可控三模纠缠相干态的产生

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(2011 年 3 月 7 日收到; 2011 年 4 月 19 日收到修改稿)

采用量子理论, 考虑边界振动的微腔与腔内的三模辐射场构成的系统, 给出了系统的时间演化算符及其变换, 得到了系统的态函数随时间的演化关系. 结果表明, 当振动边界回到初态时, 三模腔场处于纠缠相干态, 并且可以调控不同纠缠相干态的产生. 给出了九个不同的三模纠缠相干态, 这些态中含有不同的相位因子, 这些相位因子充分体现了场和振动边界的相互作用和相互影响. 这些结果为可控纠缠相干态的实验制备提供了重要的理论依据.

关键词: 可控纠缠相干态, 边界振动的微腔

PACS: 42.50.Dv

1 引 言

非经典态 (如纠缠态、薛定谔猫态、压缩态等) 的产生或制备, 不仅对于了解量子力学的基本概念有着重要的意义, 而且已经或即将在一些前沿领域中 (特别是在量子信息方面) 得到广泛的应用, 如纠缠态在量子远程通信中的应用; 压缩态和薛定谔猫态在光通信、精密计量、纳波检测及量子计算机等方面的应用等. 正因为如此, 人们对非经典态的产生或制备的研究一直没有停止过, 如边界振动的微腔中薛定谔猫态和两模纠缠态的产生 [1−3]; 无简并参量放大器中多模纠缠相干态的产生 [4]; 大失谐量的微腔中非经典态 (薛定谔猫态和纠缠相干态) 的产生 [5]; 通过两腔模与 V 型三能级原子的共振相互作用产生纠缠相干态 [6]; 两原子模型纠缠相干态的产生 [7] 等.

具有振动边界的微腔在制备非经典态 (例如薛定谔猫态) 方面有若干优越性 [1−3], 文献 [2] 在忽略了场的自由演化的情况下对于边界振动的三模微腔中纠缠相干态的产生进行了研究, 没有给出纠缠相干态的具体例子. 本文没有忽略场的自由演化对边界振动的三模微腔中纠缠相干态的产生进行了研究, 发现纠缠相干态的产生是可以调控的, 这是有非常重要的实际意义的研究. 具体步骤是采用量子理论, 对于边界振动的微腔与腔内的三模辐射场构成的系统, 给出了系统的时间演化算符及其变换, 得到了系统的态函数随时间的演化关系. 讨论了当振动边界回到初态时的腔场态, 可见三模腔场处于纠缠相干态并且可以调控不同纠缠相干态的产生. 给出了九个不同的三模纠缠相干态, 这些态中含有不同的相位因子, 这些相位因子充分体现了场和振动边界的相互作用和相互影响, 这些结果为可控纠缠相干态的实验制备提供了重要的理论依据.

2 系统的时间演化算符及其变换

考虑一个边界振动的微腔 (振动边界视为频率为 ω_m 的量子谐振子) 与腔内的三模辐射场构成的系统, 设三个腔模的频率关系为 $\omega_j = j\omega_1$ ($j = 1, 2, 3$), 则该系统的哈密顿量为 [1−3]

$$H = \hbar\omega_1\left(a_1^+a_1 + 2a_2^+a_2 + 3a_3^+a_3\right) + \hbar\omega_m b^+b - \hbar g_1\left(a_1^+a_1 + 2a_2^+a_2 + 3a_3^+a_3\right)\left(b + b^+\right), \quad (1)$$

式中 a_1^+ (a_1)、a_2^+ (a_2) 和 a_3^+ (a_3) 分别是辐射场第 1 模、第 2 模和第 3 模的产生 (湮没) 算符, b^+ (b) 是频率为 ω_m 的量子谐振子的产生 (湮没) 算符, $g_j = \frac{\omega_j}{l}\sqrt{\frac{\hbar}{2m\omega_m}}$ (l 是微腔的长度, m 是振动边界的质量, $j = 1, 2, 3$) 是腔模与振动边界的耦合常数 [2]. 该系统的时间演化算符为

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$$U(t) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}H\,t} = e^{-\mathrm{i}\left[\omega_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)+\omega_m b^+b-g_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b+b^+\right)\right]\,t}. \tag{2}$$

令 $c_1 = \omega_1/\omega_m$, $c_2 = g_1/\omega_m$, 并利用 Glauber 公式 ($\mathrm{e}^{A+B} = \mathrm{e}^A\mathrm{e}^B\mathrm{e}^{-[A,B]/2}$), 可将 (2) 式改写为

$$U(t) = \mathrm{e}^{-\mathrm{i}c_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\omega_m t}\times \mathrm{e}^{-\mathrm{i}\left[b^+b-c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b+b^+\right)\right]\omega_m t}. \tag{3}$$

引入么正算符 $T = \mathrm{e}^{-c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b^+-b\right)}$, 将 (3) 式两边同时左乘 T 右乘 T^+, 并利用 B-H 公式

$$\mathrm{e}^A B\mathrm{e}^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}[A,[A,[A,B]]] + \Lambda, \tag{4}$$

得

$$TU(t)T^+ = \mathrm{e}^{-\mathrm{i}c_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\omega_m t}\,\mathrm{e}^{\mathrm{i}c_2^2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)^2\omega_m t}\,\mathrm{e}^{-\mathrm{i}b^+b\omega_m t}. \tag{5}$$

将 (5) 式两边同时左乘 T^+ 右乘 T 得

$$U(t) = \mathrm{e}^{-\mathrm{i}c_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\omega_m t}\times \mathrm{e}^{\mathrm{i}c_2^2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)^2\omega_m t}\times \mathrm{e}^{c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b^+-b\right)}\times \mathrm{e}^{-ib^+b\omega_m t}\mathrm{e}^{-c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b^+-b\right)}. \tag{6}$$

将 (6) 式右边右乘 $\mathrm{e}^{\mathrm{i}b^+b\omega_m t}\mathrm{e}^{-\mathrm{i}b^+b\omega_m t}$, 并利用 (4) 式得

$$\begin{aligned} U(t) = {} & \mathrm{e}^{-\mathrm{i}c_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\omega_m t} \\ & \times\mathrm{e}^{\mathrm{i}c_2^2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)^2\omega_m t} \\ & \times\mathrm{e}^{c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b^+-b\right)} \\ & \times\mathrm{e}^{-c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(b^+\mathrm{e}^{-\mathrm{i}\omega_m t}-b\mathrm{e}^{\mathrm{i}\omega_m t}\right)} \\ & \times\mathrm{e}^{-\mathrm{i}b^+b\omega_m t}. \end{aligned} \tag{7}$$

利用 Glauber 公式可将 (7) 式改写为

$$\begin{aligned} U(t) = {} & \mathrm{e}^{-\mathrm{i}c_1\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\omega_m t} \\ & \times\mathrm{e}^{\mathrm{i}c_2^2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)^2\left(\omega_m t-\sin\omega_m t\right)} \\ & \times\mathrm{e}^{c_2\left(a_1^+a_1+2a_2^+a_2+3a_3^+a_3\right)\left(\eta b^+-\eta^\bullet b\right)} \\ & \times\mathrm{e}^{-\mathrm{i}b^+b\omega_m t}, \end{aligned} \tag{8}$$

式中 $\eta = 1 - \mathrm{e}^{-\mathrm{i}\omega_m t}$. (8) 式就是经过变换后的系统的时间演化算符.

3 系统的态函数随时间的演化关系

假定初始时三个腔模分别处于相干态 $|\alpha_1\rangle_{f_1}$, $|\alpha_2\rangle_{f_2}$ 和 $|\alpha_3\rangle_{f_3}$, 边界初始也处于相干态 $|\beta\rangle_m$, 且它们彼此无纠缠, 则系统的初态为

$$|\psi(0)\rangle = |\alpha_1\rangle_{f_1}|\alpha_2\rangle_{f_2}|\alpha_3\rangle_{f_3}|\beta\rangle_m, \tag{9}$$

式中相干态与 Fock 态和真空态的关系为

$$\begin{aligned} |\alpha_1\rangle_{f_1} &= \mathrm{e}^{-\frac{1}{2}|\alpha_1|^2}\sum_{n=0}^{\infty}\frac{\alpha_1^n}{\sqrt{n!}}|n\rangle_{f_1}, \\ |\alpha_2\rangle_{f_2} &= \mathrm{e}^{-\frac{1}{2}|\alpha_2|^2}\sum_{k=0}^{\infty}\frac{\alpha_2^k}{\sqrt{k!}}|k\rangle_{f_2}, \\ |\alpha_3\rangle_{f_3} &= \mathrm{e}^{-\frac{1}{2}|\alpha_3|^2}\sum_{d=0}^{\infty}\frac{\alpha_3^d}{\sqrt{d!}}|d\rangle_{f_3}, \\ |\beta\rangle_m &= \mathrm{e}^{-\frac{1}{2}|\beta|^2}\sum_{r=o}^{\infty}\frac{\beta^r}{\sqrt{r!}}|r\rangle_m \\ &= \mathrm{e}^{\beta b^+-\beta^* b}|0\rangle_m, \end{aligned} \tag{10}$$

$|n\rangle_{f_1}$, $|k\rangle_{f_2}$ 和 $|d\rangle_{f_3}$ 分别是腔场第 1 模、第 2 模和第 3 模的 Fock 态, $|r\rangle_m$ 和 $|0\rangle_m$ 分别是边界的 Fock 态和真空态. 将系统的时间演化算符 (8) 式作用于系统的初态 (9) 式上, 并利用 (10) 式可得 t 时刻的态函数

$$\begin{aligned} |\psi(t)\rangle = {} & \mathrm{e}^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)}\sum_{n,k,d=0}^{\infty}\frac{\alpha_1^n\alpha_2^k\alpha_3^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\mathrm{e}^{\mathrm{i}\left\{\left[c_2^2(n+2k+3d)^2-c_1(n+2k+3d)\right]\omega_m t-c_2^2(n+2k+3d)^2\sin\omega_m t\right\}} \\ & \times\mathrm{e}^{c_2(n+2k+3d)\left(\eta b^+-\eta^b\right)}\mathrm{e}^{\beta\mathrm{e}^{-\mathrm{i}\omega_m t}b^+-\beta^*\mathrm{e}^{\mathrm{i}\omega_m t}b}|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}|0\rangle_m. \end{aligned} \tag{11}$$

利用 Glauber 公式及 (10) 式可将 (11) 式改写为

$$\begin{aligned} |\psi(t)\rangle = {} & \mathrm{e}^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)}\sum_{n,k,d=0}^{\infty}\frac{\alpha_1^n\alpha_2^k\alpha_3^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\mathrm{e}^{\frac{1}{2}c_2(n+2k+3d)(\beta\eta-\beta^\bullet\eta^\bullet)} \\ & \times\mathrm{e}^{\mathrm{i}\left\{\left[c_2^2(n+2k+3d)^2-c_1(n+2k+3d)\right]\omega_m t-c_2^2(n+2k+3d)^2\sin\omega_m t\right\}} \\ & \times|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}\left|c_2(n+2k+3d)\eta+\beta\mathrm{e}^{-\mathrm{i}\omega_m t}\right\rangle_m. \end{aligned} \tag{12}$$

(12) 式就是系统的态函数随时间的演化关系.

4 可控三模纠缠相干态的产生

由 (13) 式可知, 当 $0 < t < 2\pi/\omega_m$ 时, 系统处于纠缠态, 且当 $t = \pi/\omega_m$ 时, 振动边界和腔场具有最大的纠缠度 [2]; 当 $t = 2\pi/\omega_m$ 时, $\left|c_2(n+2k+3d)\eta+\beta e^{-i\omega_m t}\right\rangle_m = |\beta\rangle_m$, 即振动边界回到了初态, 而此时的三模腔场态为

$$\begin{aligned}|\phi\rangle_f =& e^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)} \sum_{n,k,d=0}^{\infty}\frac{\alpha_1^n\alpha_2^k\alpha_3^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}} e^{i\left[c_2^2(n+2k+3d)^2-c_1(n+2k+3d)\right]2\pi} |n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}\\ =& e^{-\frac{1}{2}|\alpha_1|^2}\sum_{n=0}^{\infty}\frac{\left(\alpha_1 e^{-i2\pi c_1}\right)^n}{\sqrt{n!}} e^{i2\pi c_2^2 n^2}|n\rangle_{f_1} e^{-\frac{1}{2}|\alpha_2|^2} \sum_{k=0}^{\infty}\frac{\left(\alpha_2 e^{-i4\pi c_1}e^{i8\pi n c_2^2}\right)^k}{\sqrt{k!}} e^{i8\pi c_2^2 k^2}|k\rangle_{f_2}\\ &\times e^{-\frac{1}{2}|\alpha_3|^2}\sum_{d=0}^{\infty}\frac{\left(\alpha_3 e^{-i6\pi c_1}e^{i12\pi(n+2k)c_2^2}\right)^d}{\sqrt{d!}} e^{i18\pi c_2^2 d^2}|d\rangle_{f_3}. \end{aligned} \tag{13}$$

一般来说上式是一个三模纠缠相干态 [4,8–12], 通过调整 c_2(或 c_1), 可以调控不同纠缠相干态的产生, 经过复杂的推导, 我们找到了以下九个不同的纠缠相干态. 这些结果是非常有意义的, 为可控纠缠相干态的实验制备提供了重要的理论依据.

当 $c_2 = 1/2$ 时, $c_1 = lr/2$ (记 $r = \sqrt{2m\omega_m/\hbar}$, 以下同), 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f =& \frac{1+i}{2}\left|\alpha_1 e^{-i\pi lr}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\pi lr}\right\rangle_{f_2}\left|\alpha_3 e^{-i3\pi lr}\right\rangle_{f_3}\\ &+\frac{1-i}{2}\left|-\alpha_1 e^{-i\pi lr}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\pi lr}\right\rangle_{f_2}\left|-\alpha_3 e^{-i3\pi lr}\right\rangle_{f_3};\end{aligned} \tag{14}$$

当 $c_2 = \sqrt{3}/2$ 时,$c_1 = \sqrt{3}lr/2$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f =& \frac{1-i}{2}\left|\alpha_1 e^{-i\sqrt{3}\pi lr}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\sqrt{3}\pi lr}\right\rangle_{f_2}\left|\alpha_3 e^{-i3\sqrt{3}\pi lr}\right\rangle_{f_3}\\ &+\frac{1+i}{2}\left|-\alpha_1 e^{-i\sqrt{3}\pi lr}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\sqrt{3}\pi lr}\right\rangle_{f_2} \times\left|-\alpha_3 e^{-i3\sqrt{3}\pi lr}\right\rangle_{f_3};\end{aligned} \tag{15}$$

当 $c_2 = \sqrt{6}/6$ 时, $c_1 = \sqrt{6}lr/6$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f =& -i\frac{\sqrt{3}}{3}\left|-\alpha_1 e^{-i\sqrt{6}\pi lr/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\sqrt{6}\pi lr/3}\right\rangle_{f_2}\left|-\alpha_3 e^{-i\sqrt{6}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1+i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{i\pi\left(1-\sqrt{6}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{i2\pi\left(1-\sqrt{6}\,lr\right)/3}\right\rangle_{f_2}\left|-\alpha_3 e^{-i\sqrt{6}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1+i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{-i\pi\left(1+\sqrt{6}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i2\pi\left(1+\sqrt{6}\,lr\right)/3}\right\rangle_{f_2} \times\left|-\alpha_3 e^{-i\sqrt{6}\pi lr}\right\rangle_{f_3};\end{aligned} \tag{16}$$

当 $c_2 = \sqrt{3}/3$ 时, $c_1 = \sqrt{3}lr/3$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f =& i\frac{\sqrt{3}}{3}\left|\alpha_1 e^{-i2\sqrt{3}\pi lr/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i4\sqrt{3}\pi lr/3}\right\rangle_{f_2}\left|\alpha_3 e^{-i2\sqrt{3}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1-i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{i2\pi\left(1-\sqrt{3}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{i4\pi\left(1-\sqrt{3}\,lr\right)/3}\right\rangle_{f_2}\left|\alpha_3 e^{-i2\sqrt{3}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1-i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{-i2\pi\left(1+\sqrt{3}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i4\pi\left(1+\sqrt{3}\,lr\right)/3}\right\rangle_{f_2} \times\left|\alpha_3 e^{-i2\sqrt{3}\pi lr}\right\rangle_{f_3};\end{aligned} \tag{17}$$

当 $c_2 = \sqrt{6}/3$ 时, $c_1 = \sqrt{6}lr/3$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f =& -i\frac{\sqrt{3}}{3}\left|\alpha_1 e^{-i2\sqrt{6}\pi lr/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i4\sqrt{6}\pi lr/3}\right\rangle_{f_2}\left|\alpha_3 e^{-i2\sqrt{6}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1+i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{i2\pi\left(2-\sqrt{6}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{i4\pi\left(2-\sqrt{6}\,lr\right)/3}\right\rangle_{f_2}\left|\alpha_3 e^{-i2\sqrt{6}\pi lr}\right\rangle_{f_3}\\ &+\frac{1}{2}\left(1+i\frac{\sqrt{3}}{3}\right)\left|\alpha_1 e^{-i2\pi\left(2+\sqrt{6}\,lr\right)/3}\right\rangle_{f_1} \times\left|\alpha_2 e^{-i4\pi\left(2+\sqrt{6}\,lr\right)/3}\right\rangle_{f_2} \times\left|\alpha_3 e^{-i2\sqrt{6}\pi lr}\right\rangle_{f_3};\end{aligned} \tag{18}$$

当 $c_2=\sqrt{2}/4$ 时, $c_1=\sqrt{2}lr/4$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f = &\frac{\sqrt{2}}{4}(1+\mathrm{i})\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{2}\pi lr/2}\right\rangle_{f_3}\\&-\frac{\sqrt{2}}{4}(1+\mathrm{i})\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr}\right\rangle_{f_2}\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{2}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{2}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{2}\pi lr}\right\rangle_{f_2}\\&\times\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{2}\pi lr/2}\right\rangle_{f_3};\end{aligned}\tag{19}$$

当 $c_2=\sqrt{6}/4$ 时, $c_1=\sqrt{6}lr/4$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f = &-\frac{\sqrt{2}}{4}(1-\mathrm{i})\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{6}\pi lr/2}\right\rangle_{f_3}\\&+\frac{\sqrt{2}}{4}(1-\mathrm{i})\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr}\right\rangle_{f_2}\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{6}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{6}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{6}\pi lr}\right\rangle_{f_2}\\&\times\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{6}\pi lr/2}\right\rangle_{f_3};\end{aligned}\tag{20}$$

当 $c_2=\sqrt{10}/4$ 时, $c_1=\sqrt{10}lr/4$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f = &-\frac{\sqrt{2}}{4}(1+\mathrm{i})\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{10}\pi lr/2}\right\rangle_{f_3}\\&+\frac{\sqrt{2}}{4}(1+\mathrm{i})\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr}\right\rangle_{f_2}\left|-\alpha_3\mathrm{e}^{-i3\sqrt{10}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{10}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{10}\pi lr}\right\rangle_{f_2}\\&\times\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{10}\pi lr/2}\right\rangle_{f_3};\end{aligned}\tag{21}$$

当 $c_2=\sqrt{14}/4$ 时, $c_1=\sqrt{14}lr/4$, 由 (13) 式得

$$\begin{aligned}|\phi\rangle_f = &\frac{\sqrt{2}}{4}(1-\mathrm{i})\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{14}\pi lr/2}\right\rangle_{f_3}\\&-\frac{\sqrt{2}}{4}(1-\mathrm{i})\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr/2}\right\rangle_{f_1}\\&\times\left|\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr}\right\rangle_{f_2}\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{14}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{14}\pi lr/2}\right\rangle_{f_3}\\&+\frac{1}{2}\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr/2}\right\rangle_{f_1}\\&\times\left|-\alpha_2\mathrm{e}^{-\mathrm{i}\sqrt{14}\pi lr}\right\rangle_{f_2}\\&\times\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\sqrt{14}\pi lr/2}\right\rangle_{f_3}.\end{aligned}\tag{22}$$

(14)—(22) 式是九个不同的三模纠缠相干态, 这些态中含有不同的相位因子, 这些相位因子充分反映了场和振动边界的相互作用和相互影响, 这些态从实验上制备成功的话, 将有非常重要的实用意义 [11,12].

5 结 论

采用量子理论, 考虑了边界振动的微腔 (振动边界视为频率为 ω_m 的量子谐振子) 与三模辐射场构成的系统, 给出了系统的时间演化算符及其变换, 得到了系统的态函数随时间的演化关系. 结果表明, 当 $0<t<2\pi/\omega_m$ 时, 系统处于纠缠态; 当 $t=2\pi/\omega_m$ 时, 振动边界回到了初态, 而三模腔场处于纠缠相干态, 并且可以调控不同纠缠相干态的产生. 给出了九个不同的三模纠缠相干态, 这些态中含有不同的相位因子, 这些相位因子充分反映了场和振动边界的相互作用和相互影响. 这些结果是非常有意义的, 为可控纠缠相干态的实验制备提供了重要的理论依据.

附录　(14) 式的推导和证明

当 $c_2=\frac{1}{2}$ 时, $c_1=\frac{1}{2}lr$, 由 (13) 式得 (14) 式

$$
\begin{aligned}
|\phi\rangle_f &= \mathrm{e}^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{d=0}^{\infty}\frac{\left(\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right)^n\left(\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right)^k\left(\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right)^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\\
&\quad\times\mathrm{e}^{\mathrm{i}\frac{\pi}{2}n^2}\mathrm{e}^{\mathrm{i}2\pi\, n\, k}\mathrm{e}^{\mathrm{i}2\pi\, k^2}\mathrm{e}^{\mathrm{i}3\pi(n+2k)d}\mathrm{e}^{\mathrm{i}\frac{9}{2}\pi\, d^2}\,|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}\\
&= \mathrm{e}^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{d=0}^{\infty}\frac{\left(\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right)^n\left(\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right)^k\left(\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right)^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\\
&\quad\times\mathrm{e}^{\mathrm{i}\frac{\pi}{2}n^2}\mathrm{e}^{\mathrm{i}3\pi nd}\mathrm{e}^{\mathrm{i}\frac{9}{2}\pi\, d^2}\,|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3},
\end{aligned}
$$

$$
\begin{aligned}
|\phi\rangle_f &= \frac{1+\mathrm{i}}{2}\left|\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right\rangle_{f_1}\left|\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right\rangle_{f_2}\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right\rangle_{f_3}+\frac{1-\mathrm{i}}{2}\left|-\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right\rangle_{f_1}\left|\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right\rangle_{f_2}\left|-\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right\rangle_{f_3}\\
&= \mathrm{e}^{-\frac{1}{2}\left(\left|\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right|^2+\left|\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right|^2+\left|\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right|^2\right)}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{d=0}^{\infty}\frac{\left(\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right)^n\left(\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right)^k\left(\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right)^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\\
&\quad\times\left[\frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^n(-1)^d\right]|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}\\
&= \mathrm{e}^{-\frac{1}{2}\left(|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2\right)}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{d=0}^{\infty}\frac{\left(\alpha_1\mathrm{e}^{-\mathrm{i}\pi\, l\, r}\right)^n\left(\alpha_2\mathrm{e}^{-\mathrm{i}2\pi\, l\, r}\right)^k\left(\alpha_3\mathrm{e}^{-\mathrm{i}3\pi\, l\, r}\right)^d}{\sqrt{n!}\sqrt{k!}\sqrt{d!}}\\
&\quad\times\left[\frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^n(-1)^d\right]|n\rangle_{f_1}|k\rangle_{f_2}|d\rangle_{f_3}.
\end{aligned}
$$

记

$$
\begin{aligned}
A &= \mathrm{e}^{\mathrm{i}\frac{\pi}{2}n^2}\mathrm{e}^{\mathrm{i}3\pi nd}\mathrm{e}^{\mathrm{i}\frac{9}{2}\pi\, d^2}=\mathrm{e}^{\mathrm{i}\frac{\pi}{2}n^2}\mathrm{e}^{\mathrm{i}\pi nd}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}d^2}\\
&= \mathrm{e}^{\mathrm{i}\frac{\pi}{2}(n+d)^2}=\mathrm{i}^{(n+d)^2},\\
B &= \frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^n(-1)^d\\
&= \frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^{n+d}
\end{aligned}
$$

利用数学归纳法可证 $A=B$.

令 $n+d=m$, 则

$$
\begin{aligned}
A &= \mathrm{i}^{m^2},\\
B &= \frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^m,\\
m &= 0,1,2,\cdots
\end{aligned}
$$

当 $m=0$ 时, $A=B$.

假设当 $m=k$ 时 $A=B$, 即 $\mathrm{i}^{k^2}=\dfrac{1+\mathrm{i}}{2}+\dfrac{1-\mathrm{i}}{2}(-1)^k$,

当 $m=k+1$ 时,

$$
\begin{aligned}
A &= \mathrm{i}^{(k+1)^2}=\mathrm{i}^{k^2}\mathrm{i}^{2k}\mathrm{i}=\left[\frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^k\right](-1)^k\mathrm{i}\\
&= \frac{\mathrm{i}-1}{2}(-1)^k+\frac{\mathrm{i}+1}{2}\\
&= \frac{1+\mathrm{i}}{2}+\frac{1-\mathrm{i}}{2}(-1)^{k+1}=B,
\end{aligned}
$$

则 $A=B$,

故 (14) 式正确.

(15)—(22) 式可用相同的方法推导和证明.

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Preparation of controllable three-mode entangled coherent states

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(Received 7 March 2011; revised manuscript received 19 April 2011)

Abstract

For a system composed of a three-mode cavity field interacting with a movable mirror, the time evolution operator of the system and its transformation are given, and the time evolution of the system state is obtained by means of quantum theory. The result shows that the state of the three-mode cavity field is in an entangled coherent state when the mirror returns to its original state, and the preparations of different entangled coherent states can be controlled. Nine different three-mode entangled coherent states are given, and the different phase factors of these states indicate the different reactions between the field and the movable mirror, which provides an important theoretical basis for the experimental preparations of controllable entangled coherent states.

Keywords: controllable entangled coherent state, cavity with a moving mirror

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