

扰动 Nizhnik-Novikov-Veselov 系统分形孤子渐近解*

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文章研究了一类扰动 Nizhnik-Novikov-Veselov 非线性系统, 利用特殊的渐近方法得到了相应系统分形孤子渐近解.

关键词: 孤子, 渐近解, 扰动

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1. 引言

探讨孤子解的定性、定量理论是一个很重要的研究课题. 对它相应方程的求解, 目前提出了许多方法, 例如双曲正切函数法^[1]、齐次平衡法^[2-4]、Jacobi 椭圆函数展开法^[5-8]、辅助方程法^[9-12]、Riccati 函数法^[13-15]、 F 展开法^[16,17]、 G'/G 展开法^[18,19]等. 近来, 许多学者研究了高维孤子波问题, 研究了各种形式的孤子, 包括环孤子, 方孤子, 泡孤子, 峰孤子等^[20-23]. 本文从一类古典 Nizhnik-Novikov-Veselov 系统的分形孤子解出发, 得到非线性扰动系统的行波孤子渐近解^[24].

随着对非线性问题的研究不断深入, 渐近方法被发展和优化, 包括边界层法, 多重尺度法, 匹配渐近展开法和平均法. 许多学者, 诸如 D'Aprile 和 Pistoia^[25], Barbu 和 Morosanu^[26], Barbu 和 Cosma^[27], Ei, Shin-Ichiro 和 Matsuzawa^[28], Kellogg 和 Kopteva^[29], Suzuk^[30] 以及 Mo^[31,32] 等都做了大量的工作. 利用解的渐近理论, 作者等也研究了一类非线性问题^[33-40]. 本文将利用一个简单而有效的渐近方法^[41], 对相应的非线性系统构造其近似解.

2. 扰动 Nizhnik-Novikov-Veselov 系统

今考虑如下一个 $(2+1)$ 维扰动 Nizhnik-Novikov-Veselov 系统:

$$u_t + u_{xxx} - 3uv_x - 3vu_x = \varepsilon f(u, v), \quad (1)$$

$$u_x - v_y = \varepsilon g(u, v). \quad (2)$$

上述系统在天体力学, 地球物理, 理论物理, 等离子物理, 凝聚态物理等领域中有重要的应用, 其中 εf , εg 为扰动函数, 它们是关于其变量为足够光滑的函数, ε 为正的小参数.

首先考虑在无扰动的情形, 即 $f = g = 0$ 的典型系统. 由文献^[24], 利用投射方程法得到了系统解的多孤子和分形孤子结构, 其中得到如下的分形孤子解:

$$\bar{u} = \frac{1}{2} \left[-1 + \tanh \left(\frac{1}{2} (\chi + \varphi) \right) \right]^2 \chi_x \varphi_y, \quad (3)$$

$$\bar{v} = \frac{1}{6\chi_x} \left[2\chi_{xxx} - \chi_x^3 + 2\chi_t - 6\chi_x \chi_{xx} \tanh \left(\frac{1}{2} \sigma(\chi + \varphi) \right) + 3\chi_x^3 \tanh \left(\frac{1}{2} (\chi + \varphi) \right)^2 \right], \quad (4)$$

其中

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$$\chi = \frac{1}{10} + \exp \left[- (x - t) \left((x - t) + 4sn \left(\ln x^2, \frac{4}{5} \right) \right) \right], \quad (5)$$

$$\varphi = \frac{1}{10} + \exp \left[- y \left(y + 4sn \left(\ln y^2, \frac{4}{5} \right) \right) \right]. \quad (6)$$

(3)式和(4)式的图形描述的是一个典型的分形结构. 在标度的变换下, 具有相似性的几何形状. 分形在自然界中广泛地存在, 例如星系, 海岸线, 雪花, 细菌群落的生长, 以及视网膜血管分布等, 对这种整体和局部的相似性的研究, 即对这种分形理论的研究, 加深了人们对自然界现象的认识. 因此利用分形孤子结构来讨论相应的问题, 具有广阔的研究前景.

3. 扰动系统的渐近解

为了求得 $(2 + 1)$ 维扰动 Nizhnik-Novikov-Veselov 系统(1), (2)的渐近解, 今引入连续映射 $H_i[u, v, p]: R \times I \rightarrow R (i = 1, 2)^{[41]}$:

$$H_1[u, v, p] = L_1(u) - L_1(u_0) - p \left[L_1(u_0) - 3u \frac{\partial v}{\partial x} - 3v \frac{\partial u}{\partial x} - \varepsilon f(u, v) \right], \quad (7)$$

$$H_2[u, v, p] = L_2(u, v) - L_2(u_0, v_0) - p [L_2(u_0, v_0) - \varepsilon g(u, v)], \quad (8)$$

其中 (u_0, v_0) 为系统(1), (2)的初始近似, p 为人工参数, 线性算子 $L_i (i = 1, 2)$ 为

$$L_1(u) = \frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3}, \quad L_2(u, v) = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}.$$

显然, 由连续映射(7), (8)知, $H_i[u, v, 1] = 0 (i = 1, 2)$ 就是系统(1), (2). 于是 $H_i[u, v, p] = 0 (i = 1, 2)$ 的解, 当 $p \rightarrow 1$ 时就是系统(1), (2)的解.

在映射(7), (8)中, 选取初始近似 (u_0, v_0) 为古典 Nizhnik-Novikov-Veselov 系统的分形孤子解(3), (4). 即

$$u_0 = \frac{1}{2} \left[-1 + \tanh \left(\frac{1}{2} (\chi + \varphi)^2 \right) \right] \chi_x \varphi_y, \quad (9)$$

$$v_0 = \frac{1}{6\chi_x} \left[2\chi_{xxx} - \chi_x^3 + 2\chi_t - 6\chi_x \chi_{xx} \tanh \left(\frac{1}{2} \sigma (\chi + \varphi) \right) + 3\chi_x^3 \tanh \left(\frac{1}{2} (\chi + \varphi) \right)^2 \right]. \quad (10)$$

再设扰动 Nizhnik-Novikov-Veselov 系统(1), (2)的解 (u, v) 为

$$u = \sum_{i=0}^{\infty} u_i p^i,$$

$$v = \sum_{i=0}^{\infty} v_i p^i, \quad p \in I \equiv [0, 1]. \quad (11)$$

将(11)式代入 $H_i[u, v, p] = 0 (i = 1, 2)$, 按 p 展开, 比较 $p^i (i = 0, 1, 2, \dots)$ 同次幂的系数. 对于 p^0 的系数自然成立. 对于 p^1 的系数, 有

$$L_1(u_1) = \varepsilon f(u_0, v_0), \quad (12)$$

$$L_2(u_1, v_1) = \varepsilon g(u_0, v_0). \quad (13)$$

现作行波变换

$$\xi = x - t. \quad (14)$$

由(14)式, 方程(12)为

$$\frac{du_1}{d\xi} + \frac{d^3 u_1}{d\xi^3} = \varepsilon f(u_0, v_0). \quad (15)$$

因此方程(12)具有如下形式的行波解:

$$u_1(\xi, y) = C_{11}(y) \cos \xi + C_{12}(y) \sin \xi + C_{13}(y) + \varepsilon \int_0^\xi \int_0^{\xi_1} [f(u_0(\eta, y), v_0(\eta, y)) \times \sin(\xi_1 - \eta)] d\eta d\xi_1. \quad (16)$$

再由(16)式, 便可得到方程(13)的解

$$v_1(\xi, y) = \int_0^y \left[\frac{\partial u_1}{\partial \xi}(\xi, y_1) - \varepsilon g(u_0(\xi, y_1), v_0(\xi, y_1)) \right] dy_1 + C_{14}(\xi). \quad (17)$$

于是由(16), (17), 便得到系统(12), (13)的一组行波解

$$u_1(x - t, y) = C_{11}(y) \cos(x - t) + C_{12}(y) \sin(x - t) + C_{13}(y) + \varepsilon \int_0^{x-t} \int_0^{\xi_1} [f(u_0(\eta, y), v_0(\eta, y)) \sin(\xi_1 - \eta)] d\eta d\xi_1, \quad (18)$$

$$v_1(x - t, y) = \int_0^y \left[\frac{\partial u_1}{\partial \xi}(x - t, y_1) - \varepsilon g(u_0(x - t, y_1), v_0(x - t, y_1)) \right] dy_1 + C_{14}(x - t), \quad (19)$$

其中 $C_{1i}(y) (i = 1, 2, 3), C_{14}(x - t)$ 为任意函数.

将(9)–(11), (18), (19)式代入 $H_i[u, v, p] = 0$, 按 p 展开, 关于 p^2 的系数, 有

$$L_1(u_2) = 3u_0 \frac{\partial v_1}{\partial x} + 3v_0 \frac{\partial u_1}{\partial x} + \varepsilon [f_u(u_0, v_0) u_1 + f_v(u_0, v_0) v_1], \quad (20)$$

$$L_2(u_2, v_2) = \varepsilon [g_u(u_0, v_0) u_1 + g_v(u_0, v_0) v_1]. \quad (21)$$

在行波变换(14)下, 系统(20), (21)具有如下行波解:

$$\begin{aligned}
 u_2(x-t, y) = & C_{21}(y) \cos(x-t) \\
 & + C_{22}(y) \sin(x-t) + C_{23}(y) \\
 & + \int_0^{x-t} \int_0^{\xi_1} \left[(3u_0(\eta, y) \frac{\partial v_1}{\partial \eta}(\eta, y) \right. \\
 & + 3v_0(\eta, y) \frac{\partial u_1}{\partial \eta}(\eta, y) \\
 & + \varepsilon(f_u(u_0(\eta, y), v_0(\eta, y))u_1(\eta, y) \\
 & + f_v(u_0(\eta, y), v_0(\eta, y))v_1(\eta, y)) \left. \right] \\
 & \times \sin(\xi_1 - \eta) d\eta d\xi_1, \quad (22)
 \end{aligned}$$

$$\begin{aligned}
 v_2(x-t, y) = & \int_0^y \left[\frac{\partial u_2}{\partial \xi}(x-t, y_1) \right. \\
 & - \varepsilon(g_u(u_0(x-t, y), v_0(x-t, y)) \\
 & \times u_1(x-t, y) + g_v(u_0(\eta, y), \\
 & v_0(\eta, y))v_1(\eta, y)) \left. \right] dy_1 \\
 & + C_{24}(x-t). \quad (23)
 \end{aligned}$$

其中 $C_{2i}(y) (i = 1, 2, 3)$, $C_{24}(x-t)$ 为任意函数.

用相同的方法, 可以依次地得到 $u_n(x-t, y)$,

$v_n(x-t, y) (n = 3, 4, \dots)$:

$$\begin{aligned}
 u_n(x-t, y) = & C_{n1}(y) \cos(x-t) \\
 & + C_{n2}(y) \sin(x-t) + C_{n3}(y) \\
 & + \int_0^{x-t} \int_0^{\xi_1} (F_{n1}(\eta, y) + \varepsilon F_{n2}(\eta, y)) \\
 & \times \sin(\xi_1 - \eta) d\eta d\xi_1, \quad (24)
 \end{aligned}$$

$$\begin{aligned}
 v_n(x-t, y) = & \varepsilon \int_0^y \left[\frac{\partial u_n}{\partial \xi}(x-t, y_1) \right. \\
 & - \varepsilon G_n(x-t, y_1) \left. \right] dy_1 \\
 & + C_{n4}(x-t), \quad (25)
 \end{aligned}$$

其中 $C_{ni}(y) (i = 1, 2, 3)$, $C_{n4}(x-t) (n = 3, 4, \dots)$ 为任意函数, 而

$$\begin{aligned}
 F_{n1}(\eta, y) = & \sum_{i=0}^{n-1} \left[u_i(\eta, y) \frac{\partial v_{n-i-1}}{\partial \eta}(\eta, y) \right. \\
 & + v_i(\eta, y) \frac{\partial u_{n-i-1}}{\partial \eta}(\eta, y) \left. \right], \\
 F_{n2}(\eta, y) = & \frac{1}{(i-1)!} \left[\frac{\partial^{i-1}}{\partial p^{i-1}} \left(\sum_{i=0}^{\infty} u_i(\eta, y) p^i, \right. \right. \\
 & \left. \left. \sum_{i=0}^{\infty} v_i(\eta, y) p^i \right) \right]_{p=0}, \\
 G_n(\eta, y) = & \frac{1}{(i-1)!} \left[\frac{\partial^{i-1}}{\partial p^{i-1}} \left(\sum_{i=0}^{\infty} u_i(\eta, y) p^i, \right. \right. \\
 & \left. \left. \sum_{i=0}^{\infty} v_i(\eta, y) p^i \right) \right]_{p=0}.
 \end{aligned}$$

将得到的 $u_i, v_i (i = 0, 1, 2, \dots, n)$ 代入(11)式, 并令

$p = 1$, 得到系统(1), (2) 的 n 次近似的孤子解

$$\begin{aligned}
 \tilde{u}_n(t, x, y) = & \frac{1}{2} \left[-1 + \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \right] \chi_x \varphi_y \\
 & + \sum_{i=0}^n [C_{n1}(y) \cos(x-t) \\
 & + C_{n2}(y) \sin(x-t) + C_{n3}(y)] \\
 & + \varepsilon \int_0^{x-t} \int_0^{\xi_{11}} f(u_0(\eta, y), v_0(\eta, y)) \\
 & \times \sin(\xi_1 - \eta) d\eta d\xi_1 \\
 & + \int_0^{x-t} \int_0^{\xi_{11}} \left[(u_0(\eta, y) \frac{\partial v_1}{\partial \eta}(\eta, y) \right. \\
 & + v_0(\eta, y) \frac{\partial u_1}{\partial \eta}(\eta, y) \\
 & + \varepsilon[f_u(u_0(\eta, y), v_0(\eta, y))u_1(\eta, y) \\
 & + f_v(u_0(\eta, y), v_0(\eta, y))v_1(\eta, y)) \left. \right] \\
 & \times \sin(\xi_1 - \eta) d\eta d\xi_1 \\
 & + \sum_{i=3}^n \left[\int_0^{x-t} \int_0^{\xi_1} (F_{i1}(\eta, y) \right. \\
 & + \varepsilon F_{i2}(\eta, y)) \sin(\eta, y) d\eta d\xi_1 \left. \right], \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 \tilde{v}_n(t, x, y) = & \frac{1}{6\chi_\xi} \left[2\chi_{\xi\xi\xi} - \chi_\xi^3 - 2\chi_\xi \right. \\
 & - 6\chi_\xi \chi_{\xi\xi} \tanh\left(\frac{1}{2}\sigma(\chi + \varphi)\right) \\
 & + 3\chi_\xi^3 \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \left. \right] \\
 & + \int_0^y \left[\frac{\partial u_1}{\partial \xi}(x-t, y_1) \right. \\
 & - \varepsilon g(u_0(x-t, y_1), v_0(x-t, y_1)) \left. \right] dy_1 \\
 & + \int_0^y \left[\frac{\partial u_2}{\partial \xi}(x-t, y_1) \right. \\
 & - \varepsilon(g_u(u_0(x-t, y), v_0(x-t, y)) \\
 & \times u_1(x-t, y) \\
 & + g_v(u_0(\eta, y), v_0(\eta, y))v_1(\eta, y)) \left. \right] dy_1 \\
 & + \sum_{i=3}^n \left[\int_0^y \left(\frac{\partial u_n}{\partial \xi}(x-t, y_1) \right. \right. \\
 & - \varepsilon \sum_{i=3}^n [G_n(x-t, y_1)] dy_1 \left. \right] \\
 & + \sum_{i=1}^n C_{i4}(x-t), \quad (27)
 \end{aligned}$$

其中 $C_{ni}(y) (i = 1, 2, 3)$, $C_{n4}(x-t), (n = 0, 1, 2, \dots)$ 为任意常数, χ, φ 由(5), (6) 式表示. 由(26), (27) 式可得: $u(t, x, y) = \lim_{n \rightarrow \infty} \tilde{u}_n(t, x, y), v(t, x, y) = \lim_{n \rightarrow \infty} \tilde{v}_n(t, x,$

y). 因此它们就是非线性扰动 Nizhnik-Novikov-Veselov 系统(1),(2) 的一组孤子精确解.

4. 例

考虑如下一个简单的扰动 Nizhnik-Novikov-Veselov 系统

$$u_t + u_{xxx} - 3uv_x - 3vu_x = \varepsilon \sin u, \quad (28)$$

$$u_x - v_y = \varepsilon \cos v. \quad (29)$$

设扰动 Nizhnik-Novikov-Veselov 系统(28),(29) 的解(u, v)为

$$\begin{aligned} u &= \sum_{i=0}^{\infty} u_i p^i, \\ v &= \sum_{i=0}^{\infty} v_i p^i. \end{aligned} \quad (30)$$

在映射(7),(8)中,仍选取初始近似(u_0, v_0)为古典 Nizhnik-Novikov-Veselov 系统的分形孤子解(3),(4). 即(9),(10)式.

将(9),(10),(30)式代入 $H_i[u, v, p] = 0$,按 p 展开,关于 p^1 的系数,有

$$L_1(u_1) = \varepsilon \sin u_0, \quad (31)$$

$$L_2(u_1, v_1) = \varepsilon \cos v_0. \quad (32)$$

在行波变换 $\xi = x - t$ 下,系统(31),(32)具有如下行波解:

$$\begin{aligned} u_1(x-t, y) &= C_{11}(y) \cos(x-t) + C_{12}(y) \sin(x-t) \\ &+ C_{13}(y) + \varepsilon \int_0^{x-t} \int_0^{\xi_{11}} \left[\sin\left(\frac{1}{2}\right. \right. \\ &\times \left. \left. \left[-1 + \tanh\left(\frac{1}{2}(\chi + \varphi)\right)\right]^2 \right] \right. \\ &\times \left. \chi_\eta \varphi_y \right) \sin(\xi_1 - \eta) \Big] d\eta d\xi_1, \end{aligned} \quad (33)$$

$$\begin{aligned} v_1(x-t, y) &= \int_0^y \left[(-C_{11}(y_1) \sin(x-t) \right. \\ &+ C_{12}(y_1) \cos(x-t)) \\ &+ \varepsilon \int_0^y \left[\int_0^{x-t} \left[\sin\left(\frac{1}{2}\left[-1 \right. \right. \right. \right. \\ &+ \left. \left. \left. \tanh\left(\frac{1}{2}(\chi + \varphi)\right)\right]^2 \right] \chi_\eta \varphi_y \right] \right. \end{aligned}$$

$$\begin{aligned} &\times \sin(\xi - \eta) d\eta - \varepsilon \cos\left(\frac{1}{6\chi_\xi} [2\chi_{\xi\xi\xi} \right. \\ &- \chi_\xi^3 - 2\chi_\xi - 6\chi_\xi \chi_{\xi\xi} \tanh\left(\frac{1}{2}\sigma(\chi + \varphi)\right) \\ &+ 3\chi_x^3 \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \Big] dy_1 \\ &+ C_{14}(x-t). \end{aligned} \quad (34)$$

于是我们得到了扰动 Nizhnik-Novikov-Veselov 系统(28),(29)的一次近似孤子解

$$\begin{aligned} \tilde{u}_1(t, x, y) &= \frac{1}{2} \left[-1 + \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \right] \chi_x \varphi_y \\ &+ C_{11}(y) \cos(x-t) \\ &+ C_{12}(y) \sin(x-t) + C_{13}(y) \\ &+ \varepsilon \int_0^{x-t} \int_0^{\xi_{11}} \left[\sin\left(\frac{1}{2}\left[-1 + \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \right] \right. \right. \\ &\times \left. \left. \chi_\eta \varphi_y \right) \sin(\xi_1 - \eta) \Big] d\eta d\xi_1 \\ &+ O(\varepsilon^2), \quad 0 < \varepsilon \ll 1, \end{aligned}$$

$$\begin{aligned} \tilde{v}_1(t, x, y) &= \frac{1}{6\chi_\xi} \left[2\chi_{\xi\xi\xi} - \chi_\xi^3 - 2\chi_\xi - 6\chi_\xi \chi_{\xi\xi} \tanh\left(\frac{1}{2}\sigma \right. \right. \\ &\times \left. \left. (\chi + \varphi) \right) + 3\chi_x^3 \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \right] \\ &+ \int_0^y \left[(-C_{11}(y_1) \sin(x-t) + C_{12}(y_1) \right. \\ &\times \cos(x-t)) + \varepsilon \int_0^y \left[\int_0^{x-t} \left[\sin\left(\frac{1}{2}\left[-1 \right. \right. \right. \right. \\ &\times \left. \left. \left. \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \right] \chi_\eta \varphi_y \right) \right. \right. \\ &\times \sin(\xi - \eta) d\eta - \varepsilon \cos\left(\frac{1}{6\chi_\xi} [2\chi_{\xi\xi\xi} - \chi_\xi^3 \right. \\ &- 2\chi_\xi - 6\chi_\xi \chi_{\xi\xi} \tanh\left(\frac{1}{2}\sigma(\chi + \varphi)\right) \\ &+ 3\chi_x^3 \tanh\left(\frac{1}{2}(\chi + \varphi)\right)^2 \Big] dy_1 \\ &+ C_{14}(x-t) + O(\varepsilon^2), \\ &0 < \varepsilon \ll 1. \end{aligned}$$

用上述方法,我们还可进一步得到扰动 Nizhnik-Novikov-Veselov 系统(28),(29)的更高次近似孤子解.

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Asymptotic solution for fractal soliton of disturbed Nizhnik-Novikov-Veselov system^{*}

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Abstract

A class of nonlinear disturbed Nizhnik-Novikov-Veselov system is studied. Using the special asymptotic method, the asymptotic solution for fractal soliton of corresponding system is obtained.

Keywords: soliton, asymptotic solution, disturbance

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