第 58 卷 第 6 期 2009 年 6 月
1000-3290/2009/58(06)/3806-06
物 理 学 报
ACTA PHYSICA SINICA
Vol.58,No.6,June,2009

基于 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 共同本征态的相干纠缠态表象的构建及性质*

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(2008 年 9 月 4 日收到;2008 年 10 月 19 日收到修改稿)

利用有序算符乘积内的积分技术(IWOP),建立了一种称之为相干纠缠态的两粒子体系的新表象,研究了这种新表象的性质,从理论上探讨了这种相干纠缠态的产生方法.结果表明:本文建立的这种 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 的共同本征态 $|p,\beta\rangle$,既具有相干态的特性,又体现了纠缠态的特征,具有超完备性,完全可以作为一个表象使用.物理上可以用光分束器来实现 $|p,\beta\rangle$,让分束器的两个输入端分别输入理想单模压缩态 $|p=0\rangle_2=\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2$ 和真空态 $|0\rangle_1$,再经过对激光场一定的调制作用即可得到 $|p,\beta\rangle$ 态.

关键词:IWOP 技术,相干纠缠态表象,分束器

PACC:0635

1. 引 言

表象理论及表象变换已成为量子力学、量子光学等领域的一个重要课题,文献[1—11]对表象、表象变换及其应用作了深入的研究与探讨.正如狄拉克[8]所说,当一个人要解决一个具体的量子力学问题时,可以通过选择使用一种合适的表象来大大减少劳动量,在这种表象中,问题的重要物理量的表示是尽可能的简单.文献[9]给出了基于 $\hat{x}_1+\hat{x}_2$ 与 a_1-a_2 共同本征态的两体相干纠缠态新表象,本文作者也曾利用单位分解技术建立了基于组合坐标 $\hat{Q}_1$,$\hat{Q}_2$ 共同本征态的两粒子新表象[10]并研究了其性质.本文将利用有序算符乘积内的积分(IWOP)技术[11]采用文献[10]的方法,建立一种基于 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 共同本征态的两粒子体系的新表象 $|p,\beta\rangle$,并研究其性质,讨论这种量子态产生的物理方法.

2. 基于 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 共同本征态 $|p,\beta\rangle$ 的构建

在文献[10]中,对于两粒子体系,构造如下四个线性组合算符 $\hat{Q}_1,\hat{Q}_2,\hat{P}_1,\hat{P}_2$:

$$\begin{pmatrix}\hat{Q}_1\\ \hat{Q}_2\end{pmatrix}=\begin{pmatrix}A_{11}&A_{12}\\ A_{21}&A_{22}\end{pmatrix}\begin{pmatrix}\hat{q}_1\\ \hat{q}_2\end{pmatrix}=\boldsymbol{A}\begin{pmatrix}\hat{q}_1\\ \hat{q}_2\end{pmatrix},$$
$$\begin{pmatrix}\hat{P}_1\\ \hat{P}_2\end{pmatrix}=\begin{pmatrix}A_{11}&A_{12}\\ A_{21}&A_{22}\end{pmatrix}\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix}=\boldsymbol{A}\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix},\tag{1}$$

其中 $A_{11},A_{12},A_{21},A_{22}$ 均为实数,$\boldsymbol{A}=\begin{pmatrix}A_{11}&A_{12}\\ A_{21}&A_{22}\end{pmatrix}$ 为幺正矩阵,这意味着

$$\boldsymbol{A}\boldsymbol{A}^{\dagger}=\boldsymbol{A}^{\dagger}\boldsymbol{A}=\boldsymbol{I},$$
$$\sum_{k=1}^{2}A_{ki}A_{kj}=\sum_{k=1}^{2}A_{ik}A_{jk}=\delta_{ij}\tag{2}$$
$$(i,j=1,2).$$

显然 $[\hat{Q}_i,\hat{P}_j]=\mathrm{i}\delta_{ij}$,$[\hat{Q}_1,\hat{Q}_2]=0$,即变换后的组合

* 山东省自然科学基金(批准号:Y2008A16)、菏泽学院自然科学基金(批准号:XY07WL01,XY08WL03)和山东省高等学校实验技术基金(批准号:S04W138)资助的课题.

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坐标 $\hat{Q}_1,\hat{Q}_2$ 具有共同本征态，为求其 Fock 空间的显式，构造如下正规乘积内的高斯积分：

$$1=\iint_{-\infty}^{\infty}\frac{\mathrm{d}\eta_1\mathrm{d}\eta_2}{\pi}:\exp\{-(\eta_1-\hat{Q}_1)^2-(\eta_2-\hat{Q}_2)^2\}:$$

$$=\iint_{-\infty}^{\infty}\frac{\mathrm{d}\eta_1\mathrm{d}\eta_2}{\pi}:\exp\left\{-\frac{1}{2}(\eta_1^2+\eta_2^2)+\sqrt{2}(\hat{a}_1^\dagger\hat{a}_2^\dagger)\tilde{\boldsymbol{A}}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}-\frac{1}{2}(\hat{a}_1^{\dagger2}+\hat{a}_2^{\dagger2})\right\}|00\rangle,$$

$$\langle00|\exp\left\{-\frac{1}{2}(\eta_1^2+\eta_2^2)+\sqrt{2}(\hat{a}_1\hat{a}_2)\tilde{\boldsymbol{A}}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}-\frac{1}{2}(\hat{a}_1^2+\hat{a}_2^2)\right\}=\iint_{-\infty}^{\infty}\mathrm{d}\eta_1\mathrm{d}\eta_2|\eta_1\eta_2\rangle\langle\eta_1\eta_2|,\tag{3}$$

式中 $\tilde{\boldsymbol{A}}=\begin{pmatrix}A_{11}&A_{21}\\A_{12}&A_{22}\end{pmatrix}$，$|00\rangle\langle00|=:\exp\{-(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2\}:$，

$$|\eta_1,\eta_2\rangle=\pi^{-1/2}\exp\left\{-\frac{1}{2}(\eta_1^2+\eta_2^2)+\sqrt{2}(\hat{a}_1^\dagger\quad\hat{a}_2^\dagger)\tilde{\boldsymbol{A}}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}-\frac{1}{2}(\hat{a}_1^{\dagger2}+\hat{a}_2^{\dagger2})\right\}|00\rangle,\tag{4}$$

其中，$|00\rangle$是基态态矢，$\hat{a}_1,\hat{a}_2$ 及 $\hat{a}_1^\dagger,\hat{a}_2^\dagger$ 为双模湮灭算符及产生算符，它们与 $\hat{q}_1,\hat{q}_2$ 和 $\hat{p}_1,\hat{p}_2$ 的关系为

$$\hat{q}_i=(\hat{a}_i+\hat{a}_i^\dagger)/\sqrt{2},$$
$$\hat{p}_i=(\hat{a}_i-\hat{a}_i^\dagger)/\mathrm{i}\sqrt{2}\quad(i=1,2).\tag{5}$$

利用有序算符正规乘积的性质[11] $:\frac{\partial}{\partial a_i^+}f(\hat{a}_i,\hat{a}_i^\dagger):=[\hat{a}_i,:f(\hat{a}_i,\hat{a}_i^\dagger):]$及 $\hat{a}_i|00\rangle=0$，容易得到

$$\hat{Q}_i|\eta_1,\eta_2\rangle=\eta_i|\eta_1,\eta_2\rangle\quad(i=1,2).\tag{6}$$

即$|\eta_1,\eta_2\rangle$是 $\hat{Q}_1,\hat{Q}_2$ 的共同本征态，相应的本征值分别为 η_1 和 η_2.

$|\eta_1,\eta_2\rangle$的完备性如(3)式所示，正交性为

$$\langle\eta_1',\eta_2'|\eta_1,\eta_1\rangle=\delta(\eta_1'-\eta_1)\delta(\eta_2'-\eta_2),\tag{7}$$

即$|\eta_1,\eta_2\rangle$既是完备的又是正交的，完全可以构成一种表象. 因为算符 $\hat{Q}_1,\hat{Q}_2$ 是坐标算符 $\hat{q}_1,\hat{q}_2$ 的线性组合，所以称$|\eta_1,\eta_2\rangle$为组合坐标表象.

对于一两粒子体系，以 $\hat{x}_1,\hat{x}_2$ 和 $\hat{p}_1,\hat{p}_2$ 表示其坐标和动量算符，a_1,a_2 及 $a_1^\dagger,a_2^\dagger$ 为双模湮灭算符及产生算符

$$\hat{x}_j=(a_j+a_j^\dagger)/\sqrt{2},$$
$$\hat{p}_j=(a_j-a_j^\dagger)/\mathrm{i}\sqrt{2}\quad(i=1,2).\tag{8}$$

由于

$$[\hat{p}_1-\hat{p}_2,a_1+a_2]=0,\tag{9}$$

因此 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 具有共同本征态. 仿照(3)式的做法，构造如下正规乘积内的高斯积分

$$1=\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{\sqrt{\pi}}\int\frac{\mathrm{d}^2\beta}{2\pi}:\exp\left\{-\left(p-\frac{\hat{p}_1-\hat{p}_2}{\sqrt{2}}\right)^2-\frac{1}{2}[\beta-(a_1+a_2)]\cdot[\beta^*-(a_1^++a_2^+)\right\}:$$

$$=\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{\sqrt{\pi}}\int\frac{\mathrm{d}^2\beta}{2\pi}\exp\left\{-\frac{p^2}{2}-\frac{|\beta|^2}{4}+\left(\frac{\beta}{2}+\mathrm{i}p\right)a_1^++\left(\frac{\beta}{2}-\mathrm{i}p\right)a_2^++\frac{(a_1^+-a_2^+)^2}{4}\right\}$$

$$\cdot:\exp[-a_1^+a_1-a_2^+a_2]:\exp\left\{-\frac{p^2}{2}-\frac{|\beta|^2}{4}+\left(\frac{\beta^*}{2}-\mathrm{i}p\right)a_1+\left(\frac{\beta^*}{2}+\mathrm{i}p\right)a_2+\frac{(a_1-a_2)^2}{4}\right\},\tag{10}$$

式中 $\beta=\beta_1+\mathrm{i}\beta_2$ 为复数. 利用双模真空投影算符的正规乘积形式$|00\rangle\langle00|=:\exp\{-a_1^\dagger a_1-a_2^\dagger a_2\}:$可将(10)式变为

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{\sqrt{\pi}}\int\frac{\mathrm{d}^2\beta}{2\pi}|p,\beta\rangle\langle p,\beta|=1,\tag{11}$$

式中

$$|p,\beta\rangle=\exp\left\{-\frac{p^2}{2}-\frac{|\beta|^2}{4}+\left(\frac{\beta}{2}+\mathrm{i}p\right)a_1^++\left(\frac{\beta}{2}-\mathrm{i}p\right)a_2^++\frac{(a_1^+-a_2^+)^2}{4}\right\}\times|00\rangle,\tag{12}$$

其中，$|00\rangle$是基态态矢.

利用有序算符正规乘积的性质$:\frac{\partial}{\partial a_j^+}f(a_j,a_j^\dagger):=[a_j,:f(a_j,a_j^+):]$及 $a_j|00\rangle=0$ 可得

$$a_1|p,\beta\rangle=\left[\left(\frac{\beta}{2}+\mathrm{i}p\right)+\frac{1}{2}(a_1^+-a_2^+)\right]|p,\beta\rangle,\tag{13}$$

$$a_2|p,\beta\rangle=\left[\left(\frac{\beta}{2}-\mathrm{i}p\right)-\frac{1}{2}(a_1^+-a_2^+)\right]|p,\beta\rangle.\tag{14}$$

由(12)，(13)和(8)式，得

$$(a_1+a_2)|p,\beta\rangle=\beta|p,\beta\rangle,\tag{15}$$

$$(\hat{p}_1-\hat{p}_2)|p,\beta\rangle=\sqrt{2}p|p,\beta\rangle.\tag{16}$$

(15)，(16)式表明$|p,\beta\rangle$是 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 的共同本征态，相应的本征值分别为$\sqrt{2}p$ 和β.

由(11)式显见,$|p,\beta\rangle$是完备的,下面考察$|p,\beta\rangle$的正交性.首先

$$\delta(q-q')=\langle q|q'\rangle$$
$$=\pi^{-1/2}\langle 0|\exp\left(-\frac{q^2}{2}+\sqrt{2}qa-\frac{a^2}{2}\right)$$
$$\times\exp\left(-\frac{q'^2}{2}+\sqrt{2}q'a^{+}-\frac{a^{+2}}{2}\right)|0\rangle,\tag{17}$$

其中$|q\rangle=\pi^{-1/4}\exp\left(-\frac{q^2}{2}+\sqrt{2}qa^{+}-\frac{a^{+2}}{2}\right)|0\rangle$.在(17)式中插入相干态完备性$\int\frac{d^2z}{\pi}|z\rangle\langle z|=1$,并注意$a|z\rangle=z|z\rangle$及$\langle z|a^{+}=\langle z|z^{*}$,得

$$\delta(q-q')=\pi^{-1/2}\exp\left(-\frac{q^2+q'^2}{2}\right)\int\frac{d^2z}{\pi}$$
$$\times\exp\left(\sqrt{2}qz-\frac{z^2}{2}\right.$$
$$\left.+\sqrt{2}q'z^{*}-\frac{z^{*2}}{2}\right)|\langle z|0\rangle|^2,\tag{18}$$

式中$|z\rangle=\exp\left(-\frac{|z|^2}{2}+za^{+}\right)|0\rangle$.注意$a|0\rangle=0$,可得

$$\langle z|0\rangle=\exp(-|z|^2).\tag{19}$$

将(19)式代入(18)式,得

$$\delta(q-q')=\pi^{-1/2}\exp\left(-\frac{q^2+q'^2}{2}\right)\int\frac{d^2z}{\pi}$$
$$\times\exp\left(-|z|^2+\sqrt{2}qz-\frac{z^2}{2}\right.$$
$$\left.+\sqrt{2}q'z^{*}-\frac{z^{*2}}{2}\right).\tag{20}$$

利用数学公式[11]

$$\int\frac{d^2z}{\pi}\exp(-\zeta|z|^2+\xi z+\eta z^{*}+fz^2+gz^{*2})$$
$$=\frac{1}{\sqrt{\zeta^2-4fg}}\exp\left(\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}\right),\tag{21}$$

此积分的收敛条件为 $\mathrm{Re}(\xi-f-g)<0$, $\mathrm{Re}\left(\frac{\xi^2-4fg}{\xi-f-g}\right)<0$ 或 $\mathrm{Re}(\xi+f+g)<0$, $\mathrm{Re}\left(\frac{\xi^2-4fg}{\xi+f+g}\right)<0$,可得

$$\delta(q-q')=\pi^{-1/2}\exp\left(-\frac{q^2+q'^2}{2}\right)$$
$$\times\lim_{t\to1}\int\frac{d^2z}{\pi}\exp\left(-|z|^2+\sqrt{2}qz\right.$$
$$\left.+\sqrt{2}q'z^{*}-\frac{tz^2}{2}-\frac{tz^{*2}}{2}\right)$$
$$=\pi^{-1/2}\exp\left(-\frac{q^2+q'^2}{2}\right)\lim_{t\to1}\frac{1}{\sqrt{1-t^2}}$$
$$\times\exp\left(\frac{2qq'-(q^2+q'^2)t}{1-t^2}\right),$$

即

$$\delta(q-q')=\pi^{-1/2}\exp\left(-\frac{q^2+q'^2}{2}\right)\lim_{t\to1}\frac{1}{\sqrt{1-t^2}}$$
$$\times\exp\left(\frac{2qq'-(q^2+q'^2)t}{1-t^2}\right).\tag{22}$$

由(12)式得

$$\langle p,\beta|p',\beta'\rangle=\langle 00|\exp\left[-\frac{p^2}{2}-\frac{|\beta|^2}{4}\right.$$
$$+\left(\frac{1}{2}\beta^{*}-\mathrm{i}p\right)a_1+\left(\frac{1}{2}\beta^{*}+\mathrm{i}p\right)a_2$$
$$\left.+\frac{1}{4}(a_1-a_2)^2\right]$$
$$\times\exp\left[-\frac{p'^2}{2}-\frac{|\beta'|^2}{4}\right.$$
$$+\left(\frac{\beta'}{2}+\mathrm{i}p'\right)a_1^{+}+\left(\frac{\beta'}{2}-\mathrm{i}p'\right)a_2^{+}$$
$$\left.+\frac{1}{4}(a_1^{+}-a_2^{+})^2\right]|00\rangle.\tag{23}$$

在(23)式中插入双模相干态的完备性$\int\frac{d^2z_1d^2z_2}{\pi^2}|z_1,z_2\rangle\langle z_1,z_2|=1$,并注意$\langle z_1,z_2|00\rangle=\exp\left(-\frac{|z_1|^2+|z_2|^2}{2}\right)$,可得

$$\langle p,\beta|p',\beta'\rangle=\exp\left(-\frac{p^2+p'^2}{2}-\frac{|\beta|^2+|\beta'|^2}{4}\right)$$
$$\times\int\frac{d^2z_1d^2z_2}{\pi^2}\exp\left[-|z_1|^2+\left(\frac{\beta^{*}}{2}-\mathrm{i}p\right.\right.$$
$$\left.-\frac{1}{2}z_2\right)z_1+\left(\frac{\beta'}{2}+\mathrm{i}p'-\frac{1}{2}z_2^{*}\right)z_1^{*}$$
$$\left.\frac{1}{4}z_1^2+\frac{1}{4}z_1^{*2}\right]\exp\left[-|z_2|^2+\right.$$
$$\left(\frac{1}{2}\beta^{*}+\mathrm{i}p\right)z_2+\left(\frac{\beta'}{2}-\mathrm{i}p'\right)z_2^{*}$$
$$\left.+\frac{1}{4}z_2^2+\frac{1}{4}z_2^{*2}\right].\tag{24}$$

利用(21)式完成(24)式中对z_1,z_1^{*}的积分,得

$$\langle p,\beta|p'\beta'\rangle=C\int\frac{d^2z_2}{\pi}\exp\left[-\frac{2}{3}|z_2|^2\right.$$
$$+\frac{2\mathrm{i}}{3}\left[(2p-p')+\frac{\beta^{*}-\beta'}{\mathrm{i}2}\right]z_2$$
$$-\frac{2\mathrm{i}}{3}\left[(2p'-p)+\frac{\beta^{*}-\beta'}{\mathrm{i}2}\right]z_2^{*}$$
$$\left.+\frac{1}{3}z_2^2+\frac{1}{3}z_2^{*2}\right],\tag{25}$$

式中

$$C=\frac{2}{\sqrt{3}}\exp\left(-\frac{p^2+p'^2}{2}-\frac{|\beta|^2+|\beta'|^2}{4}+\frac{4}{3}\left(\frac{\beta^*}{2}-\mathrm{i}p\right)\left(\frac{\beta'}{2}+\mathrm{i}p'\right)+\frac{1}{3}\left(\frac{\beta^*}{2}-\mathrm{i}p\right)^2+\frac{1}{3}\left(\frac{\beta'}{2}+\mathrm{i}p'\right)^2\right).\quad(26)$$

利用 (21)式,完成(25)式中对 z_2, z_2^* 的积分,则有

$$\langle p,\beta|p',\beta'\rangle=\frac{3}{2}C\lim_{t\to 1}\frac{1}{\sqrt{1-t^2}}\times\exp\left\{\frac{2AA'-(A^2+A'^2)t}{1-t^2}\right\},\quad(27)$$

式中

$$A=\frac{(2p-p')+(\beta^*-\beta')/\mathrm{i}2}{\sqrt{3}},$$
$$A'=\frac{(2p'-p)+(\beta^*-\beta')/\mathrm{i}2}{\sqrt{3}}.\quad(28)$$

由 (22) 和(27)式,得

$$\langle p,\beta|p',\beta'\rangle=\frac{3}{2}C\sqrt{\pi}\exp\left(\frac{A^2+A'^2}{2}\right)\delta(A-A').\quad(29)$$

将(26),(28)式代入(29)式,并注意到 $\delta(\alpha x)=\frac{1}{|\alpha|}\delta(x)$,可得

$$\langle p,\beta|p',\beta'\rangle=\sqrt{\pi}\exp\left(-\frac{|\beta|^2+|\beta'|^2}{4}+\frac{1}{2}\beta'\beta^*\right)\times\delta(p-p').\quad(30)$$

由(30)式可见,对 β 而言,$|p,\beta\rangle$呈现相干态的非正交性,而对 p 而言则是正交的.

因此,$|p,\beta\rangle$具有超完备性,可以作为一个表象.

3. 态$|p,\beta\rangle$的纠缠性

对$|p,\beta\rangle$作如下傅里叶变换:

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\beta_1}{2\pi}|p,\beta\rangle\exp[-\mathrm{i}u\beta_1]$$
$$=\exp\left\{-\frac{p^2}{2}-\frac{\beta_2^2}{4}+\left(\frac{\beta_2}{2}+\mathrm{i}p\right)a_1^+ +\left(\frac{\beta_2}{2}-\mathrm{i}p\right)a_2^+ +\frac{1}{4}(a_1^+-a_2^+)^2\right\}\times\int_{-\infty}^{\infty}\frac{\mathrm{d}\beta_1}{2\pi}\exp\left\{-\frac{\beta_1^2}{4}+\left(\frac{a_1^++a_2^+}{2}-\mathrm{i}u\right)\beta_1\right\}|00\rangle.$$

完成上式的积分,得

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\beta_1}{2\pi}|p,\beta\rangle\exp[-\mathrm{i}u\beta_1]=\exp\left\{-\frac{\beta_2^2}{8}-\frac{u\beta_2}{2}-\frac{u^2}{2}\right\}|p_1\rangle\otimes|p_2\rangle,\quad(31)$$

式中

$$\left|p_1=\frac{1}{\sqrt{2}}\left[p+\left(\frac{\beta_2}{2}-u\right)\right]\right\rangle=\pi^{-1/4}\exp\left\{-\frac{p_1^2}{2}+\sqrt{2}\mathrm{i}p_1a_1^+ +\frac{a_1^{+2}}{2}\right\}|0\rangle,\quad(32)$$

$$\left|p_2=-\frac{1}{\sqrt{2}}\left[p-\left(\frac{\beta_2}{2}-u\right)\right]\right\rangle=\pi^{-1/4}\exp\left\{-\frac{p_2^2}{2}+\sqrt{2}\mathrm{i}p_2a_2^+ +\frac{a_2^{+2}}{2}\right\}|0\rangle.\quad(33)$$

(32),(33)式为动量本征矢. 对(31)式进行傅里叶逆变换得

$$|p,\beta\rangle=\exp\left\{-\frac{\beta_2^2}{8}\right\}\cdot\int_{-\infty}^{\infty}\mathrm{d}u\times\exp\left\{-\frac{u\beta_2}{2}-\frac{u^2}{2}+\mathrm{i}u\beta_1\right\}|p_1\rangle\otimes|p_2\rangle.\quad(34)$$

(34)式所示为$|p,\beta\rangle$的 Schmidt 分解,表明$|p,\beta\rangle$具有纠缠性.

事实上,做$|p,\beta\rangle$关于 β_2 的积分,可得

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\beta_2}{2\sqrt{\pi}}|p,\beta\rangle=\exp\left\{-\frac{p^2}{2}-\frac{\beta_1^2}{4}+\left(\frac{\beta_2}{2}+\mathrm{i}p\right)a_1^+ +\left(\frac{\beta_2}{2}-\mathrm{i}p\right)a_2^+ -a_1^+a_2^+\right\}|00\rangle.\quad(35)$$

将(35)式与双模连续变量纠缠态[11]

$$|\xi\rangle=\exp\left\{-\frac{|\xi^2|}{2}+\xi a_1^+ +\xi^*a_2^+ -a_1^+a_2^+\right\}|00\rangle\quad(\xi=\xi_1+\xi_2),\quad(36)$$

$$(\hat{x}_1+\hat{x}_2)|\xi\rangle=\sqrt{2}\xi_1|\xi\rangle,$$
$$(\hat{p}_1-\hat{p}_2)|\xi\rangle=\sqrt{2}\xi_2|\xi\rangle,\quad(37)$$

比较,我们得到

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\beta_2}{2\sqrt{\pi}}|p,\beta\rangle=\left|\xi=\frac{\beta_2}{2}+\mathrm{i}p\right\rangle,\quad(38)$$

这表明$|p,\beta\rangle$沿 β_2 超叠加为一个 EPR 纠缠态.

4. 态$|p,\beta\rangle$的物理产生方法

物理上可以用光分束器来实现$|p,\beta\rangle$态[12]. 分束器的功能由算符

$$\hat{B}=\exp[-\theta(a_1^+a_2-a_1a_2^+)]\quad(39)$$

来表示.让分束器的两个输入端分别输入理想单模压缩态 $|p=0\rangle_2=\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2$ 和真空态 $|0\rangle_1$,利用算符恒等式 $e^{\hat{A}}\hat{B}e^{-\hat{A}}=\hat{B}+[\hat{A},\hat{B}]+\frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]]+\frac{1}{3!}[\hat{A},[\hat{A},[\hat{A},\hat{B}]]]+\cdots$,我们得到

$$\hat{B}\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2\otimes|0\rangle$$
$$=\exp\left[\frac{1}{2}(a_2^+\cos\theta-a_1^+\sin\theta)^2\right]|00\rangle_1. \tag{40}$$

取 $\theta=\frac{\pi}{4}$,(40)式变为

$$\hat{B}\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2\otimes|0\rangle_1$$
$$=\exp\left[\frac{1}{4}(a_1^+-a_2^+)^2\right]|00\rangle. \tag{41}$$

再用平移算符 $D_1\left[\frac{1}{2}(\beta+ip)\right]$ 和 $D_2\left[\frac{1}{2}(\beta-ip)\right]$ 作用到上式,得

$$D_1\left[\frac{1}{2}(\beta+ip)\right]\cdot D_2\left[\frac{1}{2}(\beta-ip)\right]$$
$$\times\exp\left[\frac{1}{4}(a_1^+-a_2^+)^2\right]|00\rangle$$
$$=\exp\left[\frac{1}{4}(a_1^+-a_2^+)^2+\frac{ip}{2}(a_1^+-a_2^+)-\frac{p^2}{4}\right]$$
$$\times D_1|0\rangle\otimes D_2|0\rangle=|p,\beta\rangle, \tag{42}$$

这里平移算符的功能在实验上可以通过按照 p,β 值调制的激光场穿过分束器得以完成.

5. 结　论

本工作建立了一种基于 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 共同本征态的新表象,研究了这种表象的性质.结果表明:$|p,\beta\rangle$是 $\hat{p}_1-\hat{p}_2$ 与 a_1+a_2 的共同本征态.该本征态对 β 而言,呈现相干态的非正交性,而对 p 而言则是正交的,具有超完备性,可以作为一个表象.通过傅里叶变换及进一步的逆变换,可以得到 $|p,\beta\rangle$的 Schmidt 分解,同时发现 $|p,\beta\rangle$沿 β_2 超叠加为一个 EPR 纠缠态,是典型的相干纠缠态. 物理上可以用光分束器来实现 $|p,\beta\rangle$, 分束器的功能可由上节中的算符 $\hat{B}$ 来表示.让分束器的两个输入端分别输入理想单模压缩态 $|p=0\rangle_2=\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2$ 和真空态 $|0\rangle_1$,即可得到 $|p,\beta\rangle$态,(42)式中的平移算符表示对光场的调制作用.

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Coherent entangled state representation for the common eigenvector of $\hat{p}_1-\hat{p}_2$ and a_1+a_2 and its characteristics*

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(Received 4 September 2008; revised manuscript received 19 October 2008)

Abstract

The new two mode coherent entangled state representation $|p,\beta\rangle$ is proposed by the technique of integration within an ordered product of operators. Its characteristics and mechanism are analysed. The results show that, the common eigenvector $|p,\beta\rangle$ of $\hat{p}_1-\hat{p}_2$ and a_1+a_2 exhibits both the characteristics of the coherent state and entangled state. The $|p,\beta\rangle$ makes up a new quantum mechanical representation and possesses the completeness relation. The ideal single-mode squeezed state $|p=0\rangle_2=\exp\left[\frac{1}{2}a_2^{+2}\right]|0\rangle_2$ and the vacuum state $|0\rangle_1$ overlapping on a beam splitter may produce the coherent entangled state $|p,\beta\rangle$ at the output by the modulation of the laser field.

Keywords: IWOP technique, coherent entangled state representation, beam splitter
PACC: 0635

* Project supported by the Natural Science Foundation of Shandong Province, China (Grant No. Y2008A16), the Natural Science Foundation of Heze University of Shandong Province, China (Grant Nos. XY07WL01, XY08WL03), and the University Experimental Technology Foundation of Shandong Province, China (Grant No. S04W138).

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