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对于具有大范围运动和非线性变形的柔性梁的有限元动力学建模*

和兴锁† 邓峰岩 吴根勇 王 睿

(西北工业大学工程力学系,西安 710072)

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研究具有大范围运动和非线性变形的柔性梁的有限元动力学建模.采用有限元方法对梁结构进行离散,利用 Lagrange 方程建立系统的精确动力学方程.该方程不仅增加了新的表征纵向、横向、侧向弯曲变形,以及扭转变形的耦合项,同时包含了变形运动与大范围运动之间的相互耦合项.

关键词:大范围运动,非线性变形,柔性梁,有限元动力学建模

PACC:0320,0547,4610

1. 引 言

随着柔性结构朝着大型、复杂、大范围运动等方向发展,系统中出现了许多用线性理论无法解释的现象,从而引起人们对结构的非线性特性研究越来越重视[1—4].对于具有大范围运动和非线性变形的柔性梁,建立其刚-柔耦合系统动力学方程[5—8],可用运动弹性动力分析法(KED 法)、零次近似模型(将柔性体上任一点的位移分解为随连体坐标系的刚体运动和柔性变形两部分)、一次耦合模型(考虑了梁、板等结构的轴向、横向变形对纵向变形的影响,考虑了变形的几何非线性作用以及由此引起的刚度项的变化.但在计算变形时忽略了扭转变形的影响;在考虑变形相互耦合作用时,只考虑了横向变形(平面)或横向、侧向变形(空间)所引起的轴向缩短,而未考虑这些变形在横向、侧向的耦合作用).但以往的建模方法为了使计算简单便于应用,在建模过程中都忽略了对系统影响较小的部分,因而所考虑的耦合因素有所欠缺[9—11].为了提高求解精度,本文将对柔性梁的有限元建模方法进行研究.

2. 平面柔性梁的运动动力学分析

为了对作任意大范围的柔性梁进行运动动力学分析,建立如图 1 所示的坐标系.其中 e^0 为惯性坐标系,e^1 为固结在未变形柔性梁中性轴上的连体坐标系.

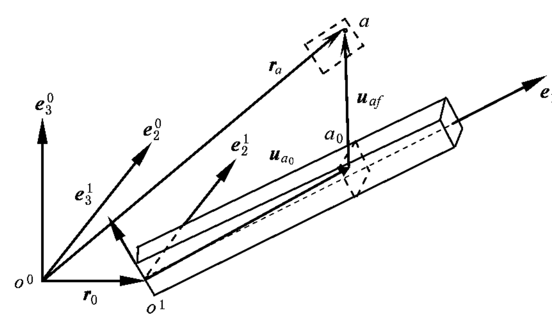

图 1 柔性梁段的坐标系选取

下面针对作大范围运动平面柔性梁的运动和变形特性(图 2),导出其动力学方程.

2.1. 系统动能

设梁非中线上任一点 a 的位置矢量在连体坐标系 e^1 下的坐标列阵[5]为

$$\boldsymbol{u}_a = \boldsymbol{u}_{a_0} + \boldsymbol{u}_{af}. \tag{1}$$

将梁分为 n 个单元,l_e 为单元长度.设 $N_{e,1}$,$N_{e,2}$为第 e 个单元的形函数阵,该单元内任一点 a 对应的位移量依次为 s_0,v_0,即轴向伸缩变形、横向弯曲变形,则

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† E-mail:xingsuoh@nwpu.edu.cn

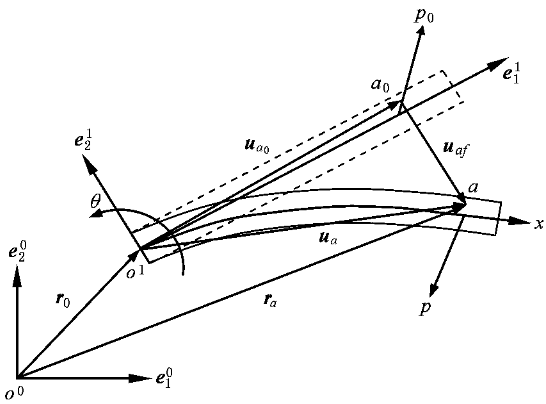

图 2 作大范围运动的平面柔性梁段

$$s_0(\bar{x},t) = \boldsymbol{N}_{e,1}(\bar{x})\,\boldsymbol{q}_{e,f}(t),$$

$$v_0(\bar{x},t) = \boldsymbol{N}_{e,2}(\bar{x})\,\boldsymbol{q}_{e,f}(t), \tag{2}$$

$$\boldsymbol{N}_{e,1}(\bar{x}) = [N_{ms} \quad 0 \quad 0 \quad N_{ks} \quad 0 \quad 0], \tag{3}$$

$$\boldsymbol{N}_{e,2}(\bar{x}) = [0 \quad N_{mv} \quad N_{m\theta 3} \quad 0 \quad N_{kv} \quad N_{k\theta 3}], \tag{4}$$

$$\boldsymbol{q}_{e,f}(t) = [u_m \quad v_m \quad \theta_{3m} \quad u_k \quad v_k \quad \theta_{3k}]^{\mathrm{T}}. \tag{5}$$

设 $\boldsymbol{q}_{f0}(t)$ 为总体变形位移列阵

$$\boldsymbol{q}_{e,f}(t) = \boldsymbol{B}_e \boldsymbol{q}_{f0}(t), \tag{6}$$

其中,$\boldsymbol{B}_e$ 为由单元编号决定的布尔(Boole)指示矩阵[12,13]. 引入转化矩阵 $\boldsymbol{R}$,$\boldsymbol{q}_{f0}(t)$ 可表示为

$$\boldsymbol{q}_{f0}(t) = \boldsymbol{R}\boldsymbol{q}_f(t),$$

$\boldsymbol{q}_f(t)$ 为独立的总体变形位移列阵,根据梁的边界条件,确定不同的转化矩阵 $\boldsymbol{R}$,例如对于一端铰支一端自由结构,

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{0}_{2\times(3n+1)} \\ \boldsymbol{I}_{(3n+1)\times(3n+1)} \end{bmatrix}, \tag{7}$$

$$\boldsymbol{q}_f(t) = [\theta_{32} \quad \cdots \quad u_{n+1} \quad v_{n+1} \quad \theta_{3n+1}]^{\mathrm{T}}. \tag{8}$$

对于两端铰支结构,$\boldsymbol{R}$,$\boldsymbol{q}_f(t)$ 分别为

$$\boldsymbol{R} = \begin{bmatrix} \multicolumn{2}{c}{\boldsymbol{0}_{2\times(3n-1)}} \\ \boldsymbol{I}_{(3n-2)\times(3n-2)} & \boldsymbol{0}_{(3n-2)\times 1} \\ \multicolumn{2}{c}{\boldsymbol{0}_{2\times(3n-1)}} \\ \boldsymbol{0}_{1\times(3n-2)} & \boldsymbol{I}_{1\times 1} \end{bmatrix}, \tag{9}$$

$$\boldsymbol{q}_f(t) = [\theta_{31} \quad u_2 \quad v_2 \quad \cdots \quad u_n \quad v_n \quad \theta_{3n} \quad \theta_{3n+1}]^{\mathrm{T}}. \tag{10}$$

将(7)式代入(2)式,可得

$$s_0(\bar{x},t) = \boldsymbol{N}_1(\bar{x})\,\boldsymbol{q}_f(t),$$

$$v_0(\bar{x},t) = \boldsymbol{N}_2(\bar{x})\,\boldsymbol{q}_f(t), \tag{11}$$

其中

$$\boldsymbol{N}_1(\bar{x}) = \boldsymbol{N}_{e,1}(\bar{x})\,\boldsymbol{B}_e\boldsymbol{R},$$

$$\boldsymbol{N}_2(\bar{x}) = \boldsymbol{N}_{e,2}(\bar{x})\,\boldsymbol{B}_e\boldsymbol{R}.$$

由上面各式可得[5] u_a 和 u_{af},这样柔性梁上任意一点 a 的矢径在惯性坐标系下可表示为

$$\boldsymbol{r}_a = \boldsymbol{r}_0 + \boldsymbol{A}\boldsymbol{u}_a = \boldsymbol{r}_0 + \boldsymbol{A}(\boldsymbol{u}_{a_0} + \boldsymbol{u}_{af}), \tag{12}$$

式中,$\boldsymbol{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 为连体坐标系相对于惯性坐标系的旋转矩阵,(12)式对时间求一次导数,得任一点 a 在惯性坐标系下的速度 $\dot{\boldsymbol{r}}_a$. 这样就可得到平面柔性梁的动能

$$T = T_f = \frac{1}{2}\int_V \rho \dot{\boldsymbol{r}}_a^{\mathrm{T}} \cdot \dot{\boldsymbol{r}}_a \mathrm{d}V$$

$$= \frac{1}{2}[\dot{\boldsymbol{r}}_0^{\mathrm{T}} \quad \dot{\theta} \quad \dot{\boldsymbol{q}}_f^{\mathrm{T}}] \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} \\ \boldsymbol{M}_{31} & \boldsymbol{M}_{32} & \boldsymbol{M}_{33} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \dot{\theta} \\ \dot{\boldsymbol{q}}_f \end{bmatrix}. \tag{13}$$

式中 ρ 为密度,设 A,l 分别为梁的横截面积和长度,有

$$\boldsymbol{M}_{11} = \rho A l \boldsymbol{I}_{2\times 2},$$

$$\boldsymbol{M}_{12} = \boldsymbol{M}_{21}^{\mathrm{T}} = \boldsymbol{A}\begin{bmatrix} -\boldsymbol{Y}_2\boldsymbol{q}_f \\ E_1 + \boldsymbol{Y}_1\boldsymbol{q}_f - \boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{C}\boldsymbol{q}_f/2 \end{bmatrix},$$

$$\boldsymbol{M}_{13} = \boldsymbol{M}_{31}^{\mathrm{T}} = \boldsymbol{A}\begin{bmatrix} \boldsymbol{Y}_1 - \boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{C} \\ \boldsymbol{Y}_2 \end{bmatrix},$$

$$\boldsymbol{M}_{22} = J_{11} + J_{22} + 2\boldsymbol{Z}_{11}\boldsymbol{q}_f - \boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{D}\boldsymbol{q}_f + \boldsymbol{q}_f^{\mathrm{T}}(\boldsymbol{W}_{11} + \boldsymbol{W}_{22} + \overline{\boldsymbol{W}}_{22})\boldsymbol{q}_f \underline{- \boldsymbol{q}_f^{\mathrm{T}}\overline{\boldsymbol{W}}_{22}\boldsymbol{q}_f},$$

$$\boldsymbol{M}_{23} = \boldsymbol{M}_{32}^{\mathrm{T}} = \overline{\boldsymbol{Y}}_2 + \boldsymbol{Z}_{12} + \boldsymbol{q}_f^{\mathrm{T}}(\boldsymbol{W}_{12} - \boldsymbol{W}_{21}) \underline{- \boldsymbol{q}_f^{\mathrm{T}}\int_V \rho y^2 \boldsymbol{V}_{12}\mathrm{d}V},$$

$$\boldsymbol{M}_{33} = \boldsymbol{W}_{11} + \boldsymbol{W}_{22} + \overline{\boldsymbol{W}}_{22},$$

$$\boldsymbol{C} = \int_V \rho \boldsymbol{H}\mathrm{d}V = \sum_{e=1}^{n}\int_0^{l_e} \rho A \boldsymbol{H}\mathrm{d}\bar{x}$$

$$= \boldsymbol{R}^{\mathrm{T}}\sum_{e=1}^{n}\boldsymbol{B}_e^{\mathrm{T}}\int_0^{l_e}\int_0^{\bar{x}} \rho A \frac{\partial \boldsymbol{N}_{e,2}^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_{e,2}}{\partial \bar{x}} \mathrm{d}\bar{x}\,\mathrm{d}\bar{x}\,\boldsymbol{B}_e\boldsymbol{R}$$

$$+ \boldsymbol{R}^{\mathrm{T}}\sum_{e=1}^{n}\sum_{i=1}^{e-1}\boldsymbol{B}_i^{\mathrm{T}}\int_0^{l_e}\int_0^{l_i} \rho A \frac{\partial \boldsymbol{N}_{i,2}^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_{i,2}}{\partial \bar{x}} \mathrm{d}\bar{x}\,\mathrm{d}\bar{x}\,\boldsymbol{B}_i\boldsymbol{R},$$

$$\boldsymbol{D} = \int_V \rho x \boldsymbol{H}\mathrm{d}V = \sum_{e=1}^{n}\int_0^{l_e} \rho A(x_e + \bar{x})\boldsymbol{H}\mathrm{d}\bar{x}$$

$$= \boldsymbol{R}^{\mathrm{T}}\sum_{e=1}^{n}\boldsymbol{B}_e^{\mathrm{T}}\int_0^{l_e}(x_e + \bar{x})\int_0^{\bar{x}} \rho A \frac{\partial \boldsymbol{N}_{e,2}^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_{e,2}}{\partial \bar{x}} \mathrm{d}\bar{x}\,\mathrm{d}\bar{x}\,\boldsymbol{B}_e\boldsymbol{R} + \boldsymbol{R}^{\mathrm{T}}\sum_{e=1}^{n}\sum_{i=1}^{e-1}\boldsymbol{B}_i^{\mathrm{T}}\int_0^{l_e}(x_e + \bar{x})$$

$$\times \int_0^{l_i} \rho A \frac{\partial \boldsymbol{N}_{i,2}^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_{i,2}}{\partial \bar{x}} \mathrm{d}\bar{x}\, \mathrm{d}\bar{x}\, \boldsymbol{B}_i \boldsymbol{R}.$$

与零次耦合模型相比较增加了耦合项 C，D 及划线项，与一次耦合模型相比增加了划线项.

2.2. 系统势能

平面柔性梁上任一点 a 的正应变为

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right) \\
&= \frac{\partial s_0}{\partial x} - \frac{1}{2}\left(\frac{\partial v_0}{\partial x}\right)^2 - \frac{\partial^2 v_0}{\partial x^2} y + \frac{\partial^2 s_0}{\partial x^2} \\
&\quad \cdot \frac{\partial v_0}{\partial x} y + \frac{\partial s_0}{\partial x} \cdot \frac{\partial^2 v_0}{\partial x^2} y \\
&\quad + \frac{1}{2}\left(\frac{\partial s_0}{\partial x} - \frac{1}{2}\left(\frac{\partial v_0}{\partial x}\right)^2 - \frac{\partial^2 v_0}{\partial x^2} y\right. \\
&\quad \left. + \frac{\partial^2 s_0}{\partial x^2} \cdot \frac{\partial v_0}{\partial x} y + \frac{\partial s_0}{\partial x} \cdot \frac{\partial^2 v_0}{\partial x^2} y\right)^2 \\
&\quad + \frac{1}{2}\left(\frac{\partial v_0}{\partial x} - \frac{\partial v_0}{\partial x} \cdot \frac{\partial^2 v_0}{\partial x^2} y\right)^2.
\end{aligned}$$

略去二次以上耦合项，有

$$\varepsilon_{xx} = \frac{\partial s_0}{\partial x} - y \frac{\partial^2 v_0}{\partial x^2}. \tag{14}$$

由于 $\varepsilon_{xy} = 0$，柔性梁为各向同性线性材料，应力-应变关系为

$$\sigma_{xx} = E\varepsilon_{xx}, \quad \tau_{xy} = 2G\varepsilon_{xy}, \tag{15}$$

梁的应变势能为

$$U = \frac{1}{2}\int_V (\sigma_{xx}\varepsilon_{xx} + 2\tau_{xy}\varepsilon_{xy})\, \mathrm{d}V. \tag{16}$$

将(14)，(15)式代入(16)式，得

$$U = \frac{1}{2}\int_V (E\varepsilon_{xx}^2 + 4G\varepsilon_{xy}^2)\, \mathrm{d}V. \tag{17}$$

代入 ε_{xx}，ε_{xy} 得

$$U = \frac{1}{2}\int_0^l \left(EA\left(\frac{\partial s_0}{\partial x}\right)^2 + EI_{zz}\left(\frac{\partial^2 v_0}{\partial x^2}\right)^2\right) \mathrm{d}x. \tag{18}$$

将(11)式代入，得

$$U = \frac{1}{2} \boldsymbol{q}_f^{\mathrm{T}} \boldsymbol{K}_f \boldsymbol{q}_f, \tag{19}$$

其中

$$\begin{aligned}
\boldsymbol{K}_f = \sum_{e=1}^{n} \boldsymbol{R}^{\mathrm{T}} \boldsymbol{B}_e^{\mathrm{T}} \int_0^{l_e} \Bigg(& EA \frac{\partial \boldsymbol{N}_{e,1}^{T}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_{e,1}}{\partial \bar{x}} \\
& + EI_{zz} \frac{\partial^2 \boldsymbol{N}_{e,2}^{\mathrm{T}}}{\partial \bar{x}^2} \cdot \frac{\partial^2 \boldsymbol{N}_{e,2}}{\partial \bar{x}^2}\Bigg) \mathrm{d}\bar{x}\, \boldsymbol{B}_e \boldsymbol{R}.
\end{aligned}$$

2.3. 外力对应的广义力

设柔性梁的分布外力(含体力)坐标列阵为

$$\boldsymbol{f}_w = [f_1 \quad f_2]^{\mathrm{T}}. \tag{20}$$

系统的广义坐标为

$$\boldsymbol{q} = [\boldsymbol{r}_0^{\mathrm{T}} \quad \theta \quad \boldsymbol{q}_f^{\mathrm{T}}]^{\mathrm{T}}. \tag{21}$$

令 $\delta \boldsymbol{r}_0 \neq \boldsymbol{0}$，$\delta\theta = 0$，$\delta \boldsymbol{q}_f = \boldsymbol{0}$，则外力做功为

$$\int_V \delta W_{f_w}^{\delta r_0} \mathrm{d}V = \int_V \delta \boldsymbol{r}_0^{\mathrm{T}} \cdot \boldsymbol{f}_w \mathrm{d}V, \tag{22}$$

得 $\boldsymbol{r}_0$ 对应的广义力为

$$\boldsymbol{Q}_{r_0} = \frac{\int_V \delta W_{f_w}^{\delta r_0} \mathrm{d}V}{\delta \boldsymbol{r}_0} = \int_V \boldsymbol{f}_w \mathrm{d}V. \tag{23}$$

令 $\delta \boldsymbol{r}_0 = \boldsymbol{0}$，$\delta\theta \neq 0$，$\delta \boldsymbol{q}_f = \boldsymbol{0}$，梁上任一点的位移矢量在惯性坐标系下为

$$\delta \boldsymbol{u}_\theta = \delta(\boldsymbol{A}\boldsymbol{u}_a) = \boldsymbol{A}\tilde{\boldsymbol{I}}\delta\theta \boldsymbol{u}_a, \tag{24}$$

则外力做功为

$$\begin{aligned}
\int_V \delta W_{f_w}^{\delta\theta} \mathrm{d}V &= \int_V \delta \boldsymbol{u}_\theta^{\mathrm{T}} \cdot \boldsymbol{f}_w \mathrm{d}V \\
&= \int_V \delta\theta (u_{a1} f_2' - u_{a2} f_1')\, \mathrm{d}V,
\end{aligned} \tag{25}$$

其中，$\boldsymbol{f}_w' = \boldsymbol{A}^{\mathrm{T}} \boldsymbol{f}_w = [f_1' \quad f_2']^{\mathrm{T}}$ 为外力在连体系下的坐标列阵.

对应于 θ 的广义力为

$$\begin{aligned}
Q_\theta &= \frac{\int_V \delta W_{f_w}^{\delta\theta} \mathrm{d}V}{\delta\theta} = \int_V \boldsymbol{u}_a^{\mathrm{T}} \boldsymbol{f}_w' \mathrm{d}V \\
&= \int_V (u_{a1} f_2' - u_{a2} f_1')\, \mathrm{d}V.
\end{aligned} \tag{26}$$

令 $\delta \boldsymbol{r}_0 = \boldsymbol{0}$，$\delta\psi = 0$，$\delta \boldsymbol{q}_f \neq \boldsymbol{0}$，则外力做功为

$$\begin{aligned}
\int_V \delta W_{f_w}^{\delta q_f} \mathrm{d}V &= \int_V (\boldsymbol{A}\delta \boldsymbol{u}_{af})^{\mathrm{T}} \cdot \boldsymbol{f}_w \mathrm{d}V \\
&= \int_V \delta \boldsymbol{q}_f^{T} \Bigg(\left(\boldsymbol{N}_1 - y \frac{\partial \boldsymbol{N}_2}{\partial \bar{x}} - \boldsymbol{q}_f^{\mathrm{T}} \boldsymbol{H} + y \boldsymbol{q}_f^{\mathrm{T}} \boldsymbol{V}_{12} \right)^{\mathrm{T}} f_1' \\
&\quad + \left(\boldsymbol{N}_2 - y \boldsymbol{q}_f^{\mathrm{T}} \frac{\partial \boldsymbol{N}_2^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_2}{\partial \bar{x}} \right)^{\mathrm{T}} f_2' \Bigg) \\
&\quad \times \mathrm{d}V.
\end{aligned} \tag{27}$$

从而得 $\boldsymbol{q}_f$ 对应的广义力为

$$\begin{aligned}
\boldsymbol{Q}_{q_f} &= \frac{\int_V \delta W_{f_w}^{\delta q_f} \mathrm{d}V}{\delta \boldsymbol{q}_f^{\mathrm{T}}} \\
&= \int_V \Bigg(\left(\boldsymbol{N}_1 - y \frac{\partial \boldsymbol{N}_2}{\partial \bar{x}} - \boldsymbol{q}_f^{\mathrm{T}} \boldsymbol{H} + y \boldsymbol{q}_f^{\mathrm{T}} \boldsymbol{V}_{12} \right)^{\mathrm{T}} f_1' \\
&\quad + \left(\boldsymbol{N}_2 - y \boldsymbol{q}_f^{\mathrm{T}} \frac{\partial \boldsymbol{N}_2^{\mathrm{T}}}{\partial \bar{x}} \cdot \frac{\partial \boldsymbol{N}_2}{\partial \bar{x}} \right)^{\mathrm{T}} f_2' \Bigg) \mathrm{d}V,
\end{aligned} \tag{28}$$

则外力对应的广义力为

$$\boldsymbol{Q} = [\boldsymbol{Q}_{r_0}^{\mathrm{T}} \quad \boldsymbol{Q}_\theta^{\mathrm{T}} \quad \boldsymbol{Q}_{q_f}^{\mathrm{T}}]^{\mathrm{T}}. \tag{29}$$

2.4. 刚-柔耦合系统的动力学方程

将(13)，(19)，(29)式代入 Lagrange 方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial \boldsymbol{q}}+\frac{\partial U}{\partial \boldsymbol{q}}=\boldsymbol{Q}, \tag{30}$$

得运动规律未知的刚-柔耦合系统的动力学方程

$$\boldsymbol{M}\ddot{\boldsymbol{q}}=\boldsymbol{Q}_v,$$

式中 M 为广义质量矩阵，Q_v 为广义力列阵，且有

$$\boldsymbol{Q}_v=\left[\boldsymbol{Q}_{f1}^{\mathrm{T}}+\boldsymbol{Q}_{r_0}^{\mathrm{T}}\quad \boldsymbol{Q}_{f2}^{\mathrm{T}}+\boldsymbol{Q}_{\theta}^{\mathrm{T}}\quad \boldsymbol{Q}_{f3}^{\mathrm{T}}+\boldsymbol{Q}_{q_f}^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{31}$$

$$\boldsymbol{Q}_{f1}=-(\boldsymbol{G}_{11}\dot{\boldsymbol{r}}_0+\boldsymbol{G}_{12}\dot{\theta}+\boldsymbol{G}_{13}\dot{\boldsymbol{q}}_f),$$

$$\boldsymbol{Q}_{f2}=-(\boldsymbol{G}_{21}\dot{\boldsymbol{r}}_0+\boldsymbol{G}_{22}\dot{\theta}+\boldsymbol{G}_{23}\dot{\boldsymbol{q}}_f),$$

$$\boldsymbol{Q}_{f3}=-(\boldsymbol{G}_{31}\dot{\boldsymbol{r}}_0+\boldsymbol{G}_{32}\dot{\theta}+\boldsymbol{G}_{33}\dot{\boldsymbol{q}}_f)-\boldsymbol{K}_f\boldsymbol{q}_f,$$

其中

$$\boldsymbol{G}_{11}=\boldsymbol{0}_{2\times 2},$$

$$\boldsymbol{G}_{12}=\boldsymbol{G}_{21}^{\mathrm{T}}=\boldsymbol{A}\tilde{\boldsymbol{I}}\begin{bmatrix}-s_2\\ s_1\end{bmatrix}\dot{\theta}+\boldsymbol{A}\begin{bmatrix}-\dot{s}_2\\ \dot{s}_1\end{bmatrix},$$

$$\boldsymbol{G}_{13}=\boldsymbol{A}\tilde{\boldsymbol{I}}\begin{bmatrix}\boldsymbol{Y}_1-\boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{C}\\ \boldsymbol{Y}_2\end{bmatrix}\dot{\theta}+\boldsymbol{A}\begin{bmatrix}-\dot{\boldsymbol{q}}_f^{\mathrm{T}}\boldsymbol{C}\\ \boldsymbol{0}\end{bmatrix},$$

$$\boldsymbol{G}_{22}=2\boldsymbol{Z}_{11}\dot{\boldsymbol{q}}_f-2\boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{D}\dot{\boldsymbol{q}}_f+2\boldsymbol{q}_f^{\mathrm{T}}(\boldsymbol{W}_{11}+\boldsymbol{W}_{22})\dot{\boldsymbol{q}}_f - \dot{\boldsymbol{r}}_0^{\mathrm{T}}\boldsymbol{A}\tilde{\boldsymbol{I}}\begin{bmatrix}-s_2\\ s_1\end{bmatrix},$$

$$\boldsymbol{G}_{23}=\dot{\boldsymbol{q}}_f^{\mathrm{T}}(\boldsymbol{W}_{12}-\boldsymbol{W}_{21})-\dot{\boldsymbol{q}}_f^{\mathrm{T}}\overline{\boldsymbol{W}}_{21}-\dot{\boldsymbol{r}}_0^{\mathrm{T}}\boldsymbol{A}\tilde{\boldsymbol{I}}\begin{bmatrix}\boldsymbol{Y}_1-\boldsymbol{q}_f^{\mathrm{T}}\boldsymbol{C}\\ \boldsymbol{Y}_2\end{bmatrix},$$

$$\boldsymbol{G}_{31}=\boldsymbol{G}_{13}-\left[-\boldsymbol{Y}_2^{\mathrm{T}}\dot{\theta}-\boldsymbol{C}\dot{\boldsymbol{q}}_f(\boldsymbol{Y}_1^{\mathrm{T}}-\boldsymbol{C}\boldsymbol{q}_f)\dot{\theta}\right]\boldsymbol{A}^{\mathrm{T}},$$

$$\boldsymbol{G}_{32}=(\boldsymbol{W}_{21}-\boldsymbol{W}_{12})\dot{\boldsymbol{q}}_f-\overline{\boldsymbol{W}}_{21}\dot{\boldsymbol{q}}_f - (\boldsymbol{Z}_{11}^{\mathrm{T}}-\boldsymbol{D}\boldsymbol{q}_f+(\boldsymbol{W}_{11}+\boldsymbol{W}_{22})\boldsymbol{q}_f)\dot{\theta},$$

$$\boldsymbol{G}_{33}=-(\boldsymbol{W}_{12}-\boldsymbol{W}_{21}-\overline{\boldsymbol{W}}_{21})\dot{\theta}.$$

3. 结 论

本文利用 Lagrange 方程建立具有大范围运动和非线性变形的柔性梁的有限元动力学方程. 该方程不仅增加了新的表征纵向、横向、侧向弯曲变形，以及扭转变形的耦合项，同时包含了变形运动与大范围运动之间的耦合项.

常规的有限元模型，是一种零次的近似模型，即用线性关系来拟合变形，对于大柔性高速多体系统，该方法的准确性值得怀疑. 而通过本项目的研究，可以用一次或二次近似模型来拟合变形，有助于研究像航天器这样的大柔性高速系统.

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Dynamic modeling of a flexible beam with large overall motion and nonlinear deformation using the finite element method *

He Xing-Suo† Deng Feng-Yan Wu Gen-Yong Wang Rui

(*Department of Engineering Mechanics*, *Northwestern Polytechnical University*, *Xi'an* 710072, *China*)

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Abstract

In this paper, the dynamic modeling theory of a flexible beam, which incorporates large overall motion and nonlinear deformation, is investigated. As we knows, in spacecraft and space station, there are a lot of flexible appendices, the dynamic modeling of a flexible beam is essential. Yet existing such models, in our opinion, lack several important coupling terms. This paper supplies these important coupling terms. It is reasonable to use geometrically nonlinear deformation fields to demonstrate and simplify a flexible beam undergoing large overall motion. In this paper, the finite element method is used for the system discretization and the coupling dynamic equations of flexible beam are obtained by Lagrange's equations. The complete expression of stiffness matrix and all coupling terms are included in the dynamic equations. The second order coupling terms among rigid large overall motion, arc length stretch, lateral flexible deformation kinematics and torsional deformation terms are concluded in the present exact coupling model to expand the theory of one-order coupling model. The dynamic properties of a planar flexible rotating beam in the non-inertial reference frame are developed.

Keywords: large overall motion, nonlinear deformation, flexible beam, finite element method dynamic modeling

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† E-mail: xingsuoh@nwpu.edu.cn