ON THE APPLICATION OF THE HILGER PRISM INTERFEROMETER TO THE RESOLUTION OF SPECTRAL LINES

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ABSTRACT

An interference pattern of two systems of fringes has been obtained by using two radiations in a Hilger prism interferometer. A system of white bands with equal spacing which are always parallel to the refracting edge of the prism is the result of the intersection of the two systems of fringes. The white bands travel as one of the reflecting mirror in the interferometer makes a parallel displacement. Let d be the displacement of mirror when the white bands travel one spacing, we have

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{2d}.$$

Two yellow doublets in sodium and mercury lamps were used. The values found for $1/\lambda_2 - 1/\lambda_1$ were 17.22 cm^{-1} and 63.29 cm^{-1} respectively.

A comparison of this method with the visibility curve by Michelson interferometer has been discussed.

Introduction

It has been a great step of advancement in the manufacturing of optical parts by the invention of interferometers for testing of optical systems. The interferometers designed for this purpose by F. Twyman [1] of Adam Hilger

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(1) F. Twyman: *Phil. Mag.*, 35, (1918), 49.
Trans. Opt. Soc., 22, (1920-21), 174.
Trans. Opt. Soc., 24, (1922-23), 187.

Ltd., are different in details, though based on one principle, according to the different optical system to be tested and corrected. There are three types so far Hilger has made, one for prisms and lenses for axial pencils only, one for camera lenses for both axial and oblique pencils and one for microscope objectives.

Many papers have dealt with the theory and pattern of the interferometers J. W. Perry[2] has given a method of coverting the interferometer indication into the expressions which are more familiar to the most optical designers. Two papers on the studying of the interferograms based on Conrady's formula for optical path difference have been written by R. Kingslake.[3] T. Smith[4] has investigated the theory of the lens-testing interferometer, especially paying attention to the importance of focussing the photographic plate on the surface of the spherical mirror. A universal lens interferometer for testing and correcting camera lens, process lens, telephoto lens and telescopic system has been described by J. H. Dowell.[5] O. G. Hay[6] has made a modification of the Hilger Interferometer for testing unusual large optical system. All these papers mentioned above were contributed to the theory and technique for measuring the aberrations of optical systems. For the application other than that purpose, D. S. Perfect[7] has applied the interferometer to the precise goniometry of prism.

The present paper describes a simple method of an application of the interferometer to the resolution of spectral lines by means of which $\frac{1}{\lambda_2} - \frac{1}{\lambda_1}$ of two neighbouring radiations in a spectrum can be measured with pretty high accuracy.

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(2) J. W. Perry: *Trans. Opt. Soc.*, *25*, (1923-24), 97.

(3) R. Kingslake: *Trans. Opt. Soc.*, *27*, (1925-26), 94.
Trans. Opt. Soc., *28*, (1926-27), 1.

(4) T. Smith: *Trans. Opt. Soc.*, *28*, (1926-27), 104.

(5) J. H. Dowell: *Proc. Opt. Convention II*, (1926), 1032.

(6) O. G. Hay: *Trans. Opt. Soc.*, *31*, (1931), 91.

(7) D. S. Perfect: *Trans. Opt. Soc.*, *30*, (1928-29), 118.

Hilger Prism Interferometer

Fig. 1 represents a plan of Hilger Prism Interferometer. Light from a mercury lamp S converging by condenser C and passing through a green filter F which absorbs all visible radiations except the green light 5461 Å is focussed on slit s_1 which is placed at the focal point of the lens L_1. The parallel beam of monochromatic light after emerging out from L_1 is divided into two beams of equal intensity by the half-silvered plate T, one beam of light reflected toward

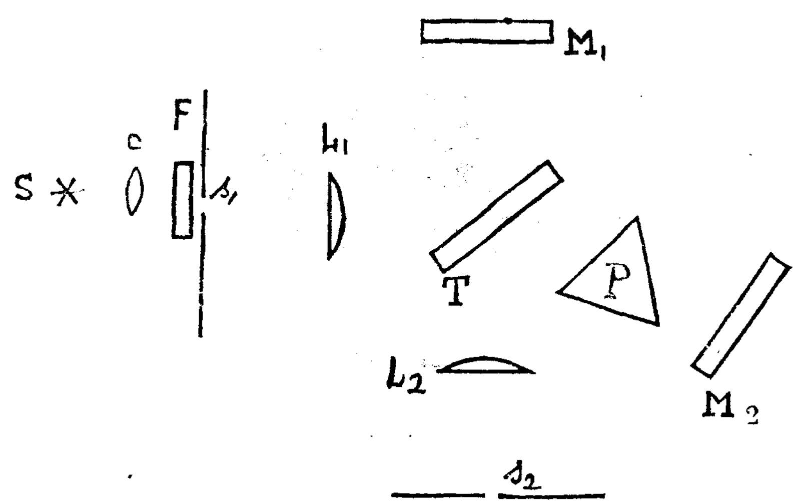

Fig. 1.

to a plane mirror M_1 by which the beam is reflected back again, and another beam of light transmitting T passing through prism P which is under examination and reflected back by a second mirror M_2. These two reflected beams of light re-unite together at T and interfere each other to give the fringes which are collected by the second lens L_2 and converge to a hole s_2 where is our eye to be placed for observation.

Truly speaking, the Hilger interferometer is a modification of the interferometer which was invented by A. A. Michelson and bears his name. It will be readily seen that the chief difference between these two interferometers lies in the use of the truly plane wave and the collecting lens L_2 for the former. In the Michelson interferometer, the interference fringes are in general curved and become a uniform intensity of field only if equal optical path of two beams of light comes to realization; while in the Hilger interferometer, the fringes will always appear, independant of the path difference, a uniform field or a system of straight lines at certain inclination, (Fig. 2). When in Fig. 1, all the optical

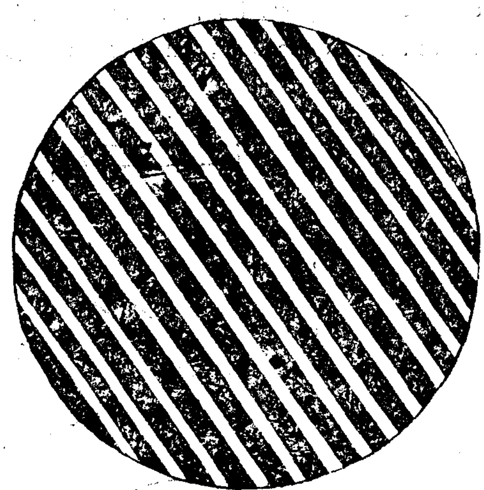

Fig. 2.

parts but the prism P are perfect, the fringes will take a form of what Twyman called "contour map" which shows the position and degree of the defects of the prism and by which the proper correction could be possibly made. The physical meaning of the fringes is the same as that of Newton's rings (or called localized fringes) and they are similar to each other in many respects.

Two Radiations

In Fig. 1, replace the green filter F by an absorption cell filling with potassium bichromate solution, which will absorb nearly all the blue and violet radiations and a large percentage of green light from the mercury light source, and consequently give out a good portion of yellow light which is known to be a doublet with a separation of about 21 Å.

In this experiment, the interferometer in our possession is made by Hilger Co. Ltd., with the reflecting mirror of 85 mm. at each side. A 60° prism of flint glass with 60 mm. each side of the base and 40 mm. height was put at minimum deviation position with respect to the beam emerging from the plate **T**. The mirror M_2 was so adjusted that it was nearly perpendicular to the beam of light emerging from P. Now, a new pattern of fringes (Fig. 3) would be

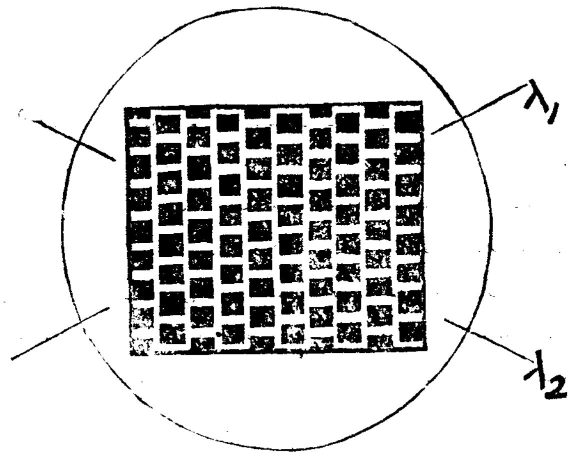

Fig. 3.

seen at slit s_2. Further adjusting the mirror M_2 by means of two micromotion screws attached on it would change the figure to a certain varieties but with the same nature.

Prism of other refractive index and with different angles had been brought in. In another case, a hot-cathode sodium lamp of 40 watts had been put in the place of the mercury lamp, the absorption cell of potassium bichromate solution being removed in this case. Interference pattern of similar nature were obtained, but they were different from each other in certain points which were important to the different source of light and different prism used, and will be discussed later.

For measuring purpose, swing out the lens L_2 and slit s_2, a reading telescope with micrometer eyepiece was brought in its position.

Some photographs had been taken with the camera supplied by Hilger with the interferometer. It seems not possible at least for the time being to reproduce a photo-engraving in this Journal, so they will be published elsewhere.

Nature of the Pattern of the Fringes

From the Fig. 3 it will be readily seen that the pattern is composed of two systems of interference fringes, intersecting at different orientations. Both the orientations of the two systems of fringes and their spacings can be changed to a great extent by adjusting the two fine screws attached on the frame which supports the mirror M_2. The dark dashes are obviously the intersecting points of the dark lines of the two systems of fringes. Between the rows of dark dashes are white bands, which show the ultimate importance in our experiment. The width of the white bands changes with the positions of the two systems of fringes. They will be reduced to fine lines as soon as the spacing of each system of fringes is broaden up. As the two systems of fringes grow finer and finer, the white bands turn out a set of broaden channels. However there are two things remaining unaltered, no matter how the fringes change, so far as the source of light and the prism are not disturbed. In the first place, the white bands are sensibly parallel to the refracting edge of the prism and secondly, the spacing of the white bands is practically constant.

Some trials were made by turning the prism P to such a position that the refracting edge of the prism was horizontal or made certain angle with the horizontal line, the mirror M_1 being, of course, turned accordingly, the direction of the white bands turned with the prism.

In one case, with a sodium lamp as light source and a crown prism of 60° and a telescope of $f = 247$ mm. fitting with a micro-meter eyepiece, some twenty measurements of the width of the white bands were made at different settings, the result came out as 1.54 ± 0.02 mm.

It was possible, after a few trials, to adjust the mirror M_2 to such a position that there was only one system of fringes which were parallel, vertical lines if the quality of the prism was perfect. The spacings between these lines were measured sensibly equal to those of white bands, when two systems of fringes were in existance. In one case, the spacings between each system of fringes were measured to be 1.52 mm. and 1.44 mm. respectively and the spacing of white bands was found 3.29 mm. Making only one system of fringes existing in the field of view, we measured its spacing at 3.3 mm.

What is more interesting is that the formation of the white bands has been found to have nothing to do with the quality of the prism at all. If we introduce a prism of rather bad quality either arising from the inhomogeneity of the glass or from the surface defects of the prism, the two systems of fringes themselves will appear distorted. The white bands, however, remain perfectly straight.

Difference of $\frac{1}{\lambda_2}$ *and* $\frac{1}{\lambda_1}$

Now the reflecting mirror M_1 was fitted on a carrier which was attached to a good screw in millimeter scale. Firstly, the cross hair in the micrometer eyepiece was placed on one of the white bands. It came in to our notice that the white bands were shifting from one side to another when the carrier screw was gradually turned in one way. If the screw was turned in the other way, the white bands were shifting to the other direction.

The moving of mirror M_1, back and forth, was also performed by Michelson when he observed what he called the visibility curve with his interferometer. By the displacement of the mirror M_1, over which one period of brightness and darkness of the fringes occured, Michelson was able to calculate the difference of the wave-length of the radiations which were under investigation. It has been found in the present experiment, that the displacement of mirror M_1 has exactly the same meaning as that in the Michelson's experiment, the periodicity now being counted by the passing of one neighbouring white band after another. With two radiations, the equation will be

$$\frac{1}{\lambda_2}-\frac{1}{\lambda_1}=\frac{1}{2d}$$

where d represents the displacement of the mirror M_1 for one periodicity of passing white band.

In this experiment, a mirror carrier with a screw in millimeter scale supplied by Hilger for lens-correcting purpose was used to suport the mirror M_1. The thread of the screw is a good one, but will not be supposed a standard. Several measurements were made with sodium lamp and mercury lamp as light source. The result were tabulated as follows:

Light source	d(cm)			mean d(cm)	$\frac{1}{\lambda_2} - \frac{1}{\lambda_1}$ (cm^{-1})
	Prism Crown 60°	Prism Flint 60°	Prism Crown 45°		
Hewittic Hg low pressure discharge lamp, A. C.	7.9×10^{-3} 7.8 7.8 8.0		8.0×10^{-3} 7.8 8.0 7.9	$7.90 \pm 0.10 \times 10^{-3}$	63.29 ± 0.80
O S C Na hot cathode discharge lamp	29.0×10^{-3} 29.1 28.9	28.9×10^{-3} 28.9 29.2	29.1×10^{-3} 29.2 29.1	$29.04 \pm 0.15 \times 10^{-3}$	17.22 ± 0.09

Each value in the second column was the mean value of about 10 readings of the displacement of the one spacing shifting of the white band. The dial attached on the thread could be read to 1/1000 cm.

For the mercury yellow doublet, the standard values of λ_1 and λ_2 are taken from the Critical Table. They are

$$\lambda_1 = 5790.66 \text{ Å}, \qquad \lambda_2 = 5769.60 \text{ Å}$$

Hence

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = 63.03 \text{ cm.}^{-1}$$

For the sodium yellow doublet, the standard values of λ_1 and λ_2 are also taken from the Critical Table. They are

$$\lambda_1 = 5895.932 \text{ Å}, \qquad \lambda_2 = 5889.965 \text{ Å}$$

Hence

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = 17.18 \text{ cm.}^{-1}$$

The determination of $\frac{1}{\lambda_2} - \frac{1}{\lambda_1}$ for the D_1, D_2 lines of sodium has recently been done by G. F. C. Searle.[8] The method he adopted was the coincidence of two bright Newton's rings due to two radiations between one surface of a plane parallel plate and the convex surface of a lens. The value in one experiment he found was 17.434 cm.$^{-1}$

Discussion

Though that the two systems of fringes in Fig. 3 are due to two radiations in the light source can be accepted without question, we have made the following experiment to support this view. Removing the collimating system in the interferometer, except lens L_1, a Zeiss Monochromator was carefully arranged in such a position that its slit from which the monochromatic rays emerged was exactly in the place of slit s_1. Sodium light was used. The slit of the monochromator was opened a little more widely than usual. At first, the monochromator prism was set at such a position that both radiations of the yellow doublet

(8) G. F. C. Searle: *Proc. Phy. Soc.* London; 53, No. 297, (1941), 265.

were emerging from the slit. An interference pattern of two systems of fringes as shown in Fig. 3 was obtained. Next, turn slowly the drum of the monochromator in one direction, only one system of the fringes was now observed. If turn the drum in the other way, another system of fringes came to our eyes.

A neon discharge tube was used instead of sodium lamp. It revealed a more complicated interference pattern with more than two systems of fringes; for more than two radiations in the red part of the neon spectrum were now in use.

The different orientations of the two systems of the fringes are, generally speaking, attributed to the different angles by which the two radiations after emerging from the prism fall on and are reflected back by the mirror M_2. For the sake of simplicity, let us suppose that the one beam of light reflected by the half-silvered mirror T is directed perpendicularly to the mirror M_1. Then, if the mirror M_2 is so adjusted that it is made perpendicular to the one of two radiations from the prism, this is the case when only one system of fringe can be seen. Since the plane made by the two radiations after the refraction from the prism is horizontal as the reflecting edge of the prism is vertical, the mirror M_2 in the above arrangement should be in vertical position also, so its intersecting line with the mirror M_1 is vertical and consequently the one system of fringe which was obtained in the above arrangement could not but be parallel to the reflecting edge of the prism. The verticality of the white bands which are formed by the intersection of the two systems of fringes involves a little more complicated nature.

However, it seems not difficult to understand what the shifting of the white bands means when the mirror M_1 is displaced. Let us assume first there is only one radiation in our question. The fringes will then be simply one system of lines at certain orientation, and the spacing between them will keep constant as long as the mirror M_1 makes a strictly parallel displacement. A second radiation comes in meeting the mirror M_2 at different angle, thus the second interference fringes will be at different orientation from the first one. Their spacing and orientation will be neither changed as the mirror M_1 makes a strictly parallel displacement. The first system of fringe will resume its exact same position when the mirror M_1 makes displacement of $1/2\,\lambda_1$ or its multiplication and $1/2\lambda_2$ or its multiplication for the second radiation. Now let the mirror M_1 have a displacement d through which the first system of fringes repeats its original position for p times, and the second radiation for $(p+1)$ times, where p is an integer. That means in the length d there are p times of 1/2 wave-length of the λ_1 radiation and $(p+1)$ times of 1/2 wave-length of the λ_2 radiation. Hence we have

$$d = \frac{1}{2} p\lambda_1 = \frac{1}{2}(p+1)\lambda_2 ,$$

or $$\frac{1}{\lambda_1} = \frac{1}{2}\frac{p}{d} , \quad \text{and} \quad \frac{1}{\lambda_2} = \frac{1}{2}\frac{p+1}{d}$$

Therefore $$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{2d} \qquad (1)$$

Only at such a position, the two systems of fringes *both* resume their own original positions respectively, and the white bands which are the result of the intersection of the two systems of fringes repeat their original position just for the first time after the moving of the mirror M_1, that is to say, the white band travels over one spacing. Equation (1) is exactly the form which is applied in the Michelson experiment of visibility curve when two radiations are in question.

By means of the visibility curve of the Michelson interferometer, Michelson was the pioneer worker in the studying of fine structure of spectral lines. With his unsurpassed technique with his interferometer, and his ingeneous invention of harmonic analyzer, Michelson achieved in the resolution of many spectral lines at that time they were generally believed to be a single line. However, the method achieved by Michelson has not hitherto been much used outside the inventor's laboratory, partly because of the utmost delicate technique involved in this experiment. It has been suggested by W. Williams (9) that if the Twyman's modification of the instrument were used, the localized fringes would keep the same separation independently of the path difference; photographs could be taken at regular intervals and the visibility measured with high precision by an instrument such as the Moll Microphotometer. Although William's suggestion would be proved helpful in the method of measuring the intensity of

(9) W. Williams: *Applications of Interferometry*, p. 51

the fringes, the photometric measurements either visual or photographic is not usually regarded as an easy work to be done so far as the accuracy is concerned. In the present experiment, however, all the photometric measurements are dispensable and only the usual judgement of shifting certain marks passing the cross-hair shall be made. This makes the measurements much easier and comfortable to be carried out. The performance of the experiment is simple enough and the accuracy of the results obtained is fairly high.

It seems possible to handle the question of more than two radiations by analyzing the arrangement of the white bands thus obtained.

Acknowdgement

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