NUCLEAR LEVEL SPACING DEDUCED FROM THE RESONANCE ABSORPTION OF NEUTRONS*

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ABSTRACT

The average nuclear level spacings of In, Ir and Au are estimated from the beta-ray activities induced by the primary photo-neutrons emitted from a Ra+Be source. A Geiger-Müller counter made of aluminum is used for measuring the induced activities. The saturated induced activities for the three elements are found to be 0.665±.023, 0.982±.031, 0.453±.021 (no./sec.) respectively. The estimation for the average nuclear level spacings is made in accordance with Breit-Wigner's one level formula, the results being 5.6, 15 and 6 volts for In, Ir, Au respectively. The spacing of Ir is probably somewhat over estimated since the values of the energy and absorption coefficient of the resonance neutron group, used in the estimation, are not very accurate.

INTRODUCTION

It is known that the average nuclear level spacing at a certain energy interval can be deduced from the experiment with resonance neutrons provided that we know the level widths and if the induced activities are measured. This paper contains the results of measurements on the induced activities of In, Ir and Au due to resonance capture of medium fast neutrons. The capturing cross-section of a nucleus for neutrons having energies close to one of the resonance levels of the compound nucleus can be obtained from the Breit-Wigner's one level formula[1]

$$\sigma=\frac{\pi}{2}\left(1\pm\frac{1}{2i+1}\right)\frac{\hbar^2}{2M\sqrt{EE_r}}\times\frac{\Gamma_n\Gamma_r}{(E-E_r)^2+\frac{1}{4}\Gamma^2} \qquad (1)$$

where M is the mass of the neutron, E and E_r are the energies of the incident neutrons and the resonance energy of the compound nucleus respectively, and Γ_n, Γ_r and Γ are the neutron, radiation and total widths ($\Gamma=\Gamma_n+\Gamma_r$), i is the angular

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1. Bethe, *Rev. Mod. Phys.* **9** (1937), 140.

momentum of the capturing nucleus. But in the energy range of the neutrons used in this experiment it is impossible to deal with individual resonance levels, and only the general variation of the capturing cross-section with energy can be found. This is obtained by averaging the above formula over the band of levels near the mean energy E_m of the incident neutrons. If D is the mean level spacing in this interval, then the average capturing cross-section is given by

$$\sigma_a = \frac{2.03 \times 10^{-18} \Gamma_n' \Gamma_r}{E_m^{\frac{1}{2}} \Gamma D} \tag{2}$$

in which D, Γ and E_m are measured in volts, σ_a in square cm and

$$\Gamma_n' = \Gamma_n E^{-\frac{1}{2}}$$

is a constant by theory and also confirmed by experiments. In deriving formula (2) the term in the bracket of expression (1) is absorbed into Γ_n as an usual practice which means neglecting the fine structure of the levels.

If an element is exposed to neutrons of suitable speed for a suitable length of time this element will become radioactive through the capture of neutrons. Applying formula (2) one can obtain the saturated induced activity A_0 of this element. It is given by

$$A_0 = \frac{2.03 \times 10^{-18} a \Gamma_n' \Gamma_r Q}{\mu E_m^{\frac{1}{2}} \Gamma D} \tag{3}$$

or

$$D = \frac{2.03 \times 10^{-18} a \Gamma_n' \Gamma_r Q}{\mu E_m^{\frac{1}{2}} \Gamma A_0} \tag{4}$$

In the above expression Q is the total number of neutrons striking at this element in one second; μ is the atomic absorption cross-section of this element for the induced beta-ray; and "a" is the abundance of the active isotope. The values of Γ are not directly observable, but they can be deduced from the measured absorption coefficients for resonance and thermal neutrons. The expressions relating these factors can be found in Bethe's review[2]. Thus knowing the quantities on

2. Bethe, *Rev. Mod. Phys.* **9** (1937), 144, eqs. (537a), (536).

the right hand side of formula (4) one can easily calculate the level spacing of this element. The level spacings of the elements stated have been estimated in accordance with this expression. Experiments of a similar nature have been performed by Chao and Wang[3] on Br, Rh and Ag and also on a large number of elements by Griffith[4]. But the latter carried out his calculations with assumed level width and spacing for deducing capturing cross-section.

EXPERIMENTAL

The neutron source of this experiment consisted of a lead spherical shell of 2.8 cm diameter, filled with 11.46 grams of Be powder at the center of which was a tiny glass tube containing 50 mg of radium. Such a source gives two groups of neutrons of energies about 1.4×10^6 ev and 5.3×10^5 ev. Of these two groups the one having lower energy is more intense and effective. Hence the source may be considered as one which emits neutrons of energy 1.4×10^6 ev. Because of the enormous obliquities of collision between the Be nuclei and the gamma-rays emitted from RaC′, the neutrons projected out were spread into a spectrum of width several kilovolts covering a large number of levels of a measured element.

The induced beta-activities of the elements were registered by a thin walled (0.015cm) Geiger-Müller tube counter, 1.8 cm in diameter and 4 cm in length. The tube was made of aluminum and the inner electrode was a thin tungsten wire. These were assembled together with a pair of amber plugs. The tube was filled with dry air to a pressure of 5 cm of Hg. To reduce the effects due to cosmic rays the tube counter was shielded with blocks of lead (5 cm on both sides and 10 cm on top) and coupled electrically to a two-stage pulse amplifier with a thyratron which operated a mechanical recording counter.

The high tension applied to the counter was delivered from a rectifying set, the voltage output of which was controlled by a thermionic pentode. This circuit was proposed by Evans[5]. Its stablization was very good.

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3. Chao and Wang, *Nature* **140** (1937), 768.
4. Griffiths, *Proc. Roy. Soc.* A, **170** (1939), 513.
5. Evans, *Rev. Sci. Inst.* **5** (1934), 371; later modified by Gengrich, *Rev. Sci. Inst.* **7** (1936), 207.

Two sheets of the elements to be investigated were cemented onto two circularly curved nickel backings which were so hinged that two pairs of these sheets could be put to surround the G-M tube counter entirely. We may here call thess sheets the detectors. Two detectors of the same material were enclosed in twe Cd-boxes in order to filter out thermal neutrons. The boxes were placed parallco to each other at a distance nearly 3.4 cm apart and the neutron source was placed at the middle point in between.

Iridium and gold detectors were exposed to the neutrons for about 15 hours each time and their induced activities were measured subsequently. The procesl was repeated many times for each material. As to indium the exposure varied from one hour to some twenty hours. The activity registration of these elements was started at suitable time after the exposures, ten minutes for indium and fifteen minutes for iridium. For gold detectors the measurements were made immediately after the exposures. The background counts of the tube counter were about ten per minute which seemed fairly high, yet such a drawback was completely over-shadowed by the long periods and large activities of the elements investigated.

RESULTS AND CONCLUSION

Since the half periods of these substances are rather long, the number of true counts in one registration is not only due to the activities induced in the exposure immediately preceding the measurements but also due to the residual activities of all the former exposures. Thus the saturated activities A_0 of these detectors are calculated by evaluating the following formula

$$A_i = \frac{\Omega\,\tau}{4\pi} \sum_{\alpha} (1-e^{-\alpha\lambda}) \sum_{\beta} e^{-\beta\lambda} (1-e^{-\gamma\lambda}) \qquad (5)$$

where A_i represents the activity registered at the ith time of γ hours, noted at β hours after the exposure of α hours; τ represents the fraction of the activity registered by the counter after an absorption due to counter wall, and Ω is the solid angle that the counter subtended with the active isotope as center. For an intimate contact of the detectors and the counter the value of Ω should be 2π.

From the experimental condition it was estimated to be 1.6π. λ is the decay constant of the period in effectiveness. Indium is known to have two periods of 13″ and 54′[6]; iridium three periods of 1.5′, 19h, 2m[7], and gold only one period of 2.7 days. Since a delay in counting the activities due to 13″ in the case of In and 1.5′ in the case of Ir were not recorded and at the same time the exposures made were not long enough so that the two-month activity of Ir was not excited appreciably. Therefore the recorded activities were due to periods of 54′, 19 hour and 2.7 days for In, Ir and Au respectively. Thus evaluating the above formula with different registrated A_i for an element it gives a series of the saturated activities of the element. The mean and its probable error is calculated.

The total number of neutrons given off per second by the source described previously is 7.4×10^4, since the number of effective gamma-rays emitted per second by the equilibrium products of one milligram of Ra is set equal to 1.22×10^7 and the photo cross-section of the Be-nucleus is put equal to 1.6×10^{-27} cm, and the Be powder of density 0.94g/cc traversed by the gammas is 1.2 cm. The number of neutrons Q striking at the detectors per second can therefore be calculated if the solid angles that these detectors subtend at the source as a center are known. These solid angles as reckoned are 2.91, 3.12 and 2.97 for indium, iridium and gold respectively, and the corresponding values of Q are obtained.

The value of τ in formula (5) and the atomic absorption cross-section μ for beta-rays in different substances are calculated from Kohlrausch's[8] empirical equation relating the mass absorption coefficients and the atomic number of the absorber:

$$\mu_m = a\,(105 + N) \tag{6}$$

where a is a constant characterized by the nature of beta-rays and N is the atomic number of the absorber. The half value thicknesses of aluminum for the induced

6. Amaldi, D'Agostins, Fermi, Pontecorvo, Rasetti and Segri, *Proc. Roy. Soc.* A, **149** (1935), 522.

7. McMillan, Kamen and Rubin, *Phys. Rev.* **52** (1937), 375; Fomin and Houtermans, *Phys. Zeits. Sowj.* **9** (1936), 273.

8. Kohlrausch, *Hand. d. Experimentalphysik* Bd. XV, S. 381.

beta-rays of In, Ir and Au were determined by Fermi[9] and his co-workers, which are 0.045, 0.12 and 0.4g/cm^2 respectively. Hence the values of "*a*" for these elements can be calculated from these data, and in turn the atomic absorption coefficients for the beta-rays induced in the elements themselves are determined. With the aid of the same empirical equation the value of τ for different detectors are also determined.

TABLE 1.

		In^{116}	Ir^{194}	Au^{198}
P	(period)	54′	19h	[illegible].7d
E_r	(volts)	1.0	1.9	2.6
K_r	(cm^2/gm)	52.0	1.0	40(or 33)
K_{th}	(cm^2/gm)	0.74	1.0	0.27
Γ_r	(volts)	0.07	0.97	0.10
Γ_n	(1000 volts)	1.1	1.07	3.25
Γ	(volts)	0.481	1.37	1.32
a	(abundance)	95.5%	61.5%	100%
Q	(10^{-4}/sec)	1.71	1.75	1.84
μ	(10^{21}cm^2/gm)	3.81	2.83	8.80
τ		1/1.82	1/1.25	1/1.96
A_0	(no./sec)	$0.665\pm.023$	$0.982\pm.031$	$0.453\pm.021$
D	(volts)	5.6	15	6.1

All the quantities necessary for the evaluation of the average spacing D, and the value of D itself are collected in Table 1, where E_r and K_r are the energy and absorption coefficient of the ordinary resonance neutron group, K_{th} is the absorption coefficient of the thermal neutrons. It is found that the deduced spacing for In, Ir and Au are 5.6, 15 and 6 volts respectively.

In the case of In and Au the values of E_r, K_r and K_{th} are taken from the papers of Hornbostel, Goldsmith and Manley[10] and Feeny, Lapointe and Rasetti[11] respectively. Values of these quantities taken from Bethe's review[12] have also been used. The corresponding values of spacing for the same elements are 12 volts in both cases which seem too high.

9. Fermi, Amaldi, D'Agostino, Rasetti and Segri, *Proc. Roy. Soc.* A, **146** (1934) 483.
10. Hornbostel, Goldsmith and Manley, *Phys. Rev.* **58** (1940).
11. Feeny, Lapointe and Rasetti, *Phys. Rev.* **61** (1942), 469.
12. Bethe, *Rev. Mod. Phys.* **9** (1937), 145, Table. XXV.

As to Ir an estimation of 15 volts is made with the data of absorption coefficients and resonance energy taken from Bethe's review. In view of the great discrepancies of estimation for In and Au, by taking the values of K_r, K_{th} and E_r from a more recent paper and those from Bethe's review, the best can be said is that 15 volts seems uncertain.

Theoretically, there are three nuclear models proposed, namely the free particle model, the free particle model with correlations (suggested by Bardeen) and the liquid drop model (suggested by Bohr and Kalckar). According to these models, one can deduce the level spacings for all the elements at different excitation energies. The level spacings of six elements (Br, Rh, Ah, In, Ir, and Au) have been calculated in accordance with these models. They are collected in the following table together with the experimental values[13].

TABLE 2.

	Br^{80}	Rh^{104}	Ag^{110}	In^{116}	Ir^{194}	Au^{198}
Atomic number	35	45	47	49	77	79
Free Particle model	7	0.8	0.6	0.3	0.002	0.0014
Bardeen's model	2200	550	400	240	10	8
Liquid drop model	80	56	51	42	17	15
Experimental values	30	8	8	6	(<)15	(<)6

Bethe has pointed out that the free particle model gives too small a spacing in comparison with slow neutron experiments. This is clearly seen in Table 2 and is also to be expected. The discrepancy should become larger the greater the atomic weight. This is actually the case. The free particle model gives a value of level spacing for Br^{80} about four times, for In^{116} twenty times and for Au^{198} about ten thousand times, smaller than those obtained experimentally. Bardeen's free particle model with correlations gives a right order of level spacings for heavy elements, but gives too large a spacing for the light elements stated in Table 2. Comparatively speaking, the experimental values follow Bohr-Kalckar's liquid drop model more closely. The discrepancies are ranging only from two to seven

13. Chao, *Sci. Report of Tsing Hua Univ.* series A, (1941).

times greater than the experimental values. This fact suggests that a nucleus is more or less like a drop of liquid rather than a collection of two kinds of Fermi gases.

In conclusion it is found that the average spacing of nuclear levels near the neutron dissociation energy of the compound nuclei In^{116}, Ir^{194} and Au^{198} are about 6, 15 and 6 volts respectively. If better values of absorption coefficients and resonance energies for Ir are known the spacings estimated should be smaller. On the whole these values agree with the model proposed by Bohr and Kalckar more closely than the other models.

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