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Weibull 分布随机波的瞬态极化统计分析——相同形状参数情形*

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Weibull 分布的雷达极化回波特征由三个分布参数来描述:波的强度、极化椭圆率角以及椭圆倾角.在瞬态极化理论的基础上,推导了三个分布参数的联合概率密度函数,标准 Stokes 矢量统计分布的联合概率密度函数及其边缘概率密度函数,并通过蒙塔卡罗方法进行了计算机仿真,结果验证了理论推导的正确性.Weibull 分布随机极化电磁波瞬态极化统计特性的研究对高分辨条件下雷达目标的检测、识别和跟踪领域都有一定的理论指导意义.

关键词:Weibull 分布,极化椭圆参数,标准 Stokes 矢量,统计

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1. 引 言

Hurwitz[1] 研究了部分极化波的统计特性. Barakat[2] 对任意极化度的部分极化波进行了研究,并给出了极化椭圆参数的一般概率密度表达式. Eliyahu[3] 给出了高斯场假设下极化椭圆参数以及 Stokes 矢量的概率密度函数,并将其应用到随机媒质的多径极化散射中. Brosseau[4] 研究了部分极化波的标准 Stokes 参数并给出了其一、二阶矩特征. Korotkova[5] 研究了空间两点的互 Stokes 矢量参数变化,并且给出了瞬态 Stokes 参数随空间传播的统计分布. Liu 等[6] 研究了空间 Stokes 参数的相干问题. Touzi 等[7] 和 Lee 等[8] 将这些结论应用到极化 SAR (synthetic aperture radar)图像的相干斑抑制领域.以上理论都是高斯场假设下的,对于 K-分布电磁波而言,Frery 等[9,10],Lee 等[11—15],Yueh 等[16,17] 已经给出了有用的统计结果,其极化椭圆参数与高斯分布下的极化椭圆参数非常接近.

然而,以上非高斯场的统计理论的获取是高斯信号经过球不变处理(SIRP[18])或蒙特卡罗仿真[16] 得到,这些都简化了极化 SAR 的数据处理. Weibull 分布概率密度函数并不是具有简单解析表达式的 SIRP 过程,现有文献基本上是以单色电磁波或部分极化电磁波为研究对象的,给出的是电磁波极化的整体统计特征,而并不能刻画非定常电磁波的极化统计特性.随着电磁环境的日趋复杂,常规定常电磁波极化的表征方法已不能有效地刻画宽带极化雷达探测系统的电磁回波,也不能完备地表征宽带雷达目标的极化散射特性,王雪松[19] 提出了"瞬态极化"的概念用以表征非定常电磁波的极化信息,并给出了高斯随机波的极化统计特性[20].在海杂波、地杂波等环境下被广泛使用的 Weibull 分布在雷达信息处理领域有非常重要的研究价值.本文是在此基础上,给出了一种描述 Weibull 分布随机电磁波的瞬态极化统计特性的方法.

本文第 2 节给出了 Weibull 分布随机极化波的无记忆非线性变换(ZMNL)物理模型,并且给出了 Weibull 分布随机极化波的极化参数的联合概率密度函数.第 3 节给出了 Weibull 分布随机极化波方差与物理模型中高斯方差的关系,便于物理意义的理解和随机波的仿真.第 4 节给出了 Weibull 分布随机极化波的椭圆参数的联合概率密度函数以及边缘概率密度函数.第 5 节给出了标准 Stokes 矢量参数的概率密度函数.最后进行了蒙特卡罗仿真,验证了理论推导的正确性.

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2. Weibull 分布随机极化波的联合概率密度函数

四维高斯随机变量的概率密度表达式为

$$f(\boldsymbol{e}_{\mathrm{HV}}) = \frac{1}{\pi^2 |\Sigma_{\mathrm{HV}}|} \exp\{- \boldsymbol{e}_{\mathrm{HV}}^{\mathrm{H}} \Sigma_{\mathrm{HV}}^{-1} \boldsymbol{e}_{\mathrm{HV}}\}, \quad (1)$$

这里 $\boldsymbol{e}_{\mathrm{HV}} = (E_{\mathrm{H}}, E_{\mathrm{V}})^{\mathrm{T}}$，$E_{\mathrm{H}} = x_{\mathrm{H}} + \mathrm{j} y_{\mathrm{H}}$，$E_{\mathrm{V}} = x_{\mathrm{V}} + \mathrm{j} y_{\mathrm{V}}$，$\sum_{\mathrm{HV}} = \langle \boldsymbol{e}_{\mathrm{HV}} \boldsymbol{e}_{\mathrm{HV}}^{\mathrm{H}} \rangle = \begin{bmatrix} \sigma_{\mathrm{HH}} & \sigma_{\mathrm{HV}} \\ \sigma_{\mathrm{VH}} & \sigma_{\mathrm{VV}} \end{bmatrix}$. $E_{\mathrm{H}}, E_{\mathrm{V}}$ 是复高斯矢量. 并且满足独立 Feynman 路径条件（$E(\cdot)$为均值算子）

$$\begin{aligned} E(x_{\mathrm{H}} x_{\mathrm{H}}) &= E(y_{\mathrm{H}} y_{\mathrm{H}}) = \sigma_{\mathrm{H}}^2, \\ E(x_{\mathrm{H}} y_{\mathrm{H}}) &= E(x_{\mathrm{H}} y_{\mathrm{H}}) = 0, \\ E(x_{\mathrm{V}} x_{\mathrm{V}}) &= E(y_{\mathrm{V}} y_{\mathrm{V}}) = \sigma_{\mathrm{V}}^2, \\ E(x_{\mathrm{V}} y_{\mathrm{V}}) &= E(x_{\mathrm{V}} y_{\mathrm{V}}) = 0, \\ E(y_{\mathrm{H}} y_{\mathrm{V}}) &= E(x_{\mathrm{H}} x_{\mathrm{V}}), \\ E(x_{\mathrm{H}} y_{\mathrm{V}}) &= - E(x_{\mathrm{V}} y_{\mathrm{H}}). \end{aligned}$$

Weibull 分布随机变量可以通过高斯变量的无记忆非线性变换（ZMNL）得到[21]. 即 Weibull 分布变量 $w = u + \mathrm{j} v$ 可以由高斯随机变量 $g = x + \mathrm{j} y$ 经过 ZMNL 得到

$$\begin{aligned} u &= x(x^2 + y^2)^{1/p - 1/2}, \\ v &= y(x^2 + y^2)^{1/p - 1/2}. \end{aligned} \quad (2)$$

Weibull 分布随机波在水平垂直极化基下的分布变量为

$$\begin{aligned} u_{\mathrm{H}} &= x_{\mathrm{H}}(x_{\mathrm{H}}^2 + y_{\mathrm{H}}^2)^{1/p - 1/2}, \\ v_{\mathrm{H}} &= y_{\mathrm{H}}(x_{\mathrm{H}}^2 + y_{\mathrm{H}}^2)^{1/p - 1/2}, \\ u_{\mathrm{V}} &= x_{\mathrm{V}}(x_{\mathrm{V}}^2 + y_{\mathrm{V}}^2)^{1/p - 1/2}, \\ v_{\mathrm{V}} &= y_{\mathrm{V}}(x_{\mathrm{V}}^2 + y_{\mathrm{V}}^2)^{1/p - 12}. \end{aligned}$$

这样，相应极化基下的 Weibull 分布随机极化波的幅度和相位为

$$\begin{aligned} a_{\mathrm{H}} &= (u_{\mathrm{H}}^2 + v_{\mathrm{H}}^2)^{1/2}, \\ \phi_{\mathrm{H}} &= \operatorname{arctg} \frac{v_{\mathrm{H}}}{u_{\mathrm{H}}}; \\ a_{\mathrm{V}} &= (u_{\mathrm{V}}^2 + v_{\mathrm{V}}^2)^{1/2}, \\ \phi_{\mathrm{V}} &= \operatorname{arctg} \frac{v_{\mathrm{V}}}{u_{\mathrm{V}}}. \end{aligned} \quad (3)$$

将(2)式代入(3)式，得到高斯变量与 Weibull 分布幅相特性的关系为

$$\begin{aligned} x_{\mathrm{H}} &= a_{\mathrm{H}}^{p/2} \cos\phi_{\mathrm{H}}, \\ y_{\mathrm{H}} &= a_{\mathrm{H}}^{p/2} \sin\phi_{\mathrm{H}}, \\ x_{\mathrm{V}} &= a_{\mathrm{V}}^{p/2} \cos\phi_{\mathrm{V}}, \\ y_{\mathrm{V}} &= a_{\mathrm{V}}^{p/2} \sin\phi_{\mathrm{V}}. \end{aligned} \quad (4)$$

将 $\boldsymbol{e}_{\mathrm{HV}} = \begin{bmatrix} E_{\mathrm{H}} \\ E_{\mathrm{V}} \end{bmatrix} = \begin{bmatrix} x_{\mathrm{H}} + \mathrm{j} y_{\mathrm{H}} \\ x_{\mathrm{V}} + \mathrm{j} y_{\mathrm{V}} \end{bmatrix}$ 代入(1)式，得

$$\begin{aligned} f(\boldsymbol{e}_{\mathrm{HV}}) &= f(E_{\mathrm{H}}, E_{\mathrm{V}}) \\ &= f(x_{\mathrm{H}}, y_{\mathrm{H}}, x_{\mathrm{V}}, y_{\mathrm{V}}) \\ &= \frac{1}{\pi^2 |\Sigma_{\mathrm{HV}}|} \exp\Big\{ - \frac{1}{|\Sigma_{\mathrm{HV}}|} \Big[\sigma_{\mathrm{VV}} |E_{\mathrm{H}}|^2 \\ &\quad + \sigma_{\mathrm{HH}} |E_{\mathrm{V}}|^2 - 2\mathrm{Re}(\sigma_{\mathrm{HV}} E_{\mathrm{H}}^{*} E_{\mathrm{V}}) \Big] \Big\}. \end{aligned} \quad (5)$$

由(4)式得到雅可比行列式值为 $J\left(\dfrac{x_{\mathrm{H}}, y_{\mathrm{H}}, x_{\mathrm{V}}, y_{\mathrm{V}}}{a_{\mathrm{H}}, \phi_{\mathrm{H}}, a_{\mathrm{V}}, \phi_{\mathrm{V}}}\right) = \dfrac{p^2}{4} a_{\mathrm{H}}^{p-1} a_{\mathrm{V}}^{p-1}$，那么

$$\begin{aligned} & f(a_{\mathrm{H}}, a_{\mathrm{V}}, \phi_{\mathrm{H}}, \phi_{\mathrm{V}}) \\ &= \frac{p^2}{4} a_{\mathrm{H}}^{p-1} a_{\mathrm{V}}^{p-1} f(x_{\mathrm{H}}, y_{\mathrm{H}}, x_{\mathrm{V}}, y_{\mathrm{V}}). \end{aligned} \quad (6)$$

将(5)式代入(6)式，得到

$$f(a_{\mathrm{H}}, \phi_{\mathrm{H}}, a_{\mathrm{V}}, \phi_{\mathrm{V}}) = \frac{p^2 a_{\mathrm{H}}^{p-1} a_{\mathrm{V}}^{p-1}}{4\pi^2 |\Sigma_{\mathrm{HV}}|} \exp\left\{ - \frac{\sigma_{\mathrm{VV}} a_{\mathrm{H}}{}^{p} + \sigma_{\mathrm{HH}} a_{\mathrm{V}}{}^{p} - 2|\sigma_{\mathrm{HV}}| a_{\mathrm{H}}^{p/2} a_{\mathrm{V}}^{p/2} \cos(\phi_{\mathrm{V}} - \phi_{\mathrm{H}} + \beta_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|} \right\}, \quad (7)$$

这里 $\beta_{\mathrm{HV}} = \mathrm{Arg}(\sigma_{\mathrm{HV}})$表示 σ_{HV}的相位.

令 $\phi = \phi_{\mathrm{V}} - \phi_{\mathrm{H}}$ 和 $\Delta = \phi_{\mathrm{H}}$，得

$$f(a_{\mathrm{H}}, a_{\mathrm{V}}, \phi, \Delta) = \frac{p^2 a_{\mathrm{H}}^{p-1} a_{\mathrm{V}}^{p-1}}{4\pi^2 |\Sigma_{\mathrm{HV}}|} \exp\left\{ - \frac{\sigma_{\mathrm{VV}} a_{\mathrm{H}}{}^{p} + \sigma_{\mathrm{HH}} a_{\mathrm{V}}{}^{p} - 2|\sigma_{\mathrm{HV}}| a_{\mathrm{H}}^{p/2} a_{\mathrm{V}}^{p/2} \cos(\phi + \beta_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|} \right\}. \quad (8)$$

将(8)式对 Δ 积分，得到 Weibull 分布随机极化波的幅度和相位差的联合概率密度函数为

$$f(a_{\mathrm{H}}, a_{\mathrm{V}}, \phi) = \frac{p^2 a_{\mathrm{H}}^{p-1} a_{\mathrm{V}}^{p-1}}{2\pi |\Sigma_{\mathrm{HV}}|} \exp\left\{ - \frac{\sigma_{\mathrm{VV}} a_{\mathrm{H}}{}^{p} + \sigma_{\mathrm{HH}} a_{\mathrm{V}}{}^{p} - 2|\sigma_{\mathrm{HV}}| a_{\mathrm{H}}^{p/2} a_{\mathrm{V}}^{p/2} \cos(\phi + \beta_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|} \right\}. \quad (9)$$

为确认(9)式的正确性，我们可以求其边缘分布来验证.

将(9)式对 ϕ 积分,得

$$
\begin{aligned}
f(a_{\rm H},a_{\rm V}) &= \int_0^{2\pi} f(a_{\rm H},a_{\rm V},\phi)\,{\rm d}\phi \\
&= \frac{p^2 a_{\rm H}^{p-2} a_{\rm V}^{p-1}}{2\pi|\Sigma_{\rm HV}|} \\
&\quad\times \exp\left\{-\frac{\sigma_{\rm VV} a_{\rm H}{}^p + \sigma_{\rm HH} a_{\rm V}{}^p}{|\Sigma_{\rm HV}|}\right\} \\
&\quad\times \int_0^{2\pi} \exp\left\{\frac{2|\sigma_{\rm HV}| a_{\rm H}^{p/2} a_{\rm V}^{p/2}}{|\Sigma_{\rm HV}|}\cos\delta\right\}{\rm d}\delta,
\end{aligned}
\tag{10}
$$

其中 $\delta = \phi + \beta_{\rm HV}$.应用零阶 Bessel 函数的定义,(10)式可以简化为

$$
\begin{aligned}
f(a_{\rm H},a_{\rm V}) &= \frac{p^2 a_{\rm H}^{p-1} a_{\rm V}^{p-1}}{|\Sigma_{\rm HV}|} \\
&\quad\times \exp\left\{-\frac{\sigma_{\rm VV} a_{\rm H}{}^p + \sigma_{\rm HH} a_{\rm V}{}^p}{|\Sigma_{\rm HV}|}\right\} I_0 \\
&\quad\times \left[\frac{2|\sigma_{\rm HV}| a_{\rm H}^{p/2} a_{\rm V}^{p/2}}{|\Sigma_{\rm HV}|}\right].
\end{aligned}
\tag{11}
$$

利用积分公式

$$
\int_0^\infty \exp(-x) I_0(a\sqrt{x})\,{\rm d}x = \exp\left(\frac{a}{2}\right)^2, \tag{12}
$$

可知(11)式的边缘分布为

$$
f(a_{\rm H}) = \int_0^\infty f(a_{\rm H},a_{\rm V})\,{\rm d}a_{\rm V} = \frac{p a_{\rm H}^{p-1}}{\sigma_{\rm HH}} {\rm e}^{\frac{a_{\rm H}^p}{\sigma_{\rm HH}}}, \tag{13}
$$

$$
f(a_{\rm V}) = \frac{p a_{\rm V}^{p-1}}{\sigma_{\rm VV}} {\rm e}^{\frac{a_{\rm V}^p}{\sigma_{\rm VV}}}. \tag{14}
$$

(13),(14)式就是 Weibull 分布随机变量的表达式,这也肯定了以上理论推导的正确性.

3. Weibull 分布参数与高斯分布参数的关系

由(9)式,得到 Weibull 分布随机分布参数之间的关系为

$$
\begin{aligned}
u_{\rm H} &= a_{\rm H}\cos\phi_{\rm H}, \\
v_{\rm H} &= a_{\rm H}\sin\phi_{\rm H}; \\
u_{\rm V} &= a_{\rm V}\cos\phi_{\rm V}, \\
v_{\rm V} &= a_{\rm V}\sin\phi_{\rm V},
\end{aligned}
\tag{15}
$$

易得

$$
\begin{aligned}
E(u_{\rm H}u_{\rm H}) &= E(v_{\rm H}v_{\rm H}) \\
&= \frac{1}{2}\sigma_{\rm HH}^{2/p}\Gamma\left(1+\frac{2}{p}\right),
\end{aligned}
\tag{16}
$$

$$
E(u_{\rm H}v_{\rm H}) = E(v_{\rm H}u_{\rm H}) = 0, \tag{17}
$$

$$
\begin{aligned}
E(u_{\rm V}u_{\rm V}) &= E(v_{\rm V}v_{\rm V}) \\
&= \frac{1}{2}\sigma_{\rm VV}^{2/p}\Gamma\left(1+\frac{2}{p}\right),
\end{aligned}
\tag{18}
$$

$$
E(u_{\rm V}v_{\rm V}) = E(v_{\rm V}u_{\rm V}) = 0. \tag{19}
$$

正交极化基下水平、垂直极化数据之间的相关系数为[22,24]

$$
\begin{aligned}
E(u_{\rm H}u_{\rm V}) &= E(v_{\rm H}v_{\rm V}) = {\rm Cov}(p,\Sigma_{\rm HV})\cos(\beta_{\rm HV}) \\
&= \frac{\pi|\sigma_{\rm HV}|^{-2-\frac{2}{p}}\Sigma_{\rm HV}^{1+\frac{2}{p}}\cos(\beta_{\rm HV})}{4p}\Bigg\{2p\,{\rm MeijerG}\left[\left\{\left\{-\frac{3}{2}-\frac{1}{p}\right\},\left\{-2-\frac{1}{p},\frac{1}{2}-\frac{1}{p}\right\}\right\},\right. \\
&\quad \left.\left\{\{-1,0\},\left\{-2-\frac{1}{p}\right\}\right\},\frac{\sigma_{\rm HH}\sigma_{\rm VV}}{|\sigma_{\rm HV}|^2}\right] + (2+p)\,{\rm MeijerG}\left[\left\{\left\{-\frac{2+p}{2p}\right\},\left\{\frac{1}{2}-\frac{1}{p},-\frac{1+p}{p}\right\}\right\},\right. \\
&\quad \left.\left\{\{-1,0\},\left\{-\frac{1+p}{p}\right\}\right\},\frac{\sigma_{\rm HH}\sigma_{\rm VV}}{|\sigma_{\rm HV}|^2}\right]\Bigg\},
\end{aligned}
\tag{20}
$$

$$
E(u_{\rm H}v_{\rm V}) = -E(u_{\rm V}v_{\rm H}) = {\rm Cov}(p,\Sigma_{\rm HV})\sin(\beta_{\rm HV}), \tag{21}
$$

其中 MeijerG[{{$a_1,\cdots,a_p$}}, {{$b_1,\cdots,b_m$}}, {{$b_{m+1},\cdots,b_q$}}, z] 为梅杰函数,记为 $G_{pq}^{mn}\left(z\,\middle|\,\begin{matrix}a_1,\cdots,a_p\\ b_1,\cdots,b_q\end{matrix}\right)$,

$$
\begin{aligned}
{\rm Cov}(p,\Sigma_{\rm HV}) &= \frac{\pi|\sigma_{\rm HV}|^{-2-\frac{2}{p}}\Sigma_{\rm HV}^{1+\frac{2}{p}}}{4p}\Bigg\{2p\,{\rm MeijerG}\left[\left\{\left\{-\frac{3}{2}-\frac{1}{p}\right\},\left\{-2-\frac{1}{p},\frac{1}{2}-\frac{1}{p}\right\}\right\},\left\{\{-1,0\},\left\{-2-\frac{1}{p}\right\}\right\},\frac{\sigma_{\rm HH}\sigma_{\rm VV}}{|\sigma_{\rm HV}|^2}\right] \\
&\quad + (2+p)\,{\rm MeijerG}\left[\left\{\left\{-\frac{2+p}{2p}\right\},\left\{\frac{1}{2}-\frac{1}{p},-\frac{1+p}{p}\right\}\right\},\left\{\{-1,0\},\left\{-\frac{1+p}{p}\right\}\right\},\frac{\sigma_{\rm HH}\sigma_{\rm VV}}{|\sigma_{\rm HV}|^2}\right]\Bigg\}.
\end{aligned}
\tag{22}
$$

如果 $p=2$,由(16)—(22)式得到相关系数为 $E(u_{\rm H}v_{\rm V}) = E(v_{\rm H}v_{\rm V}) = \frac{1}{2}{\rm Re}(\sigma_{\rm HV})$,$E(u_{\rm H}v_{\rm V}) = -E(u_{\rm V}v_{\rm H}) = \frac{1}{2}{\rm Im}(\sigma_{\rm HV})$,这与 Eliyahu[3] 的结论是一致.这样我们就从 Weibull 分布随机极化波的方差矩

阵得到了高斯方差矩阵,可以从中提取出(9)式的高斯分布参数.

4. 极化椭圆参数的概率密度分布

(9)式中,将 a_H, a_V 用 I_H, I_V 代替,这里 $I_H = a_H^2, I_V = a_V^2$. 得雅可比行列式为

$$J\left(\frac{a_H, a_V, \phi}{I_H, I_V, \phi}\right) = \frac{1}{4\sqrt{I_H I_V}}, \tag{23}$$

易得

$$\begin{aligned} f(J_H, I_V, \phi) &= J\left(\frac{a_H, a_V, \phi}{I_H, I_V, \phi}\right) f(a_H, a_V, \phi) \\ &= \frac{p^2 I_H^{p/2-1} I_V^{p/2-1}}{8\pi|\Sigma_{HV}|} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV} I_H^{p/2} - \sigma_{HH} I_V^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{\frac{2|\sigma_{HV}| I_H^{p/4} I_V^{p/4} \cos(\phi + \beta_{HV})}{|\Sigma_{HV}|}\right\}. \end{aligned} \tag{24}$$

为得到极化椭圆参数的统计分布，我们引入一组物理变量 x, y, τ,满足以下条件[3,23]：

$$x = I_H + I_V, y = [(I_H - I_V)^2 + 4 I_H I_V \cos^2\phi]^{1/2},$$

$$-\pi \leqslant \phi \leqslant \pi,$$

$$\mathrm{tg}2\tau = \frac{2\sqrt{I_H I_V}\cos\phi}{I_H - I_V},$$

$$-\pi/4 \leqslant \tau \leqslant \pi/4.$$

其中 $|\tau| \leqslant \pi/4, I_H \geqslant I_V$,得相应的雅可比行列式值为

$$J\left(\frac{I_H, I_V, \phi}{x, y, \tau}\right) = \frac{y}{\sqrt{x^2 - y^2}}.$$

这样我们得到其联合概率密度函数为

$$\begin{aligned} f_1(x, y, \tau) &= \frac{y}{\sqrt{x^2 - y^2}} \frac{p^2 \left(\frac{x + y\cos 2\tau}{2}\right)^{p/2-1} \left(\frac{x - y\cos 2\tau}{2}\right)^{p/2-1}}{4\pi|\Sigma_{HV}|} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV}\left(\frac{x + y\cos 2\tau}{2}\right)^{p/2} + \sigma_{HH}\left(\frac{x - y\cos 2\tau}{2}\right)^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{\frac{y\sin 2\tau \left(\frac{x + y\cos 2\tau}{2}\right)^{p/4-1/2} \left(\frac{x - y\cos 2\tau}{2}\right)^{p/4-1/2} \mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}\right\} \\ &\quad \times \cosh\left\{-\frac{\sqrt{x^2 - y^2}\left(\frac{x + y\cos 2\tau}{2}\right)^{p/4-1/2} \left(\frac{x - y\cos 2\tau}{2}\right)^{p/4-1/2} \mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}. \end{aligned} \tag{25}$$

为了记录简便，令 $a = \left(\frac{x + y\cos 2\tau}{2}\right)$，$b = \left(\frac{x - y\cos 2\tau}{2}\right)$,(25)式变为

$$\begin{aligned} f_1(x, y, \tau) &= \frac{y}{\sqrt{x^2 - y^2}} \frac{p^2 a^{p/2-1} b^{p/2-1}}{4\pi|\Sigma_{HV}|} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV} a^{p/2} + \sigma_{HH} b^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{\frac{y\sin 2\tau a^{p/4-1/2} b^{p/4-1/2} \mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}\right\} \\ &\quad \times \cosh\left\{-\frac{\sqrt{x^2 - y^2} a^{p/4-1/2} b^{p/4-1/2} \mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}. \end{aligned} \tag{26}$$

当 $|\tau| \leqslant \pi/4$ 时，$I_H \leqslant I_V$,相应的雅可比行列式值为

$$J\left(\frac{I_H, I_V, \phi}{x, y, \tau}\right) = -\frac{y}{\sqrt{x^2 - y^2}},$$

其概率密度函数为

$$\begin{aligned} f_2(x, y, \tau) &= \frac{y}{\sqrt{x^2 - y^2}} \frac{p^2 b^{p/2-1} a^{p/2-1}}{4\pi|\Sigma_{HV}|} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV} b^{p/2} + \sigma_{HH} a^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{-\frac{y\sin 2\tau b^{p/4-1/2} b^{p/4-1/2} \mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}\right\} \\ &\quad \times \cosh\left\{-\frac{\sqrt{x^2 - y^2} b^{p/4-1/2} a^{p/4-1/2} \mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}. \end{aligned} \tag{27}$$

所以 x, y, τ 的联合概率密度函数为(26)式和(27)式之和,即

$$f(x,y,\tau)=f_1(x,y,\tau)+f_2(x,y,\tau). \quad (28)$$

令极化椭圆长短轴分别为 I_a, I_b，其与 x, y 的关系为 $I_a=\frac{1}{2}(x,y), I_b=\frac{1}{2}(x-y)$[23].

令 $I=I_a+I_b, \xi_\pm=\pm I_b/I_a, \xi=I_b/I_a$[23]，相应的雅可比行列式值为

$$J\left(\frac{x,y,\tau}{I,\xi,\tau}\right)=-\frac{2I}{(1+\xi)^2}.$$

设

$$c=\left(\frac{1}{2}+\frac{(1-\xi)\cos 2\tau}{2(1+\xi)}\right),$$

$$d=\left(\frac{1}{2}-\frac{(1-\xi)\cos 2\tau}{2(1+\xi)}\right).$$

则相应的概率密度函数为

$$\begin{aligned}f_1(I,\xi_\pm,\tau)=&\frac{p^2(1-\xi)I^{p-1}c^{p/2-1}d^{p/2-1}}{8\pi\sqrt{\xi}(1+\xi)^2|\Sigma_{HV}|}\\&\times\exp\left\{-\frac{\sigma_{VV}I^{p/2}c^{p/2}+\sigma_{HH}I^{p/2}d^{p/2}}{|\Sigma_{HV}|}\right\}\\&\times\exp\left\{\frac{(1-\xi)\sin 2\tau I^{p/2}c^{p/4-1/2}d^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}\right\}\\&\times\exp\left\{\mp\frac{2\sqrt{\xi}I^{p/2}c^{p/4-1/2}d^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}\right\},\end{aligned} \quad (29)$$

$$\begin{aligned}f_2(I,\xi_\pm,\tau)=&\frac{p^2(1-\xi)I^{p-1}d^{p/2-1}c^{p/2-1}}{8\pi\sqrt{\varepsilon}(1+\xi)^2|\Sigma_{HV}|}\\&\times\exp\left\{-\frac{\sigma_{VV}I^{p/2}d^{p/2}+\sigma_{HH}I^{p/2}c^{p/2}}{|\Sigma_{HV}|}\right\}\\&\times\exp\left\{-\frac{(1-\xi)\sin 2\tau I^{p/2}d^{p/4-1/2}c^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}\right\}\\&\times\exp\left\{\mp\frac{2\sqrt{\xi}I^{p/2}d^{p/4-1/2}c^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}\right\}.\end{aligned} \quad (30)$$

令 $A=\frac{p^2(1-\xi)c^{p/2-1}d^{p/2-1}}{8\pi\sqrt{\xi}(1+\xi)^2|\Sigma_{HV}|}$，$h=\frac{\sigma_{VV}}{|\Sigma_{HV}|}$，$l=\frac{\sigma_{HH}}{|\Sigma_{HV}|}$，$f=\frac{(1-\xi)\sin 2\tau c^{p/4-1/2}d^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}$，

$k=\frac{2\sqrt{\xi}d^{p/4-1/2}c^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}$，(29)，30)式变为

$$\begin{aligned}f_1(I,\xi_\pm,\tau)=&AI^{p-1}\exp\{-hc^{p/2}I^{p/2}-ld^{p/2}I^{p/2}\}\\&\times\exp\{fI^{p/2}\}\exp\{\mp kI^{p/2}\},\end{aligned} \quad (31)$$

$$\begin{aligned}f_2(I,\xi_\pm,\tau)=&AI^{p-1}\exp\{-hd^{p/2}I^{p/2}-lc^{p/2}I^{p/2}\}\\&\times\exp\{-fI^{p/2}\}\exp\{\mp kI^{p/2}\},\end{aligned} \quad (32)$$

(31)，(32)式对 I 积分，得到

$$\begin{aligned}f_1(\xi_\pm,\tau)=&A\int_0^\infty I^{p-1}\exp\{-hc^{p/2}I^{p/2}-ld^{p/2}I^{p/2}\}\\&\times\exp\{fI^{p/2}\}\exp\{\mp kI^{p/2}\}\mathrm{d}I,\\=&-\frac{2A}{p}\frac{\partial}{\partial(hc^{p/2}+ld^{p/2})}\\&\times\int_0^\infty\exp\{-(hc^{p/2}+ld^{p/2})I^{p/2}\}\\&\times\exp\{(f\mp k)I^{p/2}\}\mathrm{d}I^{p/2}\\=&\frac{2A}{p(hc^{p/2}+ld^{p/2}\pm k-f)^2}.\end{aligned}$$

故

$$\begin{aligned}f_1(\xi_\pm,\tau)&=\frac{2A}{p(hc^{p/2}+ld^{p/2}\pm k-f)^2},\\f_2(\xi_\pm,\tau)&=\frac{2A}{p(hd^{p/2}+lc^{p/2}\pm k+f)^2}.\end{aligned} \quad (33)$$

如果设 $|\varepsilon|=\sqrt{\xi}$，$k=\frac{2\varepsilon d^{p/4-1/2}c^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|(1+\xi)}$，$B=2\varepsilon A$，椭圆参数$(\varepsilon,\tau)$ $(-1\leqslant\varepsilon\leqslant 1)$的联合概率密度函数为

$$\begin{aligned}f(\varepsilon,\tau)=&\frac{2B}{p}\left[\frac{1}{(hc^{p/2}+ld^{p/2}+k-f)^2}\right.\\&\left.+\frac{1}{(hd^{p/2}+lc^{p/2}+k+f)^2}\right].\end{aligned} \quad (34)$$

当 $p=2$ 时，(34)式与文献[3]的(18)式是一致的.

如果 τ 定义为波的矢量端点与椭圆长轴的夹角，那么(34)式变为

$$f_+(\varepsilon,\tau_+)=\frac{2B}{p}\left[\frac{1}{(hc^{p/2}+ld^{p/2}+k-f)^2}\right]. \quad (35)$$

如果 τ 定义为波的矢量端点与椭圆短轴的夹角，那么(34)式变为

$$f_-(\varepsilon,\tau_-)=\frac{2\beta}{p}\left[\frac{1}{(hd^{p/2}+lc^{p/2}+k+f)^2}\right]. \quad (36)$$

由(34)式可以得到其边缘分布分别为

$$\begin{aligned}f(\varepsilon)&=\int_{-\pi/2}^{\pi/2}f(\varepsilon,\tau_\pm)\mathrm{d}\tau,\\f(\tau)&=\int_{-1}^{1}f(\varepsilon,\tau_\pm)\mathrm{d}\varepsilon.\end{aligned} \quad (37)$$

如果 $p=2$，ε 和 $\tau_\pm$ 的概率密度函数与 Barakat[2] 和 Eliyahu[3] 的结论是一致的.

令 $I_H=a_H^2, I_V=a_V^2$，(11)式变为

$$\begin{aligned}f(I_H,I_V)&=\left|J\left(\frac{a_H,a_V}{I_H,I_V}\right)\right|f(a_H,a_V)\\&=\frac{1}{4\sqrt{I_HI_V}}f(\sqrt{I_H},\sqrt{I_V}),\end{aligned}$$

即

$$f(I_H, I_V) = \frac{p^2 I_H^{(p-2)/2} I_V^{(p-2)/2}}{4|\Sigma_{HV}|} \times \exp\left\{-\frac{\sigma_{VV} I_H^{p/2} + \sigma_{HH} I_V^{p/2}}{|\Sigma_{HV}|}\right\} \times I_0\left[\frac{2|\sigma_{HV}| I_H^{p/4} I_V^{p/4}}{|\Sigma_{HV}|}\right],$$

其边缘积分为

$$f(I_H) = \int_0^\infty f(I_H, I_V)\,dI_V = \frac{pI_H^{p/2-1}}{2\sigma_{HH}}\exp\left(-\frac{I_H^{p/2}}{\sigma_{HH}}\right),$$

$$f(I_V) = \int_0^\infty f(I_H, I_V)\,dI_H = \frac{pI_V^{p/2-1}}{2\sigma_{VV}}\exp\left(-\frac{I_V^{p/2}}{\sigma_{VV}}\right).$$

它们都是 Weibull 分布随机分布的.

随机波的总强度为 $I = I_H + I_V$，得到波的强度的概率密度函数表达式为

$$f(I) = \int_0^\infty f(I_H, I - I_H)\,dI_H = \int_0^I f(I_H, I - I_H)\,dI_H.$$

易得

$$f(I) = \int_0^I \frac{p^2 I_H^{(p-2)/2}(I - I_H)^{(p-2)/2}}{4|\Sigma_{HV}|} \times \exp\left\{-\frac{\sigma_{VV} I_H^{p/2} + \sigma_{HH}(I - I_H)^{p/2}}{|\Sigma_{HV}|}\right\} \times I_0\left[\frac{2|\sigma_{HV}| I_H^{p/4}(I - I_H)^{p/4}}{|\Sigma_{HV}|}\right] dI_H.$$

令 $I_H = Ix$，可以得到波强的概率密度函数为

$$f(I) = \frac{p^2 I^{p-1}}{4|\Sigma_{HV}|}\int_0^1 x^{p/2-1}(1-x)^{p/2-1} \times \exp\left\{-\frac{\sigma_{VV} I^{p/2} x^{p/2} + \sigma_{HH} I^{p/2}(1-x)^{p/2}}{|\Sigma_{HV}|}\right\} \times I_0\left[\frac{2|\sigma_{HV}| I^{p/2} x^{p/4}(1-x)^{p/4}}{|\Sigma_{HV}|}\right] dx. \tag{38}$$

当 $p = 2$ 时，所得结果与文献[3]的(15a)式的结果是一致的.可见，波的总的强度的分布是受水平垂直极化数据的方差影响的.

5. 标准 Stokes 参数的联合概率密度函数

电磁波的 Stokes 矢量定义为[3]

$$g_{HV0} = E_H E_H^H + E_V E_V^H,$$
$$g_{HV1} = E_H E_H^H - E_V E_V^H,$$
$$g_{HV2} = E_H E_V^H + E_V E_H^H,$$
$$g_{HV3} = j[E_H E_V^H - E_V E_H^H].$$

那么 Stokes 子矢量 $\boldsymbol{g}_{HV} = (g_{HV1}, g_{HV2}, g_{HV3})$ 与幅度相位差 (a_H, a_V, ϕ) 的关系为

$$a_H^2 = \frac{1}{2}(g_{HV0} + g_{HV1}),\quad a_V^2 = \frac{1}{2}(g_{HV0} - g_{HV1}),\quad \phi = \arg(g_{HV2} + jg_{HV3}), \tag{39}$$

其中 $\arg(g_{HV2} + jg_{HV3})$ 是 $g_{HV2} + jg_{HV3}$ 的相位，因此对应的雅可比行列式值为

$$J\left(\frac{g_{HV1}, g_{HV2}, g_{HV3}}{a_H, a_V, \phi}\right) = 4g_{HV0}\sqrt{g_{HV2}^2 + g_{HV3}^2} = 4g_{HV0}\sqrt{g_{HV0}^2 - g_{HV1}^2}. \tag{40}$$

这样就得到 Stokes 子矢量 $\boldsymbol{g}_{HV}$ 的概率密度函数为

$$f(\boldsymbol{g}_{HV}) = \frac{f(a_H, a_V, \phi)}{4g_{HV0}\sqrt{g_{HV0}^2 - g_{HV1}^2}} = \frac{p^2}{16\pi g_{HV0}|\Sigma_{HV}|}\left(\frac{g_{HV0} + g_{HV1}}{2}\right)^{p/2-1}\left(\frac{g_{HV0} - g_{HV1}}{2}\right)^{p/2-1} \times \exp\left\{-\frac{\sigma_{VV}\left(\frac{g_{HV0} + g_{HV1}}{2}\right)^{p/2-1} + \sigma_{HH}\left(\frac{g_{HV0} - g_{HV1}}{2}\right)^{p/2}}{|\Sigma_{HV}|}\right\} \times \exp\left\{\frac{\left(\frac{g_{HV0} + g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{g_{HV0} - g_{HV1}}{2}\right)^{p/4-1/2}(g_{HV2}\mathrm{Re}(\sigma_{HV}) - g_{HV3}\mathrm{Im}(\sigma_{HV}))}{|\Sigma_{HV}|}\right\}. \tag{41}$$

对于 Stokes 子矢量$(g_{HV0}, g_{HV1}, g_{HV2})$,有以下变换关系：

$$\psi\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \varphi(\boldsymbol{g}_{HV}) = \begin{bmatrix} g_{HV0} \\ g_{HV1} \\ g_{HV2} \end{bmatrix} = \begin{bmatrix} \sqrt{g_{HV1}^2 + g_{HV2}^2 + g_{HV3}^2} \\ g_{HV1} \\ g_{HV2} \end{bmatrix}. \qquad (42)$$

由(42)式知其反变换有两个解,记作 $\boldsymbol{g}_{HV}^{(i)} = h^{(i)}(\psi), i = 1, 2$.

$i = 1$ 时,

$$\boldsymbol{g}_{HV} = \begin{bmatrix} g_{HV1} \\ g_{HV2} \\ g_{HV3} \end{bmatrix} = \begin{bmatrix} y \\ z \\ \sqrt{x^2 - y^2 - z^2} \end{bmatrix}, \qquad (43)$$

对应的雅可比行列式为

$$J_1\left(\frac{g_{HV1}, g_{HV2}, g_{HV3}}{x, y, z}\right) = \frac{x}{\sqrt{x^2 - y^2 - z^2}}. \qquad (44)$$

$i = 2$ 时,逆变换为

$$\boldsymbol{g}_{HV} = [x, y, -\sqrt{x^2 - y^2 - z^2}]^{\mathrm{T}}, \qquad (45)$$

对应的雅可比行列式为

$$J_2\left(\frac{g_{HV1}, g_{HV2}, g_{HV3}}{x, y, z}\right) = -\frac{x}{\sqrt{x^2 - y^2 - z^2}}. \qquad (46)$$

那么 ψ 的概率密度函数为

$$\begin{aligned} f_\psi(\psi) &= f_{G_{HV}}(y, z, \sqrt{x^2 - y^2 - z^2})\left|\frac{x}{\sqrt{x^2 - y^2 - z^2}}\right| \\ &\quad + f_{G_{HV}}(y, z, -\sqrt{x^2 - y^2 - z^2})\left|\frac{-x}{\sqrt{x^2 - y^2 - z^2}}\right| \\ &= \frac{x}{\sqrt{x^2 - y^2 - z^2}}[f_{G_{HV}}(y, z, \sqrt{x^2 - y^2 - z^2}) \\ &\quad + f_{G_{HV}}(y, z, -\sqrt{x^2 - y^2 - z^2})] \\ &= \frac{pq\left(\frac{x+y}{2}\right)^{p/2-1}\left(\frac{x-y}{2}\right)^{p/2-1}}{8\pi|\Sigma_{HV}|\sqrt{x^2 - y^2 - z^2}} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV}\left(\frac{x+y}{2}\right)^{p/2} + \sigma_{HH}\left(\frac{x-y}{2}\right)^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{\frac{z\left(\frac{x+y}{2}\right)^{p/4-1/2}\left(\frac{x-y}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}\right\} \\ &\quad \times \cosh\left\{\frac{\sqrt{x^2 - y^2 - z^2}\left(\frac{x+y}{2}\right)^{p/4-1/2}\left(\frac{x-y}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}. \end{aligned} \qquad (47)$$

为求标准 Stokes 参数的概率密度分布函数,令 $x = g_{HV0}$,故有 $g_{HV1} = \tilde{g}_{HV1}x, g_{HV2} = \tilde{g}_{HV2}x$. 代入(47)式易得

$$\begin{aligned} f_{X\tilde{G}_1\tilde{G}_2}(x, \tilde{g}_{HV1}, \tilde{g}_{HV2}) &= \frac{p^2\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{p/2-1}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{p/2-1}}{8\pi|\Sigma_{HV}|\sqrt{1 - \tilde{g}_{HV1}^2 - \tilde{g}_{HV2}^2}}x^{p-1} \\ &\quad \times \exp\left\{-\frac{\sigma_{VV}\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{p/2}x^{p/2} + \sigma_{HH}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{p/2}x^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\quad \times \exp\left\{\frac{\tilde{g}_{HV2}\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}x^{\frac{p}{2}}\right\} \\ &\quad \times \cosh\left\{\frac{\sqrt{1 - \tilde{g}_{HV1}^2 - \tilde{g}_{HV2}^2}\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}x^{\frac{p}{2}}\right\} \\ &= Ax^{p-1}\exp(-hx^{p/2} - lx^{p/2})\exp(fx^{p/2})\cosh(kx^{\frac{p}{2}}). \end{aligned} \qquad (48)$$

这里

$$A = \frac{p^2\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{p/2-1}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{p/2-1}}{8\pi|\Sigma_{HV}|\sqrt{1 - \tilde{g}_{HV1}^2 - \tilde{g}_{HV2}^2}}, \quad B = \frac{A}{p},$$

$$h=\frac{\sigma_{VV}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/2}}{|\Sigma_{HV}|},\ l=\frac{\sigma_{HH}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/2}}{|\Sigma_{HV}|},$$

$$f=\frac{\tilde g_{HV2}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|},$$

$$k=\frac{\sqrt{1-\tilde g_{HV1}^2-\tilde g_{HV2}^2}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}.$$

将(48)式对 x 积分得到

$$\begin{aligned}f_{\tilde G_1\tilde G_2}(\tilde g_{HV1},\tilde g_{HV2})&=A\int_0^\infty x^{p-1}\exp(-(h+l-f)x^{p/2})\cosh(kx^{p/2})\mathrm{d}x\\&=-\frac{2A}{p}\frac{\partial}{\partial h}\int_0^\infty\exp(-(h+l-f)x^{p/2})\cosh(kx^{p/2})\mathrm{d}x^{p/2}\\&=\frac{A}{p}\left[\frac{1}{(h+l-f-k)^2}+\frac{1}{(h+l-f+k)^2}\right]\\&=B\left[\frac{1}{(h+l-f-k)^2}+\frac{1}{(h+l-f+k)^2}\right].\end{aligned}\tag{49}$$

令

$$k=\frac{\tilde g_{HV3}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|},$$

因此,标准 Stokes 参数的联合概率密度函数为

$$f_{\tilde G_1\tilde G_2\tilde G_3}(\tilde g_{HV1},\tilde g_{HV2},\tilde g_{HV2})=C\delta\left(1-\sqrt{\tilde g_{HV1}^2+\tilde g_{HV2}^2+\tilde g_{HV3}^2}\right)\frac{1}{(h+l-f-k)^2},\tag{50}$$

记

$$C=\frac{p\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/2-1}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/2-1}}{8\pi|\Sigma_{HV}|}.$$

$p=2$ 时,(50)式与 Eliyahu[3] 的结果是一致的.

同样我们可以得到($\tilde g_{HV1}$,$\tilde g_{HV3}$)的联合概率密度函数为

$$f_{\tilde G_1\tilde G_3}(\tilde g_{HV1},\tilde g_{HV3})=B\left[\frac{1}{(h+l-f-k)^2}+\frac{1}{(h+l+f-k)^2}\right],\tag{51}$$

记

$$f=\frac{\sqrt{1-\tilde g_{HV1}^2-\tilde g_{HV3}^2}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|},$$

$$k=\frac{\tilde g_{HV3}\left(\frac{1+\tilde g_{HV1}}{2}\right)^{p/4-1/2}\left(\frac{1-\tilde g_{HV1}}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}.$$

现在推导($\tilde g_{HV2}$,$\tilde g_{HV3}$)的联合概率密度函数.类似的方法,可知(47)式变为

$$\begin{aligned}f_\alpha(\alpha)&=f_{G_{HV}}\left(\sqrt{x^2-y^2-z^2},y,z,\right)\left|\frac{x}{\sqrt{x^2-y^2-z^2}}\right|+f_{G_{HV}}\left(-\sqrt{x^2-y^2-z^2},y,z,\right)\left|\frac{-x}{\sqrt{x^2-y^2-z^2}}\right|\\&=\frac{x}{\sqrt{x^2-y^2-z^2}}\left[f_{G_{HV}}\left(\sqrt{x^2-y^2-z^2},y,z,\right]+f_{G_{HV}}\left(-\sqrt{x^2-y^2-z^2},y,z,\right)\right]\\&=\frac{p^2\left(\frac{y^2+z^2}{2}\right)^{p/2-1}}{16\pi|\Sigma_{HV}|\sqrt{x^2-y^2-z^2}}\exp\left\{\frac{y\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}\exp\left\{\frac{z\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}\right\}\end{aligned}$$

$$\times\left\{\exp\left\{-\frac{\sigma_{\mathrm{VV}}\left(\frac{x+\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}+\sigma_{\mathrm{HH}}\left(\frac{x-\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}}{|\Sigma_{\mathrm{HV}}|}\right\}\right.$$

$$\left.+\exp\left\{-\frac{\sigma_{\mathrm{VV}}\left(\frac{x-\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}+\sigma_{\mathrm{HH}}\left(\frac{x+\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}}{|\Sigma_{\mathrm{HV}}|}\right\}\right\}, \tag{52}$$

α 定义为矢量

$$\boldsymbol{\psi}\begin{bmatrix}x\\y\\z\end{bmatrix}=\varphi(\boldsymbol{g}_{\mathrm{HV}})=\begin{bmatrix}g_{\mathrm{HV0}}\\g_{\mathrm{HV2}}\\g_{\mathrm{HV3}}\end{bmatrix}=\begin{bmatrix}\sqrt{g_{\mathrm{HV1}}^2+g_{\mathrm{HV2}}^2+g_{\mathrm{HV3}}^2}\\g_{\mathrm{HV2}}\\g_{\mathrm{HV3}}\end{bmatrix}. \tag{53}$$

由(52)式易知

$$f_\alpha(\alpha_+)=\frac{p^2\left(\frac{y^2+z^2}{2}\right)^{p/2-1}}{16\pi|\Sigma_{\mathrm{HV}}|\sqrt{x^2-y^2-z^2}}\exp\left\{\frac{y\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}\right\}\exp\left\{\frac{z\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}\right\}$$

$$\times\exp\left\{-\frac{\sigma_{\mathrm{VV}}\left(\frac{x+\sqrt{x^2-y^2+z^2}}{2}\right)^{p/2}+\sigma_{\mathrm{HH}}\left(\frac{x-\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}}{|\Sigma_{\mathrm{HV}}|}\right\}, \tag{54}$$

$$f_\alpha(\alpha_-)=\frac{p^2\left(\frac{y^2+z^2}{2}\right)^{p/2-1}}{16\pi|\Sigma_{\mathrm{HV}}|\sqrt{x^2-y^2-z^2}}\exp\left\{\frac{y\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}\right\}\exp\left\{\frac{z\left(\frac{y^2+z^2}{2}\right)^{p/4-1/2}\mathrm{IM}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}\right\}$$

$$\times\exp\left\{-\frac{\sigma_{\mathrm{VV}}\left(\frac{x-\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}+\sigma_{\mathrm{HH}}\left(\frac{x+\sqrt{x^2-y^2-z^2}}{2}\right)^{p/2}}{|\Sigma_{\mathrm{HV}}|}\right\}, \tag{55}$$

这里 α_+ 定义为 g_{HV1} 为正时的矢量，α_- 为 g_{HV1} 为负时的矢量.

令 $x=g_{\mathrm{HV0}}$，得 $g_{\mathrm{HV2}}=\tilde{g}_{\mathrm{HV2}}x$，$g_{\mathrm{HV3}}=\tilde{g}_{\mathrm{HV3}}x$. 由(54)式得

$$f_{X\tilde{G}_2\tilde{G}_3+}(x,\tilde{g}_{\mathrm{HV2}},\tilde{g}_{\mathrm{HV3}}=\frac{p^2\left(\frac{\tilde{g}_{\mathrm{HV2}}^2+\tilde{g}_{\mathrm{HV3}}^2}{4}\right)^{p/2-1}}{16\pi|\Sigma_{\mathrm{HV}}|\sqrt{1-\tilde{g}_{\mathrm{HV2}}^2-\tilde{g}_{\mathrm{HV3}}^2}}x^{p-1}$$

$$\times\exp\left\{-\frac{\sigma_{\mathrm{VV}}\left(\frac{1+\sqrt{1-\tilde{g}_{\mathrm{HV2}}^2-\tilde{g}_{\mathrm{HV3}}^2}}{2}\right)^{p/2}x^{p/2}+\sigma_{\mathrm{HH}}\left(\frac{1-\sqrt{1-\tilde{g}_{\mathrm{HV2}}^2-\tilde{g}_{\mathrm{HV3}}^2}}{2}\right)^{p/2}x^{p/2}}{|\Sigma_{\mathrm{HV}}|}\right\}$$

$$\times\exp\left\{\frac{\tilde{g}_{\mathrm{HV2}}\left(\frac{\tilde{g}_{\mathrm{HV2}}^2+\tilde{g}_{\mathrm{HV3}}^2}{4}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}x^{\frac{p}{2}}\right\}\exp\left\{\frac{\tilde{g}_{\mathrm{HV3}}\left(\frac{\tilde{g}_{\mathrm{HV2}}^2+\tilde{g}_{\mathrm{HV3}}^2}{4}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{\mathrm{HV}})}{|\Sigma_{\mathrm{HV}}|}x^{\frac{p}{2}}\right\}$$

$$=Ax^{p-1}\exp(-hx^{p/2}-lx^{p/2})\exp(fx^{\frac{p}{2}})\exp(kx^{\frac{p}{2}}), \tag{56}$$

其中

$$A=\frac{p^2\left(\frac{\tilde{g}_{\mathrm{HV2}}^2+\tilde{g}_{\mathrm{HV3}}^2}{4}\right)^{p/2-1}}{16\pi|\Sigma_{\mathrm{HV}}|\sqrt{1-\tilde{g}_{\mathrm{HV2}}^2-\tilde{g}_{\mathrm{HV3}}^2}},$$

$$h_+=\frac{\sigma_{\mathrm{VV}}\left(\frac{1+\sqrt{1-\tilde{g}_{\mathrm{HV2}}^2-\tilde{g}_{\mathrm{HV3}}^2}}{2}\right)^{p/2}}{|\Sigma_{\mathrm{HV}}|},$$

$$l_+ = \frac{\sigma_{HH}\left(\frac{1-\sqrt{1-\tilde g_{HV2}^2-\tilde g_{HV3}^2}}{2}\right)^{p/2}}{|\Sigma_{HV}|},$$

$$f = \frac{\tilde g_{HV2}\left(\frac{\tilde g_{HV2}^2+\tilde g_{HV3}^2}{4}\right)^{p/4-1/2}\mathrm{Re}(\sigma_{HV})}{|\Sigma_{HV}|},$$

$$k = \frac{\tilde g_{HV3}\left(\frac{\tilde g_{HV2}^2+\tilde g_{HV3}^2}{4}\right)^{p/4-1/2}\mathrm{Im}(\sigma_{HV})}{|\Sigma_{HV}|}.$$

(56)式对 x 积分得到

$$\begin{aligned} &f_{\tilde G_2\tilde G_3+}(\tilde g_{HV2},\tilde g_{HV3}) \\ &= A\int_0^\infty x^{p-1}\exp(-(h_+ + l_+ - f - k)x^{p/2})\mathrm{d}x \\ &= -\frac{2A}{p}\frac{\partial}{\partial h}\int_0^\infty \exp(-(h_+ + l_+ - f - k)x^{p/2})\mathrm{d}x^{p/2} \\ &= \frac{2A}{p}\frac{1}{(h_+ + l_+ - f - k)^2}. \end{aligned} \tag{57}$$

同理可得

$$\begin{aligned} &f_{\tilde G_2\tilde G_3-}(\tilde g_{HV2},\tilde g_{HV3}) \\ &= A\int_0^\infty x^{p-1}\exp(-(h_- + l_- - f - k)x^{p/2})\mathrm{d}x \\ &= -\frac{2A}{p}\frac{\partial}{\partial h}\int_0^\infty \exp(-(h_- + l_- - f - k)x^{p/2})\mathrm{d}x^{p/2} \\ &= \frac{2A}{p}\frac{1}{(h_- + l_- - f - k)^2}. \end{aligned} \tag{58}$$

因此$(\tilde g_{HV2},\tilde g_{HV3})$的联合概率密度为

$$\begin{aligned} &f_{\tilde G_2\tilde G_3}(\tilde g_{HV2},\tilde g_{HV3}) \\ &= \frac{2A}{p}\left(\frac{1}{(h_+ + l_+ - f - k)^2} + \frac{1}{(h_- + l_- - f - k)^2}\right), \end{aligned} \tag{59}$$

其中

$$h_- = \frac{\sigma_{VV}\left(\frac{1-\sqrt{1-\tilde g_{HV2}^2-\tilde g_{HV3}^2}}{2}\right)^{p/2}}{|\Sigma_{HV}|},$$

$$l_- = \frac{\sigma_{HH}\left(\frac{1+\sqrt{1-\tilde g_{HV2}^2-\tilde g_{HV3}^2}}{2}\right)^{p/2}}{|\Sigma_{HV}|}.$$

由(11)式得

$$\begin{aligned} f(A_H,A_V) &= \frac{pqA_H^{(p-2)/2}A_V^{(p-2)/2}}{4|\Sigma_{HV}|} \\ &\times \exp\left\{-\frac{\sigma_{VV}A_H^{p/2}+\sigma_{HH}A_V^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\times I_0\left[\frac{2|\sigma_{HV}|A_H^{p/4}A_V^{p/4}}{|\Sigma_{HV}|}\right], \end{aligned} \tag{60}$$

A_H,A_V 分别为水平垂直极化数据的强度变量. 易得其与 Stokes 参数的关系为

$$\begin{aligned} g_{HV0} &= A_H + A_V, \\ g_{HV1} &= A_H - A_V \end{aligned} \tag{61}$$

所以

$$\begin{aligned} &f(g_{HV0},g_{HV1}) \\ &= \frac{p^2\left(\frac{g_{HV0}+g_{HV1}}{2}\right)^{p/2-1}\left(\frac{g_{HV0}-g_{HV1}}{2}\right)^{p/2-1}}{8|\Sigma_{HV}|} \\ &\times \exp\left\{-\frac{\sigma_{VV}\left(\frac{g_{HV0}+g_{HV1}}{2}\right)^{p/2}+\sigma_{HH}\left(\frac{g_{HV0}+g_{HV1}}{2}\right)^{p/2}}{|\Sigma_{HV}|}\right\} \\ &\times I_0\left[\frac{2|\sigma_{HV}|\left(\frac{g_{HV0}+g_{HV1}}{2}\right)^{p/4}\left(\frac{g_{HV0}+g_{HV1}}{2}\right)^{p/4}}{|\Sigma_{HV}|}\right]. \end{aligned} \tag{62}$$

令 $a = g_{HV0}, b = g_{HV1}/g_{HV0}$，得

$$\begin{aligned} f_{AB}(a,b) &= g_{HV0}f(g_{HV0},g_{HV1}) \\ &= af(a,ab), \end{aligned} \tag{63}$$

故

$$\begin{aligned} f(\tilde g_{HV1}) &= A\int_0^\infty x^{p-1}\exp\{-(h+l)x^{p/2}\}I_0[kx^{p/2}]\mathrm{d}x \\ &= A\left(-\frac{2}{p}\right)\frac{\partial}{\partial h}\int_0^\infty \exp[-(h+l)x^{p/2}]I_0[kx^{p/2}]\mathrm{d}x^{p/2} \\ &= \frac{-2A}{p}\frac{\partial}{\partial h}\int_0^\infty \exp[-(h+l)t]I_0[kt]\mathrm{d}t \\ &= \frac{2A(h+l)}{p[(h+l)^2-k^2]^{3/2}}, \end{aligned} \tag{64}$$

其中

$$A = \frac{p^2\left(\frac{1+b}{2}\right)^{p/2-1}\left(\frac{1-b}{2}\right)^{p/2-1}}{8|\Sigma_{HV}|},$$

$$h = \frac{\sigma_{VV}\left(\frac{1+b}{2}\right)^{p/2}}{|\Sigma_{HV}|},$$

$$l = \frac{\sigma_{HH}\left(\frac{1-b}{2}\right)^{p/2}}{|\Sigma_{HV}|},$$

$$k = \frac{2|\sigma_{HV}|\left(\frac{1+b}{2}\right)^{p/4}\left(\frac{1-b}{2}\right)^{p/4}}{|\Sigma_{HV}|}.$$

令 $B = \frac{A}{p}$，(64)式简化为

$$\begin{aligned} f(\tilde g_{HV1}) &= \frac{2A(h+l)}{p[(h+l)^2-k^2]^{3/2}} \\ &= \frac{2B(h+l)}{[(h+l)^2-k^2]^{3/2}}. \end{aligned} \tag{65}$$

另外两个标准 Stokes 参数的概率密度函数为

$$f(\tilde{g}_{HV2}) = \int_{-1}^{1} f_{\tilde{G}_1\tilde{G}_2}(\tilde{g}_{HV1},\tilde{g}_{HV2})\mathrm{d}\tilde{g}_{HV1},$$

$$f(\tilde{g}_{HV3}) = \int_{-1}^{1} f_{\tilde{G}_1\tilde{G}_3}(\tilde{g}_{HV1},\tilde{g}_{HV3})\mathrm{d}\tilde{g}_{HV1}. \quad (66)$$

因为 Weibull 分布变量不是球不变变量,所以(66)式不能简化为(65)式的形式.

6. 计算机仿真验证

为了验证以上理论公式推导的正确性,我们用蒙特卡罗方法来仿真 Weibull 分布随机波的极化参数分布,得出 Weibull 分布随机极化波的概率密度分布统计直方图,并与理论值进行比较.当然,实际的 Weibull 分布参数要通过具体的环境获得,然后通过(16)—(22)式获取对应高斯分布参数.这里取 Weibull 分布形状参数为 $p=3$,其水平垂直尺度参数分别为 $q_H = 1$, $q_V = 2$,且其方差阵为 $W_{HV} = \begin{bmatrix} 0.902745 & 0.89186(1+\mathrm{j}) \\ 0.89186(1-\mathrm{j}) & 3.61098 \end{bmatrix}$.其对应的高斯分布参数为 $\Sigma_{HV}\begin{bmatrix} \sigma_{HH} & \sigma_{HV} \\ \sigma_{VH} & \sigma_{VV} \end{bmatrix} = \begin{bmatrix} 1 & \sqrt{2}(1+\mathrm{j}) \\ \sqrt{2}(1-\mathrm{j}) & 8 \end{bmatrix}$.

椭圆参数的概率密度分布函数的参数为

$$c = \left(\frac{1}{2} + \frac{(1-\varepsilon^2)\cos 2\tau}{2(1+\varepsilon^2)}\right),$$

$$d = \left(\frac{1}{2} - \frac{(1-\varepsilon^2)\cos 2\tau}{2(1+\varepsilon^2)}\right),$$

$$B = \frac{9(1-\varepsilon^2)c^{1/2}d^{1/2}}{16\pi(1+\varepsilon^2)^2},$$

$$f = \frac{\sqrt{2}(1-\varepsilon^2)\sin(2\tau)c^{1/4}d^{1/4}}{4(1+\varepsilon^2)},$$

$$h = 2, l = \frac{1}{4}, k = \frac{\sqrt{2}\varepsilon d^{1/4}c^{1/4}}{2(1+\varepsilon^2)}.$$

如果 τ 定义为波的矢量端点与椭圆长轴的夹角,那么(35)式变为

$$f(\varepsilon,\tau_+) = \frac{2B}{3}\left[\frac{1}{(2c^{3/2} + d^{3/2}/4 + k - f)^2}\right]. \quad (67)$$

如果 τ 定义为波的矢量端点与椭圆短轴的夹角,那么(34)式变为

$$f(\varepsilon,\tau_-) = \frac{2B}{3}\left[\frac{1}{(2d^{3/2} + c^{3/2}/4 + k + f)^2}\right]. \quad (68)$$

很明显,二者之间满足 $f(\varepsilon,\tau_+) = f(\varepsilon,\frac{\pi}{2}-\tau_-)$.

其强度的概率密度分布函数为

$$f(I) = \frac{9I^2}{4|\Sigma_{HV}|}\int_0^1 x^{3/2-1}(1-x)^{1/2} \times \exp\left\{-\frac{8I^{3/2}x^{3/2} + I^{3/2}(1-x)^{3/2}}{4}\right\} \times I_0[I^{3/2}x^{3/4}(1-x)^{3/4}]\mathrm{d}x. \quad (69)$$

标准 Stokes 参数的概率密度分布的参数为

$$A = \frac{9\left(\frac{\tilde{g}_{HV2}^2 + \tilde{g}_{HV3}^2}{4}\right)^{1/2}}{64\pi\sqrt{1-\tilde{g}_{HV2}^2-\tilde{g}_{HV3}^2}},$$

$$h_+ = 2\left(\frac{1+\sqrt{1-\tilde{g}_{HV2}^2-\tilde{g}_{HV3}^2}}{2}\right)^{3/2},$$

$$l_+ = \frac{\left(\frac{1-\sqrt{1-\tilde{g}_{HV2}^2-\tilde{g}_{HV3}^2}}{2}\right)^{3/2}}{4},$$

$$f = \frac{\tilde{g}_{HV2}(\tilde{g}_{HV2}^2+\tilde{g}_{HV3}^2)^{1/4}}{4},$$

$$k = \frac{\tilde{g}_{HV3}(\tilde{g}_{HV2}^2+\tilde{g}_{HV3}^2)^{1/4}}{4},$$

$$h_- = 2\left(\frac{1-\sqrt{1-\tilde{g}_{HV2}^2-\tilde{g}_{HV3}^2}}{2}\right)^{3/2},$$

$$l_- = \frac{\left(\frac{1+\sqrt{1-\tilde{g}_{HV2}^2-\tilde{g}_{HV3}^2}}{2}\right)^{3/2}}{4}.$$

因此($\tilde{g}_{HV2}$,$\tilde{g}_{HV3}$)的联合概率密度为

$$f_{\tilde{G}_2\tilde{G}_3}(\tilde{g}_{HV2},\tilde{g}_{HV3}) = \frac{2A}{3}\left(\frac{l}{(h_+ + l_+ - f - k)^2} + \frac{1}{(h_- + l_- - f - k)^2}\right). \quad (70)$$

若

$$B = \frac{3\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{1/2}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{1/2}}{32},$$

$$h = 2\left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{3/2},$$

$$l = \frac{\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{3/2}}{4},$$

$$k = \left(\frac{1+\tilde{g}_{HV1}}{2}\right)^{3/4}\left(\frac{1-\tilde{g}_{HV1}}{2}\right)^{3/4},$$

那么标准 Stokes 参数的第一分量的概率密度函数为

$$f(\tilde{g}_{HV1}) = \frac{2B(h+l)}{[(h+l)^2 - k^2]^{3/2}}. \quad (71)$$

通过以上分析,做出其仿真值(10000 次蒙特卡罗仿真结果)与理论值曲线如图 1—6.

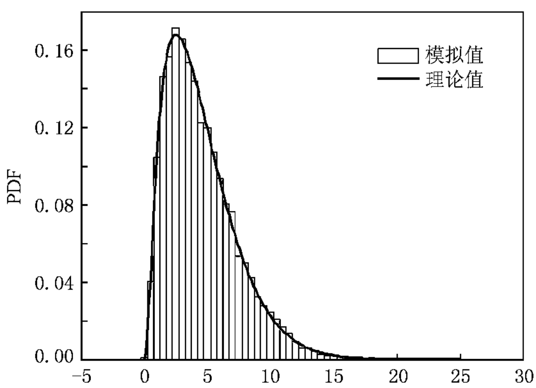

图 1　随机波的强度

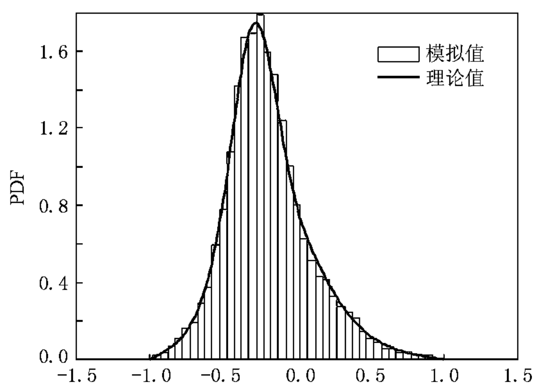

图 2　瞬态极化椭圆参数

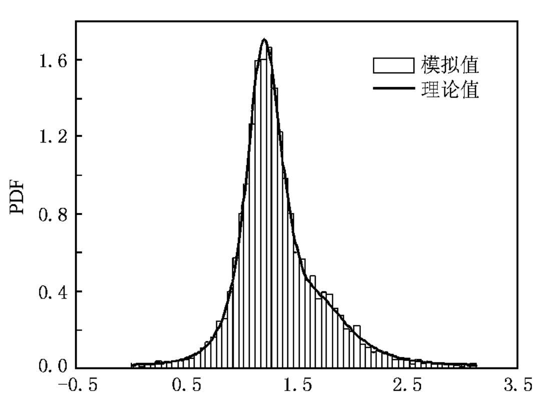

图 3　瞬态极化椭圆倾角

由图 1 可以看出在此 Weibull 分布下其椭圆倾角集中在 1.2 rad 左右，这对于目标的检测、识别等后续信息处理是相当有利的. 由图 2 可见极化椭圆率角正切值基本位于 − 0.2 左右，电磁波的椭圆率角特性还是比较集中. 图 4—6 的理论值稍带锯齿，这是由于我们数值积分不够准确造成的，理论上它

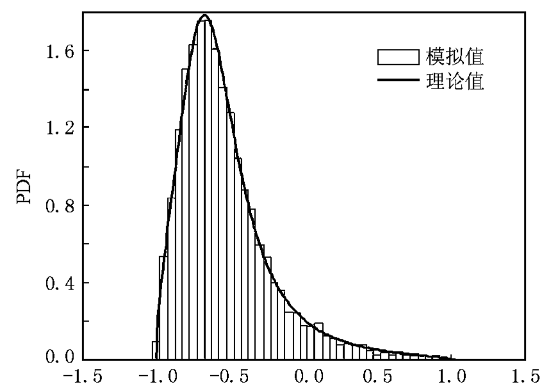

图 4　标准 Stokes 矢量第一参数

是光滑曲线，这并不影响统计直方图与理论值的比较. 可以看出，仿真值与理论值还是符合得相当好.

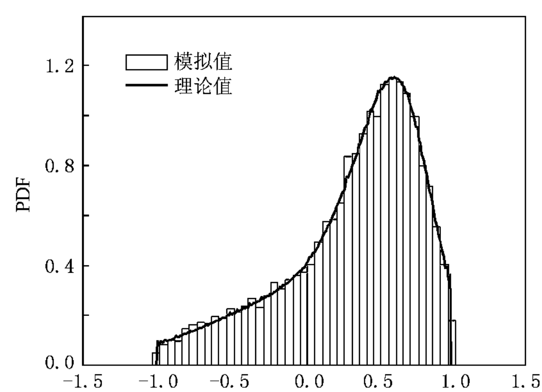

图 5　标准 Stokes 矢量第二参数

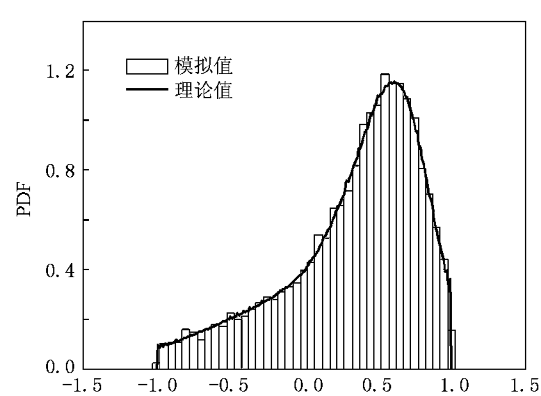

图 6　标准 Stokes 矢量第三参数

从图 1 至图 6 可知，我们的理论分析是简单有效的，对 Weibull 分布瞬态极化电磁波的统计特性描述提供了简洁切实有效的表征方法.

7. 结 论

本文研究了 Weibull 分布随机极化波的瞬态极化统计特性.给出了表征 Weibull 分布随机极化波特征的三个参数 强度、极化椭圆角以及椭圆倾角的联合概率密度函数和各参数的边缘分布函数.给出了标准 Stokes 三个参数的联合概率密度函数以及边缘概率密度函数,及其数值计算仿真.验证了理论推导的正确性.Weibull 分布随机波瞬态极化统计特性的研究结论在高分辨条件下雷达目标的检测、识别和跟踪领域的实际应用将是我们下一步研究的重点.

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Statistics of the instantaneous polarization in Weibull ——distributed fields-the same shape parameter case *

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Abstract

The statistical properties of partially polarized light in Gaussian stochastic plane wave fields have been studied. In this paper，the statistics of the stochastic plane electromagnetic（EM）wave fields with Weibull distributed is examined. The polarimetric Weibull-distributed wave is characterized by three parameters：the sum of the amplitudes of the two electric vector components，the angle which the major axis makes with the reference coordinate system，and the ratio of the minor to the major axis. The joint and marginal probability densities of these random variables are determined as a function of the covariance matrix. The main properties of this important distribution are shown. Then the statistical properties of the normalized Stokes parameters are described in detail in Weibull-distributed stochastic fields. The joint and marginal probability density functions（PDF）of the three components of the normalized Stokes parameters are presented. Results of some numerical calculation are obtained. The description of the Weibull polarimetric wave field may be useful to random medium scattering and speckle filtering.

Keywords：Weibull distribution，ellipse parameters，normalized Stokes parameters，statistics

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