第 59 卷 第 12 期 2010 年 12 月
1000-3290/2010/59(12)/8311-05

物 理 学 报
ACTA PHYSICA SINICA

Vol. 59, No. 12, December, 2010

广义(3+1)维非线性 Burgers 系统孤波级数解*

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(2010 年 1 月 11 日收到;2010 年 6 月 17 日收到修改稿)

研究了一类广义(3+1)维非线性 Burgers 系统. 首先,利用同伦映射方法构造了相应的映射关系式. 其次,利用迭代方法得到了扰动系统的一个孤波非行波的级数解.

关键词: Burgers 系统, 非线性, 孤波解

PACC: 0230

1. 引 言

探讨非线性方程的解是非线性孤波定性和定量研究中一个很重要的课题. 目前提出了许多方法,例如双曲正切函数法[1]、齐次平衡法[2—4]、Jacobi 椭圆函数展开法[5—8]、辅助方程法[9—12]、Riccati 函数法[13—15]、F 展开法[16,17]、G'/G 展开法[18,19]等. 本文利用同伦映射方法[20,21]研究了一类广义非线性 Burgers 系统孤波的解析解.

近年来,许多学者研究了非线性理论[22—30]. 文献[31—47]也利用奇异摄动等方法研究了一类反应扩散、大气物理、生态环境、流行性传染病、激波和激光脉冲等问题. 本文首先构造一组同伦映射,然后进行迭代运算,由此便可得到一个收敛级数,从而得到相应的广义非线性系统的孤波级数解.

2. 广义非线性 Burgers 系统与同伦映射

考虑如下一类广义非线性 Burgers 系统[19]:

$$\begin{aligned}&\frac{\partial u}{\partial t}-\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\&\quad-2\left(u\frac{\partial u}{\partial y}+v\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)\\&=f_1(x,y,z,u,v,w)\\&\qquad(t\in[0,\infty);\ x,y,z\in R),\end{aligned}\tag{1}$$

$$\begin{aligned}&\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=f_2(x,y,z,u,v,w)\\&\qquad(t\in[0,\infty);\ x,y,z\in R),\end{aligned}\tag{2}$$

$$\begin{aligned}&\frac{\partial u}{\partial z}-\frac{\partial w}{\partial y}=f_3(x,y,z,,u,v,w)\\&\qquad(t\in[0,\infty);\ x,y,z\in R),\end{aligned}\tag{3}$$

其中扰动项 $f_i(i=1,2,3)$ 为关于其变量在相应的变化区域内充分光滑的有界函数.

为了使用同伦映射方法[20,21]得到系统(1)—(3)的解, 引入参数 p, 并定义如下一组同伦映射 $H_i(u,v,z,p):R^3\times I\rightarrow R(i=1,2,3)$:

$$\begin{aligned}H_i(u,v,w,p)=&L_i(u,v,w)-L_i(\bar{u}_0,\bar{v}_0,\bar{w}_0)\\&+p(L_i(\bar{u}_0,\bar{v}_0,\bar{w}_0)-F_i(u,v,w))\\&(i=1,2,3).\end{aligned}\tag{4}$$

这里 $R=(-\infty,+\infty)$;$I=[0,1]$;$(\bar{u}_0,\bar{v}_0,\bar{w}_0)$ 为原系统(1)—(3)的首次近似;

$$F_1(u,v,w)=2\left(u\frac{\partial u}{\partial y}+v\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)+f_1,$$

$$F_2(u,v,w)=f_2,$$

$$F_3(u,v,w)=f_3;$$

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* 国家自然科学基金(批准号: 40876010)、中国科学院知识创新工程重要方向性项目(批准号: KZCX2-YW-Q03-08)、大气科学和地球流体力学数值模拟国家重点实验室基金和浙江省自然科学基金(批准号: Y6090164) 资助的课题.

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而 L_i 为线性算子，

$$L_1 = \frac{\partial u}{\partial t} - \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right),$$

$$L_2 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \tag{5}$$

$$L_3 = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial y}.$$

设 u, v, w 可写为如下的待定形式：

$$u(x,y,z,p) = \sum_{i=0}^{\infty} u_i(x,y,z)p^i, \tag{6}$$

$$v(x,y,z,p) = \sum_{i=0}^{\infty} v_i(x,y,z)p^i, \tag{7}$$

$$w(x,y,z,p) = \sum_{i=0}^{\infty} w_i(x,y,z)p^i, \tag{8}$$

对应于系统(1)—(3)无扰动情形下的系统

$$\frac{\partial u}{\partial t} - \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \left(u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = 0, \tag{9}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial y} = 0. \tag{11}$$

首先选取首次近似 $(\bar{u}_0, \bar{v}_0, \bar{w}_0)$ 为系统(9)—(11)的解 $(\bar{u}, \bar{v}, \bar{w})$. 再将(6)—(8)式代入

$$H_i(u,v,w,p) = 0 \quad (i = 1,2,3), \tag{12}$$

比较 p 的同次幂的系数，可以依次求得 (u_i, v_i, w_i) $(i = 1,2,\cdots)$，于是可以确定级数(6)—(8)式. 可以证明[20,48]，在广义非线性 Burgers 系统(1)—(3)的假设和选取系统(1)—(3)的首次近似为(12)式的解 $(\bar{u}, \bar{v}, \bar{w})$ 的情况下，由上述确定的级数(6)—(8)式在区域 $t \in [0, \infty)$，$x, y, z \in R$，$p \in [0,1]$ 上一致收敛. 详细证明在此从略.

显然，由(4)，(12)式可知，$H_i(u,v,w,1) = 0$ $(i = 1,2,3)$ 就是系统(1)—(3). 故系统(1)—(3)的解 (u,v,w) 就是 $H_i(u,v,w,p) = 0$ $(i = 1,2,3)$ 的解在 $p \to 1$ 时的极限情形.

3. 孤波非行波解的计算

由文献[19]可知，利用 G'/G 展开法得到系统(9)—(11)的孤波精确非行波解

$$\bar{u} = \frac{\lambda + 2}{2}\frac{\partial q}{\partial y} + \delta\frac{\partial q}{\partial y}\tanh(\delta q),$$

$$\bar{v} = \frac{\lambda + 2}{2}\frac{\partial q}{\partial x} + \delta\frac{\partial q}{\partial x}\tanh(\delta q),$$

$$\bar{w} = \frac{\lambda + 2}{2}\frac{\partial q}{\partial z} + \delta\frac{\partial q}{\partial z}\tanh(\delta q),$$

其中 $\lambda \neq -2$ 为常数，$\delta = \frac{|\lambda + 2|}{2}$，$q(x,y,z,t)$ 为任意函数. 因此，我们可把广义非线性 Burgers 系统首次近似 $(\bar{u}_0, \bar{v}_0, \bar{w}_0)$ 选取为无扰动非线性 Burgers 系统(9)—(11)的孤波精确非行波解 $(\bar{u}, \bar{v}, \bar{w})$.

将(6)—(8)式代入 $H_i(u,v,w,p) = 0$ $(i = 1,2,3)$，比较 p 的零次幂系数，得

$$L_i(u_0, v_0, w_0) = L_i(\bar{u}_0, \bar{v}_0, \bar{w}_0) \quad (i = 1,2,3). \tag{13}$$

显然，由首次近似 $(\bar{u}_0, \bar{v}_0, \bar{w}_0)$ 的选取可得到系统(13)的解 (u_0, v_0, w_0)，

$$u_0 = \frac{\lambda + 2}{2}\frac{\partial q}{\partial y} + \delta\frac{\partial q}{\partial y}\tanh(\delta q), \tag{14}$$

$$v_0 = \frac{\lambda + 2}{2}\frac{\partial q}{\partial x} + \delta\frac{\partial q}{\partial x}\tanh(\delta q), \tag{15}$$

$$w_0 = \frac{\lambda + 2}{2}\frac{\partial q}{\partial z} + \delta\frac{\partial q}{\partial z}\tanh(\delta q). \tag{16}$$

将(6)—(8)式代入 $H_i(u,v,w,p) = 0$ $(i = 1,2,3)$，比较 p 的一次幂系数，得

$$L_i(u_1, v_1, w_1) = f_i(x,y,z,t,u_0,v_0,w_0) \quad (i = 1,2,3). \tag{17}$$

由系统(17)不难得到其解为

$$\begin{aligned} u_1 = {} & \frac{1}{8\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(t - t_1)^{-1/2} \\ & \times f_1(x_1,y_1,z_1,t_1,u_0,v_0,w_0) \\ & \times \exp\left(-\frac{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}{4(t - t_1)}\right) \\ & \times \mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1, \end{aligned} \tag{18}$$

$$\begin{aligned} v_1 = {} & \frac{-1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(x - x_1)(t - t_1)^{-3/2} \\ & \times f_2(x_1,y_1,z_1,t_1,u_0,v_0,w_0) \\ & \times \exp\left(-\frac{(x - x_1)^2 + (y' - y_1)^2 + (z - z_1)^2}{4(t - t_1)}\right) \\ & \times \mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1, \end{aligned} \tag{19}$$

$$\begin{aligned} w_1 = {} & \frac{-1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(z - z_1) \\ & \times (t - t_1)^{-3/2}f_3(x_1,y_1,z_1,t_1,u_0,v_0,w_0) \\ & \times \exp\left(-\frac{(x - x_1)^2 + (y' - y_1)^2 + (z - z_1)^2}{4(t - t_1)}\right) \\ & \times \mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1, \end{aligned} \tag{20}$$

其中 u_0, v_0, w_0 分别由(14)—(16)式决定.

将(6)—(8)式代入 $H_i(u,v,w,p)=0\,(i=1,2,3)$，展开关于 p 的幂级数，分别比较 p 的 $i\,(i=2,3,\cdots)$ 次幂系数，得

$$L_i(u_j,v_j,w_j)=F_{ij}\qquad(i=1,2,3;j=2,3,\cdots),\tag{21}$$

其中

$$\begin{aligned}F_{1j}=&2\sum_{k=0}^{j-1}\left(u_{j-k}\frac{\partial u_k}{\partial y}+v_{j-k}\frac{\partial u_k}{\partial x}+w_{j-k}\frac{\partial u_k}{\partial z}\right)\\&+\bar{f}_{1j}(x,y,z,t,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&\qquad(j=2,3,\cdots),\\F_{2j}=&\bar{f}_{2j}(x,y,z,t,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&\qquad(j=2,3,\cdots),\\F_{3j}=&\bar{f}_{3j}(x,y,z,t,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&\qquad(j=2,3,\cdots),\end{aligned}$$

而

$$\begin{aligned}&\bar{f}_{ij}(x,y,z,t,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&=\frac{1}{(j-1)!}\left[\frac{\partial^{j-1}f}{\partial p^{j-1}}\left(\sum_{i=0}^{\infty}u_i(x,y,z)p^i,\right.\right.\\&\left.\left.\sum_{i=0}^{\infty}u_i(x,y,z)p^i,\sum_{i=0}^{\infty}u_i(x,y,z)p^i\right)\right]_{p=0}\\&\qquad(i=1,2,3;j=2,3,\cdots).\end{aligned}$$

由(21)式可得

$$\begin{aligned}u_j=&\frac{1}{8\pi^3}\\&\times\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[2\sum_{k=0}^{j-1}\left(u_{j-k}\frac{\partial u_k}{\partial y}+v_{j-k}\frac{\partial u_k}{\partial x}\right.\right.\\&\left.+w_{j-k}\frac{\partial u_k}{\partial z}\right)+(t-t_1)^{-1/2}\bar{f}_{1j}(x_1,y_1,z_1,\\&\left.t_1,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\right]\\&\times\exp\left(-\frac{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)\\&\times\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1\qquad(j=2,3,\cdots),\end{aligned}\tag{22}$$

$$\begin{aligned}v_j=&\frac{-1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(x-x_1)(t-t_1)^{-3/2}\\&\times f_{2j}(x_1,y_1,z_1,t_1,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&\times\exp\left(-\frac{(x-x_1)^2+(y'-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)\\&\times\mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1\qquad(j=2,3,\cdots),\end{aligned}\tag{23}$$

$$\begin{aligned}w_j=&\frac{-1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(z-z_1)(t-t_1)^{-3/2}\\&\times f_{3j}(x_1,y_1,z_1,t_1,u_0,v_0,w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\\&\times\exp\left(-\frac{(x-x_1)^2+(y'-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)\\&\times\mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1\qquad(j=2,3,\cdots),\end{aligned}\tag{24}$$

其中 u_0,v_0,w_0 分别由(14)—(16)式决定，$u_{i-1},v_{i-1},w_{i-1},\cdots(i=2,3,\cdots,j)$ 分别为逐次已知的函数. 因此，我们便得到了广义非线性 Burgers 系统一个孤波的一组非行波解的级数通解

$$\begin{aligned}&u(x,y,z,t)\\&=\frac{\lambda+2}{2}\frac{\partial q}{\partial y}+\delta\frac{\partial q}{\partial y}\tanh(\delta q)\\&+\frac{1}{8\pi^{1/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[2\sum_{k=0}^{j-1}\left(u_{j-k}\frac{\partial u_k}{\partial y}\right.\right.\\&\left.+v_{j-k}\frac{\partial u_k}{\partial x}+w_{j-k}\frac{\partial u_k}{\partial z}\right)\\&+(t-t_1)^{-1/2}\left(f_1(x_1,y_1,z_1,t_1,u_0,v_0,w_0)\right.\\&+\sum_{j=2}^{\infty}\bar{f}_{1j}(x_1,y_1,z_1,t_1,u_0,v_0,w_0,\cdots,\\&\left.\left.u_{j-1},v_{j-1},w_{j-1})\right)\right]\\&\times\exp\left(-\frac{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)\\&\times\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1,\end{aligned}$$

$$\begin{aligned}&v(x,y,z,t)\\&=\frac{\lambda+2}{2}\frac{\partial q}{\partial x}+\delta\frac{\partial q}{\partial x}\tanh(\delta q)\\&-\frac{1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(x-x_1)\\&\times(t-t_1)^{-3/2}\left[f_2(x_1,y_1,z_1,t_1,u_0,v_0,w_0)\right.\\&+\sum_{j=2}^{\infty}\bar{f}_{2j}(x_1,y_1,z_1,t_1,u_0,v_0,w_0,\cdots,\\&\left.u_{j-1},v_{j-1},w_{j-1})\right]\\&\times\exp\left(-\frac{(x-x_1)^2+(y'-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)\\&\times\mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1,\end{aligned}$$

$$\begin{aligned}&w(x,y,z,t)\\&=\frac{\lambda+2}{2}\frac{\partial q}{\partial z}+\delta\frac{\partial q}{\partial z}\tanh(\delta q)\\&-\frac{1}{16\pi^{3/2}}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{y}(z-z_1)(t-t_1)^{-3/2}\\&\times\left[f_3(x_1,y_1,z_1,t_1,u_0,v_0,w_0)\right.\\&+\sum_{j=2}^{\infty}\bar{f}_{3j}(x_1,y_1,z_1,t_1,u_0,v_0,\\&\left.w_0,\cdots,u_{j-1},v_{j-1},w_{j-1})\right]\end{aligned}$$

$$\times \exp\left(-\frac{(x-x_1)^2+(y'-y_1)^2+(z-z_1)^2}{4(t-t_1)}\right)$$

$$\times \mathrm{d}y'\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}t_1,$$

其中常数 $\lambda \neq -2, \delta = \frac{|\lambda+2|}{2}$, $q(x,y,z,t)$ 为任意函数, u_0, v_0, w_0 分别由(14)—(16)式决定, $u_{i-1}, v_{i-1}, w_{i-1}, \cdots (i = 2,3,\cdots,j)$ 分别为逐次已知的函数.

4. 结 论

由广义非线性 Burgers 系统(1)—(3)及同伦映射(4)式的构造,可以得到一个该系统的孤波级数通解. 由此我们能得到相应系统的任意次精度的近似解. 本方法不同于数值方法或模拟方法得到的解. 本方法还可以对得到的级数解进行微分或积分运算, 从而对系统解的性态作进一步的研究. 在本方法中选取了同伦映射的首次近似 $(\bar{u}_0, \bar{v}_0, \bar{w}_0)$ 为对应的无扰动状态下的 Burgers 系统(9)—(11)的解 $(\bar{u}, \bar{v}, \bar{w})$, 这保证了得到相应级数解具有较快的收敛速度.

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Solitary wave series solution for generalized (3+1)-dimensional nonlinear Burgers system*

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(Received 11 January 2010; revised manuscript received 17 June 2010)

Abstract

A class of generalized (3 + 1)-dimensional nonlinear Burgers system is studied. Using the homotopic mapping method, the corresponding mapping expansions are constructed and using the iteration method the series solution of travelling wave for a solitary wave is obtained.

Keywords: Burgers system, nonlinear, solitary wave solution

PACC: 0230

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* Project supported by the National Natural Science Foundation of China (Grant No. 40876010), the Main Direction Program of the Knowledge Innovation of Chinese Academy of Sciences (Grant No. KZCX2-YW-Q03-08), the Foundation of State Key Laboratory of Numerical Modeling for Atmospheric Science and Geophysical Fluid Dynamic, China and the Natural Science Foundation of Zhejiang Province, China (Grant No. Y6090164).

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