2^2\tanh(d_1)^3-4\tanh(d_1)\beta a_0 a_2^2\\
&-4\beta a_0 a_2^2\tanh(d_1)^3+4\beta a_0 a_1^2\tanh(d_1)^3\\
&-2\tanh(d_1)\alpha a_2^2-2\alpha a_2 a_1\tanh(d_1)^3=0,\\
&c_2 a_2\tanh(d_1)^2+2a_1\tanh(d_1)^3\\
&+c_2 a_1\tanh(d_1)^2+2\tanh(d_1)\beta a_1 a_2^2\\
&+c_2 a_2\tanh(d_1)^4+4\beta a_1^2 a_2\tanh(d_1)^3\\
&+2\tanh(d_1)\alpha a_0 a_2+2a_2\tanh(d_1)\\
&+c_2 a_2-2\beta a_2^3\tanh(d_1)^3-2\tanh(d_1)\beta a_2^3\\
&-2\beta a_2^2 a_1\tanh(d_1)^3+2\tanh(d_1)\beta a_0^2 a_2\\
&+2\beta a_0^2 a_1\tanh(d_1)^3+2\alpha a_0 a_1\tanh(d_1)^3=0,\\
&4\beta a_0 a_2 a_1\tanh(d_1)^3+4\tanh(d_1)\beta a_0 a_2^2\\
&+2\alpha a_2 a_1\tanh(d_1)^3+2\tanh(d_1)\alpha a_2^2=0,\\
&2\beta a_2^2 a_1\tanh(d_1)^3-c_2 a_2\tanh(d_1)^2\\
&+2\tanh(d_1)\beta a_2^3=0. \qquad (12)
\end{aligned}
$$

解上述非线性代数方程组可得解为

$$a_0=-\frac{\alpha}{2\beta},a_1=0,a_2=\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta},$$

$$c_2=\frac{(\alpha^2-4\beta)\tanh(d_1)}{2\beta};$$

$$a_0=-\frac{\alpha}{2\beta},a_1=\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta}$$

$$a_2=0,c_2=\frac{(\alpha^2-4\beta)\tanh(d_1)}{2\beta};$$

$$a_0=-\frac{\alpha}{2\beta},a_1=\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(1+\tanh(d_1)^2)},$$

$$a_2=\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(1+\tanh(d_1)^2)},$$

$$c_2=\frac{(\alpha^2-4\beta)\tanh(d_1)}{2\beta(1+\tanh(d_1)^2)};$$

$$a_0=-\frac{\alpha}{2\beta},a_1=\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(\tanh(d_1)^2-1)},$$

$$a_2=\mp\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(\tanh(d_1)^2-1)},$$

$$c_2=-\frac{(\alpha^2-4\beta)\tanh(d_1)}{2\beta(\tanh(d_1)^2-1}.$$

式中$(\alpha^2-4\beta)>0$,可得到相应的方程的 8 组孤波解为

$$u_{1,2}=-\frac{\alpha}{2\beta}\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta\tanh\left(d_1 n+c_1 x+\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta}+\xi_0\right)}; \qquad (13)$$

$$u_{3,4}=-\frac{\alpha}{2\beta}\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}\tanh\left(d_1 n+c_1 x+\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta}+\xi_0\right)}{2\beta}; \qquad (14)$$

$$u_{5,6}=-\frac{\alpha}{2\beta}\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}\tanh\left(d_1 n+c_1 x+\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta(\tanh(d_1)^2+1)}+\xi_0\right)}{2\beta(\tanh(d_1)^2+1)}\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(\tanh(d_1)^2+1)\tanh\left(d_1 n+c_1 x+\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta(\tanh(d_1)^2+1)}+\xi_0\right)}; \qquad (15)$$

$$u_{7,8}=-\frac{\alpha}{2\beta}\pm\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}\tanh\left(d_1 n+c_1 x-\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta(\tanh(d_1)^2-1)}+\xi_0\right)}{2\beta(\tanh(d_1)^2-1)}\mp\frac{\sqrt{(\alpha^2-4\beta)\tanh(d_1)}}{2\beta(\tanh(d_1)^2-1)\tanh\left(d_1 n+c_1 x-\frac{t(\alpha^2-4\beta)\tanh(d_1)}{2\beta(\tanh(d_1)^2-1)}+\xi_0\right)}. \qquad (16)$$

3.2.(2+1)维 Hybrid-Lattice 系统的周期波解

依据上述类似的思路,用 tan 函数对方程(9)进行行波约化和参量变换可得

$$c_2(1+T_n^2)u_n'-(1+\alpha u_n+\beta u_n^2)(u_{n-1}-u_{n+1})=0, \qquad (17)$$

式中所有变量都是 $T_n=\tan(\xi)$的函数.设方程的形式解为

$$u_n(T_n) = a_0 + a_1 T_n + a_2 T^{-1},$$
$$u_{n+p_s}(T_n) = a_0 + a_1 \frac{T_n + \tan(\phi_s)}{1 - T_n \tan(\phi_s)} + a_2\left(\frac{T_n + \tan(\phi_s)}{1 - T_n\tan(\phi_s)}\right)^{-1}, \quad (18)$$

式中 a_0，a_1，a_2 是待定常数.

将式(18)式代入(17)式也可得到一非线性代数方程组，利用吴消元法，可得方程组的解为

$$a_0 = -\frac{\alpha}{2\beta}, a_1 = 0, a_2 = \pm\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta},$$
$$c_2 = \frac{(\alpha^2 - 4\beta)\tan(d_1)}{2\beta};$$
$$a_0 = -\frac{\alpha}{2\beta}, a_1 = \pm\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta},$$
$$a_2 = 0, c_2 = \frac{(\alpha^2 - 4\beta)\tan(d_1)}{2\beta};$$
$$a_0 = -\frac{\alpha}{2\beta}, a_1 = \pm\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(1 + \tan(d_1)^2)},$$
$$a_2 = \pm\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(1 + \tan(d_1)^2)},$$
$$c_2 = \frac{(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(1 + \tan(d_1)^2)};$$
$$a_0 = -\frac{\alpha}{2\beta}, a_1 = \mp\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(\tan(d_1)^2 - 1)},$$
$$a_2 = \pm\frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(\tan(d_1)^2 - 1)},$$
$$c_2 = -\frac{(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(\tan(d_1)^2 - 1)},$$

式中 $(\alpha^2 - 4\beta) > 0$，可得到相应的方程的 8 组三角函数周期波解为

$$u_{9,10} = -\frac{\alpha}{2\beta} \pm \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta\left(\tan\left(d_1 n + c_1 x + \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta} + \xi_0\right)\right)}; \quad (19)$$

$$u_{11,12} = -\frac{\alpha}{2\beta} \pm \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)\tan\left(d_1 n + c_1 x + \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta} + \xi_0\right)}{2\beta}; \quad (20)$$

$$u_{13,14} = -\frac{\alpha}{2\beta} \pm \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)\tan\left(d_1 n + c_1 x + \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(\tan(d_1)^2 + 1)} + \xi_0\right)}{2\beta(\tan(d_1)^2 + 1)} \pm \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(\tan(d_1)^2 + 1)\tan\left(d_1 n + c_1 x + \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(\tan(d_1)^2 + 1)} + \xi_0\right)}; \quad (21)$$

$$u_{15,16} = -\frac{\alpha}{2\beta} \mp \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)\tan\left(d_1 n + c_1 x - \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(\tan(d_1)^2 - 1)} + \xi_0\right)}{2\beta(\tan(d_1)^2 - 1)} \pm \frac{\sqrt{(\alpha^2 - 4\beta)}\tan(d_1)}{2\beta(\tan(d_1)^2 - 1)\tan\left(d_1 n + c_1 x - \frac{t(\alpha^2 - 4\beta)\tan(d_1)}{2\beta(\tan(d_1)^2 + 1)} + \xi_0\right)}. \quad (22)$$

除解 $u_{3,4}$ 在文献[9]中曾提出外，其他的解都是新得到的.

4.(2+1)维 Ablowitz-Ladik-Lattice 系统的孤波解和周期波解

(2+1)维 Ablowitz-Ladik-Lattice 系统可表示为

$$\dot{u}_n(t) = (\alpha + u_n v_n)(u_{n+1} + u_{n-1}) - 2\alpha u_n,$$
$$\dot{v}_n(t) = -(\alpha + u_n v_n)(v_{n+1} + v_{n-1}) + 2\alpha v_n. \quad (23)$$

依据上面同样的思路，用 tanh 函数和 tan 函数分别对方程(23)进行行波约化和参量变换，进行类似的求解可得(2+1)维 Ablowitz-Ladik-Lattice 系统的解为

$$u_{1,2}=\mp\frac{\alpha\tanh(d_1)^2}{b_2(\tanh(d_1)^2-1)}+\frac{\alpha\tanh(d_1)^2}{b_2(\tanh(d_1)^2-1)\tanh\left(d_1n+c_1x\pm\frac{2\alpha\tanh(d_1)^2t}{(\tanh(d_1)^2-1)}+\xi_0\right)},\tag{24}$$

$$v_{1,2}=\pm b_2+\frac{b_2}{\tanh\left(d_1n+c_1x\pm\frac{2\alpha\tanh(d_1)^2t}{(\tanh(d_1)^2-1)}+\xi_0\right)},\tag{25}$$

式中 b_2 为任意常数；

$$u_{3,4}=\pm a_1+a_1\tanh\left(d_1n+c_1x\mp\frac{2\alpha\tanh(d_1)^2t}{(\tanh(d_1)^2-1)}+\xi_0\right),\tag{26}$$

$$v_{3,4}=\mp\frac{\alpha\tanh(d_1)^2}{a_1(\tanh(d_1)^2-1)}+\frac{\alpha\tanh(d_1)^2}{a_1(\tanh(d_1)^2-1)}\tanh\left(d_1n+c_1x\mp\frac{2\alpha\tanh(d_1)^2t}{(\tanh(d_1)^2-1)}+\xi_0\right),\tag{27}$$

式中 a_1 为任意常数；

$$u_{5,6}=\mp\frac{2\alpha\tanh(d_1)^2}{b_1(\tanh(d_1)^2-1)^2}-\frac{\alpha\tanh(d_1)^2}{b_1(\tanh(d_1)^2-1)^2}\tanh\left(d_1n+c_1x\pm\frac{4\alpha\tanh(d_1)^2t}{(\tanh(d_1)^2-1)^2}+\xi_0\right)$$
$$-\frac{\alpha\tanh(d_1)^2}{b_1(\tanh(d_1)^2-1)^2\tanh\left(d_1n+c_1x\pm\frac{4\alpha\tanh(d_1)^2t}{b_1(\tanh(d_1)^2-1)^2}+\xi_0\right)},\tag{28}$$

$$v_{5,6}=\mp b_1+b_1\tanh\left(d_1n+c_1x\pm\frac{4\alpha\tanh(d_1)^2t}{b_1(\tanh(d_1)^2-1)^2}+\xi_0\right)+\frac{b_1}{\tanh\left(d_1n+c_1x\pm\frac{4\alpha\tanh(d_1)^2t}{b_1(\tanh(d_1)^2-1)^2}+\xi_0\right)},\tag{29}$$

式为 b_1 为任意常数；

$$u_{7,8}=\pm\frac{i\alpha\tan(d_1)^2}{b_1\tan(d_1)^2+1)}-\frac{\alpha\tan(d_1)^2\tan\left(d_1n+c_1x\pm\frac{2i\alpha\tan(d_1)^2t}{\tan(d_1)^2+1}+\xi_0\right)}{b_1(\tan(d_1)^2+1)},\tag{30}$$

$$v_{7,8}=\pm ib_1+b_1\tan\left(d_1n+c_1x\pm\frac{2i\alpha\tan(d_1)^2t}{\tan(d_1)^2+1}+\xi_0\right),\tag{31}$$

式中 b_1 为任意常数；

$$u_{9,10}=\pm\frac{i\alpha\tan(d_1)^2}{b_2(\tan(d_1)^2+1)}-\frac{\alpha\tan(d_1)^2}{b_2(\tan(d_1)^2+1)\tan\left(d_1n+c_1x\mp\frac{2i\alpha\tan(d_1)^2t}{\tan(d_1)^2+1}+\xi_0\right)},\tag{32}$$

$$v_{9,10}=\pm ib_2+\frac{b_2}{\tan\left(d_1n+c_1x\mp\frac{2i\alpha\tan(d_1)^2t}{\tan(d_1)^2+1}+\xi_0\right)},\tag{33}$$

式中 b_2 为任意常数；

$$u_{11,12}=\pm\frac{2i\alpha\tan(d_1)^2}{b_2(\tan(d_1)^2+1)^2}+\frac{\alpha\tan(d_1)^2\tan\left(d_1n+c_1x\mp\frac{4i\alpha\tan(d_1)^2t}{(\tan(d_1)^2+1)^2}+\xi_0\right)}{b_2(\tan(d_1)^2+1)^2}$$
$$-\frac{\alpha\tan(d_1)^2}{b_2(\tan(d_1)^2+1)^2\tan\left(d_1n+c_1x\mp\frac{4i\alpha\tan(d_1)^2t}{(\tan(d_1)^2+1)^2}+\xi_0\right)},\tag{34}$$

$$v_{11,12}=\pm 2ib_2-b_2\tan\left(d_1n+c_1x\mp\frac{4\alpha\tan(d_1)^2}{(\tan(d_1)^2+1)^2}+\xi_0\right)+\frac{b_2}{\tan\left(d_1n+c_1x\mp\frac{4\alpha\tan(d_1)^2t}{(\tan(d_1)^2+1)^2}+\xi_0\right)},\tag{35}$$

式中 b_2 为任意常数.

从方程的解我们可得,除解 $u_{3,4}$, $v_{3,4}$ 在文献[9]中曾提出外,其他的解都是新得到的.

5. 结 论

tanh 函数方法在连续的非线性物理演化方程的求解中得到了多方面的改进,并且得到了广泛的应用.我们进一步对 tanh 方法进行了对称延拓,并将它应用到非线性离散系统,得到了 Hybrid-Lattice 系统和 Ablowitz-Ladik-Lattice 系统的精确孤波解和周期波解.我们也使用该方法求解了 mKdV-Lattice、Toda-Lattice 等非线性离散系统,得到了较好的结果.我们认为 tanh 函数方法在非线性编微分方程的求解过程中得到了广泛的应用并且得到了多方面的改进和拓广,能否进一步加以改进,使其在离散、高维、高阶非线性系统中得到更广泛的应用还值得深入研究.

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Improved tanh-function method and the exact solutions for the hybrid-lattice and Ablowitz-Ladik-lattice*

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Abstract

In this paper, by using the improved tanh-method, the hybrid-lattice system and Ablowitz-ladik-lattice system are reduced to nonlinear algebraic equations, and then the new exact solutions for these equations, which include exact soliton wave solutions and periodic solutions, are obtained through solving these nonlinear algebraic equations.

Keywords: im proved tanh-method, differential-difference system, soliton wave solutions, periodic solutions
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