第 58 卷 第 8 期 2009 年 8 月
1000-3290/2009/58(08)/5150-05
物 理 学 报
ACTA PHYSICA SINICA
Vol.58,No.8,August,2009

变质量完整动力学系统的共形不变性与守恒量*

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(2009 年 2 月 12 日收到;2009 年 2 月 16 日收到修改稿)

研究变质量完整力学系统在无限小变换下微分方程的共形不变性.提出了该系统共形不变性的概念,推导出变质量完整力学系统运动微分方程具有共形不变性,同时又是 Lie 对称性的充分必要条件.得到由共形不变性导致的 Noether 守恒量.

关键词:变质量系统,无限小变换,共形不变,守恒量
PACC:0320

1.引 言

动力学系统的对称性和守恒量的研究在现代数理科学中占有重要地位,也是分析力学的一个近代发展方向[1—11].近年来我国学者深入而广泛地研究了动力学系统的 Noether 对称性、Lie 对称性和形式不变性[4—10,12—21].共形不变性理论是 20 世纪 60—70 年代在规范场论,特别是引力规范场论中的热点课题[22,23],近期在动力学系统中有了新的应用研究[24,25].文献[24]利用几何方法研究了 Hamilton 系统的共形不变性,讨论了 Hamilton 系统共形不变性的几何结构及其与一般对称性的关系.文献[26]研究了 Birkhoff 系统的共形不变性并导出了 Noether 守恒量.由于近年来空间技术和其他工业技术的进步,变质量动力学系统的研究越来越受到重视[21],例如喷气飞机、火箭、卫星、航天器等,一般都是变质量系统.由于系统质量的变化,系统的动力学方程变得复杂,对其共形不变性的研究也比常质量系统要困难,并且常质量系统仅仅是其特殊情况,因此变质量系统共形不变性的理论自然适合于常质量系统.本文研究变质量完整动力学系统的共形不变性,给出变质量完整动力学系统共形不变性的定义和确定方程,并且找到了系统的守恒量,最后给出一个例子来验证本文研究成果的应用.

2.系统的运动微分方程

假设系统由 N 个质点组成,在瞬时 t,第 i 个质点的质量为 $m_i(i=1,\cdots,N)$,在瞬时 $t+\mathrm{d}t$,有质点分离(或并入)的微粒质量为 $\mathrm{d}m_i$.同时假设系统的位形由 n 个广义坐标 $q_s(s=1,\cdots,n)$ 来确定,并设质点质量依赖时间和广义坐标

$$m_i = m_i(t,\boldsymbol{q}) \quad (i=1,2,\cdots,N), \tag{1}$$

变质量完整力学系统的运动微分方程可表示为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = Q_s + P_s \quad (s=1,2,\cdots,n). \tag{2}$$

这里 $L=L(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 为系统的 Lagrange 函数,$Q_s=Q_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 为非势广义力,P_s 为广义反推力,有

$$P_s = \dot{m}(\boldsymbol{u}_i+\dot{\boldsymbol{r}}_i)\cdot\frac{\partial \boldsymbol{r}_i}{\partial q_s} - \frac{1}{2}\dot{\boldsymbol{r}}_i\cdot\dot{\boldsymbol{r}}_i\frac{\partial m_i}{\partial q_s}, \tag{3}$$

其中 $\boldsymbol{r}_i$ 和 $\dot{\boldsymbol{r}}_i$ 分别为第 i 个质点的矢径和速度,u_i 为微粒相对第 i 个质点的相对速度.

变质量完整动力学系统的运动微分方程(2)可以进一步表示为

$$A_{sk}\ddot{q}_k + B_s - Q_s - P_s = 0 \quad (s,k=1,2,\cdots,n), \tag{4}$$

其中

* 国家自然科学基金(批准号:10372053,10572021)和河南省自然科学基金(批准号:0311010900)资助的课题.
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$$A_{sk}=\frac{\partial^2 L}{\partial\dot q_s\partial\dot q_k},$$

$$B_s=\frac{\partial^2 L}{\partial\dot q_s\partial q_k}\dot q_k+\frac{\partial^2 L}{\partial\dot q_s\partial t}-\frac{\partial L}{\partial q_s}.$$

假设系统(2)非奇异,即

$$\det\left(\frac{\partial^2 L}{\partial\dot q_s\partial\dot q_k}\right)\neq 0,\tag{5}$$

由方程(2)可求出

$$\ddot q_s=\alpha_s(t,\boldsymbol q,\dot{\boldsymbol q}).\tag{6}$$

3. 变质量系统的共形不变性

令

$$F_s=A_{sk}\ddot q_k+B_s-Q_s-P_s.\tag{7}$$

定义　二阶微分方程 F_s,在无限小生成元 $\xi_0(t,\boldsymbol q,\dot{\boldsymbol q})$,$\xi_0(t,\boldsymbol q,\dot{\boldsymbol q})$ 的变换下,若满足

$$X^{(2)}F_s=\ell_s^kF_k,\tag{8}$$

则称二阶微分方程为共形不变.这里

$$t^*=t+\varepsilon\xi_0(t,\boldsymbol q,\dot{\boldsymbol q}),$$

$$x^*=x+\varepsilon\xi_0(t,\boldsymbol q,\dot{\boldsymbol q}),$$

$$X^{(0)}=\xi_k\frac{\partial}{\partial q_k}+\xi_0\frac{\partial}{\partial t},$$

$$X^{(1)}=X^{(0)}+(\dot\xi_k-\dot q_k\dot\xi_0)\frac{\partial}{\partial\dot q_k},$$

$$X^{(2)}=X^{(1)}+[(\dot\xi_k-\dot q_k\dot\xi_0)^{\cdot}-\ddot q_k\dot\xi_0]\frac{\partial}{\partial\ddot q_k}.$$

所以

$$\begin{aligned}X^{(2)}F_s&=A_{sk}[\ddot\xi_k-2\ddot q_k\dot\xi_0-\dot q_k\ddot\xi_0]\\&\quad+X^{(1)}(B_s-Q_s-P_s)+X^{(0)}(A_{sk})\ddot q_k\\&=A_{sk}\Big[\frac{\partial^2\xi_k}{\partial t\partial\dot q_m}\ddot q_m+\frac{\partial^2\xi_k}{\partial t\partial q_m}\dot q_m+\frac{\partial^2\xi_k}{\partial t^2}\\&\quad+\frac{\partial^2\xi_k}{\partial q_l\partial\dot q_m}\ddot q_m\dot q_l+\frac{\partial^2\xi_k}{\partial q_l\partial t}\dot q_l+\frac{\partial^2\xi_k}{\partial q_l\partial q_m}\dot q_m\dot q_l\\&\quad+\frac{\partial\xi_k}{\partial q_l}\ddot q_l+\frac{\mathrm d}{\mathrm dt}\left(\frac{\partial\xi_k}{\partial\dot q_l}\right)\ddot q_l+\frac{\partial\xi_k}{\partial\dot q_l}\dddot q_l\\&\quad-2\ddot q_k\xi_0-\dot q_k\Big(\frac{\partial^2\xi_0}{\partial t^2}+\frac{\partial^2\xi_0}{\partial t\partial q_m}\dot q_m+\frac{\partial^2\xi_0}{\partial t\partial\dot q_m}\ddot q_m\\&\quad+\frac{\partial^2\xi_0}{\partial q_l\partial t}\dot q_l+\frac{\partial^2\xi_0}{\partial q_l\partial q_m}\dot q_m\dot q_l+\frac{\partial^2\xi_0}{\partial q_l\partial\dot q_m}\ddot q_m\dot q_l\\&\quad+\frac{\partial\xi_0}{\partial q_l}\ddot q_l+\frac{\mathrm d}{\mathrm dt}\left(\frac{\partial\xi_0}{\partial\dot q_l}\right)\ddot q_l+\frac{\partial\xi_0}{\partial q_l}\dddot q\Big)\Big]\\&\quad+X^{(0)}(B_s-Q_s-P)\\&\quad+\Big[\frac{\partial\xi_k}{\partial q_l}\dot q_l+\frac{\partial\xi_k}{\partial t}+\frac{\partial\xi_k}{\partial\dot q_l}\ddot q_l\\&\quad-\dot q_k\left(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_l}\dot q_l+\frac{\partial\xi_0}{\partial\dot q_l}\ddot q_l\right)\Big]\\&\quad\times\frac{\partial(B_s-Q_s-P_s)}{\partial\dot q_k}+X^{(0)}(A_{sk})\ddot q_k,\end{aligned}\tag{9}$$

$$\begin{aligned}X^{(2)}F_s|_{F_s=0}&=A_{sk}\Big[\frac{\partial^2\xi_k}{\partial t\partial\dot q_m}\alpha_m+\frac{\partial^2\xi_k}{\partial t\partial q_m}\dot q_m+\frac{\partial^2\xi_k}{\partial t^2}\\&\quad+\frac{\partial^2\xi_k}{\partial q_l\partial\dot q_m}\alpha_m\dot q_l+\frac{\partial^2\xi_k}{\partial q_l\partial t}\dot q_l+\frac{\partial^2\xi_k}{\partial q_l\partial q_m}\dot q_m\\&\quad+\frac{\partial\xi_k}{\partial q_l}\alpha_l+\frac{\mathrm d}{\mathrm dt}\left(\frac{\partial\xi_k}{\partial\dot q_l}\right)\alpha_l+\frac{\partial\xi_k}{\partial\dot q_l}\dddot q_l-2\alpha_k\dot\xi_0\\&\quad-\dot q_k\Big(\frac{\partial^2\xi_0}{\partial t^2}+\frac{\partial^2\xi_0}{\partial t\partial q_m}\dot q_m+\frac{\partial^2\xi_0}{\partial t\partial\dot q_m}\alpha_m\\&\quad+\frac{\partial^2\xi_0}{\partial q_l\partial t}\dot q_l+\frac{\partial^2\xi_0}{\partial q_l\partial q_m}\dot q_m\dot q_l+\frac{\partial^2\xi_0}{\partial q_l\partial\dot q_m}\alpha_m\dot q_l\\&\quad+\frac{\partial\xi_0}{\partial q_l}\alpha_l+\frac{\mathrm d}{\mathrm dt}\left(\frac{\partial\xi_0}{\partial\dot q_l}\right)\alpha_l+\frac{\partial\xi_0}{\partial q_l}\dddot q_l\Big)\Big]\\&\quad+X^{(0)}(B_s-Q_s-P_s)\\&\quad+\Big[\frac{\partial\xi_k}{\partial q_l}\dot q_l+\frac{\partial\xi_k}{\partial t}+\frac{\partial\xi_k}{\partial\dot q_l}\alpha_l\\&\quad-\dot q_k\left(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_l}\dot q_l+\frac{\partial\xi_0}{\partial\dot q_l}\alpha_l\right)\Big]\\&\quad\times\frac{\partial(B_s-Q_s-P_s)}{\partial\dot q_k}+X^{(0)}(A_{sk})\alpha_k.\end{aligned}\tag{10}$$

因此,将(9)式与(10)式相减后可得

$$\begin{aligned}X^{(2)}F_s-X^{(2)}F_s|_{F_s=0}&=A_{sk}\Big[\frac{\partial^2\xi_k}{\partial t\partial\dot q_m}(\ddot q_m-\alpha_m)\\&\quad+\frac{\partial^2\xi_k}{\partial q_l\partial\dot q_m}(\ddot q_m-\alpha_m)\dot q_l\\&\quad+\frac{\partial\xi_k}{\partial q_l}(\ddot q_l-\alpha_l)\\&\quad+\frac{\mathrm d}{\mathrm dt}\left(\frac{\partial\xi_k}{\partial\dot q_l}\right)(\ddot q_l-\alpha_l)\\&\quad-2(\ddot q_k-\alpha_k)\dot\xi_0\\&\quad-\dot q_k\Big(\frac{\partial^2\xi_0}{\partial t\partial\dot q_m}(\ddot q_m-\alpha_m)\\&\quad+\frac{\partial^2\xi_0}{\partial q_l\partial\dot q_m}(\ddot q_m-\alpha_m)\dot q_l\end{aligned}$$

$$+\frac{\partial\xi_0}{\partial q_l}(\ddot{q}_l-\alpha_l)+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_0}{\partial\dot{q}_l}\right)(\ddot{q}_l-\alpha_l)\Big)\Big]$$

$$+\left[\frac{\partial\xi_k}{\partial q_l}(\ddot{q}_l-\alpha_l)-\dot{q}_k\frac{\partial\xi_0}{\partial q_l}(\ddot{q}_l-\alpha_l)\right]$$

$$\times\frac{\partial(B_s-Q_s-P_s)}{\partial\dot{q}_k}$$

$$+X^{(0)}(A_{sk})(\ddot{q}_k-\alpha_k).\tag{11}$$

因为

$$\begin{aligned}\ddot{q}_m-\alpha_m&=\ddot{q}_m+A^{mr}(B_r-Q_r-P_r)\\&=A^{mr}(A_{rm}\ddot{q}_m+B_r-Q_r-P_r)\\&=A^{mr}F_r,\end{aligned}\tag{12}$$

所以(11)式可变为

$$X^{(2)}F_s-X^{(2)}|_{F_s=0}=\left\{A_{sk}\left[\frac{\partial^2\xi_k}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_k}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_k}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_k}{\partial\dot{q}_m}\right)-\dot{q}_k\left(\frac{\partial^2\xi_0}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_0}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_0}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_0}{\partial\dot{q}_m}\right)\right)\right]A^{mr}-2\delta_s^r\dot{\xi}_0+\left(\frac{\partial\xi_k}{\partial\dot{q}_m}-\dot{q}_k\frac{\partial\xi_0}{\partial q_m}\right)A^{mr}\frac{\partial(B_s-Q_s-P_s)}{\partial\dot{q}_k}+X^{(0)}(A_{sk})A^{kr}\right\}F_r.\tag{13}$$

令

$$B_s^r=A_{sk}\left[\frac{\partial^2\xi_k}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_k}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_k}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_k}{\partial\dot{q}_m}\right)-\dot{q}_k\left(\frac{\partial^2\xi_0}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_0}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_0}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_0}{\partial\dot{q}_m}\right)\right)\right]A^{mr}-2\delta_s^r\dot{\xi}_0+\left(\frac{\partial\xi_k}{\partial\dot{q}_m}-\dot{q}_k\frac{\partial\xi}{\partial q_m}\right)A^{mr}\frac{\partial(B_s-Q_s-P_s)}{\partial\dot{q}_k}+X^{(0)}(A_{sk})A^{kr},\tag{14}$$

则

$$X^{(2)}F_s-X^{(2)}F_s|_{F_s=0}=B_s^rF_r.\tag{15}$$

若变质量动力学系统具有共形不变性和 Lie 对称性,可得

$$\ell_s^rF_r-B_s^rF_r=X^{(2)}F_s|_{F_s=0}=0,\tag{16}$$

即

$$(\ell_s^r-B_s^r)F_r=0.\tag{17}$$

由(17)式可得

$$\ell_s^r=B_s^r.\tag{18}$$

命题 1 对于变质量完整动力学系统,其共形不变性同时又是 Lie 对称性的充分必要条件是生成元满足

$$\begin{aligned}\ell_s^r&=B_s^r\\&=A_{sk}\Big[\frac{\partial^2\xi_k}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_k}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_k}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_k}{\partial\dot{q}_m}\right)\\&\quad-\dot{q}_k\left(\frac{\partial^2\xi_0}{\partial t\partial\dot{q}_m}+\frac{\partial^2\xi_0}{\partial q_l\partial\dot{q}_m}\dot{q}_l+\frac{\partial\xi_0}{\partial q_m}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_0}{\partial\dot{q}_m}\right)\right)\Big]A^{mr}\\&\quad-2\delta_s^r\dot{\xi}_0+\left(\frac{\partial\xi_k}{\partial\dot{q}_m}-\dot{q}_k\frac{\partial\xi_0}{\partial q_m}\right)A^{mr}\frac{\partial(B_s-Q_s-P_s)}{\partial\dot{q}_k}\\&\quad+X^{(0)}(A_{sk})A^{kr}.\end{aligned}\tag{19}$$

4. 共形不变性与守恒量

由变质量完整动力学系统的共形不变性,通过 Noether 对称性可以导出 Noether 守恒量.

命题 2 对于变质量动力学系统,如果共形不变性的无限小生成元和规范函数 G_N 满足 Noether 等式

$$L\dot{\xi}_0+X^{(1)}(L)+(Q_s+P_s)(\xi_s-\dot{q}_s\xi_0)+\dot{G}_N=0,\tag{20}$$

则共形不变性导致 Noether 守恒量

$$I_N=L\xi_0+\frac{\partial L}{\partial\dot{q}_s}(\xi_s-\dot{q}_s\xi_0)+G_N=\mathrm{const}.\tag{21}$$

证明

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t}&=\dot{L}\xi_0+L\dot{\xi}_0+\frac{\partial L}{\partial\dot{q}_s}(\dot{\xi}_s-\ddot{q}_s\xi_0-\dot{q}_s\dot{\xi}_0)\\&\quad+\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}_s}(\xi_s-\dot{q}_s\xi_0)\\&\quad-L\dot{\xi}_0-X^{(1)}(L)-(Q_s+P_s)(\xi_s-\dot{q}_s\xi_0)\\&=(\xi_s-\dot{q}_s\xi_0)\left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}_s}-\frac{\partial L}{\partial q_s}-Q_s-P_s\right)\\&=0.\end{aligned}$$

5. 算　例

设一变质量动力学系统的 Lagrange 函数为

$$L = \frac{1}{2}m(t)(\dot{q}_1^2 + \dot{q}_2^2), \tag{22}$$

其中

$$m(t) = m_0(1 - \gamma t) \quad (m_0, \gamma = \text{const.}), \tag{23}$$

非势广义力为

$$Q_1 = Q_2 = 0. \tag{24}$$

同时设微粒分离的绝对速度为零.

由假设微粒分离的绝对速度为零,利用(3)式得

$$P_1 = P_2 = 0. \tag{25}$$

所以其等价的微分方程为

$$F = \begin{pmatrix} m\ddot{q}_1 - m_0\gamma\dot{q}_1 \\ m\ddot{q}_2 - m_0\gamma\dot{q}_2 \end{pmatrix} = 0. \tag{26}$$

当无限小变量

$$\begin{aligned} \xi_0 &= 0, \\ \xi_1 &= q_2 + m\dot{q}_1, \\ \xi_2 &= -q_1 \end{aligned} \tag{27}$$

时,

$$\begin{aligned} X^{(2)} &= \xi_0\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q_s} + (\dot{\xi}_s - \dot{q}_s\dot{\xi}_0)\frac{\partial}{\partial \dot{q}_s} \\ &\quad + (\ddot{\xi}_s - 2\ddot{q}_s\dot{\xi}_0 - \dot{q}_s\ddot{\xi}_0)\frac{\partial}{\partial \ddot{q}_s} \\ &= \xi_s\frac{\partial}{\partial q_s} + \dot{\xi}_s\frac{\partial}{\partial \dot{q}_s} + \ddot{\xi}_s\frac{\partial}{\partial \ddot{q}_s}, \end{aligned}$$

所以

$$\begin{aligned} X^{(2)}F &= \left(\xi_s\frac{\partial}{\partial q_s} + \dot{\xi}_s\frac{\partial}{\partial \dot{q}_s} + \ddot{\xi}_s\frac{\partial}{\partial \ddot{q}_s}\right)\begin{pmatrix} m\ddot{q}_1 - m_0\gamma\dot{q}_1 \\ m\ddot{q}_2 - m_0\gamma\dot{q}_2 \end{pmatrix} \\ &= \begin{pmatrix} m\ddot{q}_2 - m_0\gamma\dot{q}_2 - m_0\gamma(m\ddot{q}_1 - m_0\gamma\dot{q}_1) \\ -m\ddot{q}_1 + m_0\gamma\dot{q}_1 \end{pmatrix} \\ &= \begin{pmatrix} -m_0\gamma & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} m\ddot{q}_1 - m_0\gamma\dot{q}_1 \\ m\ddot{q}_2 - m_0\gamma\dot{q}_2 \end{pmatrix}. \end{aligned}$$

因此共形因子为

$$\ell_s^r = \begin{pmatrix} -m_0\gamma & 1 \\ -1 & 0 \end{pmatrix}. \tag{28}$$

由(19)式可得变质量完整动力学系统共形不变的充分必要条件为

$$\ell_s^r = \begin{pmatrix} -m_0\gamma & 1 \\ -1 & 0 \end{pmatrix}. \tag{29}$$

显然其结果与(28)式相同. 共形不变的确定方程为

$$X^{(2)}F = \begin{pmatrix} -m_0\gamma & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} m\ddot{q}_1 - m_0\gamma\dot{q}_1 \\ m\ddot{q}_2 - m_0\gamma\dot{q}_2 \end{pmatrix}. \tag{30}$$

此时系统是 Lie 对称的,同时是共形不变的. 把(27)式代入结构方程(20)得

$$G_{\mathrm{N}} = -\frac{1}{2}(m\dot{q}_1)^2.$$

由(21)式得到系统的 Noether 守恒量为

$$I = \frac{1}{2}(m\dot{q}_1)^2. \tag{31}$$

6. 结　论

变质量完整力学系统在无限小变换下微分方程的共形不变性，可通过 Lie 对称性找到确定方程中的共形因子 ℓ_s^r ,该共形因子也就是系统的共形不变性同时是 Lie 对称性的充分必要条件. 并且得到该系统由共形不变性导致的 Noether 守恒量.

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Conformal invariance and conserved quantity for holonomic mechanical systems with variable mass*

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(Received 12 February 2009; revised manuscript received 16 February 2009)

Abstract

In this paper, the conformal invariance of holonomic mechanical systems with variable mass is studied. The necessary and the sufficient conditions of conformal invariance of the system are obtained and shown to be also those of Lie symmetry simultaneously. Finally the Noether conserved quantities of conformal invariance are presented.

Keywords: variable mass systems, infinitesimal transformations, conformal invariance, conserved quantities

PACC: 0320

* Project supported by the National Natural Sciencc Foundation (Grant Nos. 10372053, 10572021) and the Natural Science Foundation of Henan Province, China (Grant No. 0311010900).

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