

關於 ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$ 諸原子核 結合能之問題

第三部

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本文仍用根據介子學說之位能, 但除去其與電荷無關之假定, 來研究 ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$ 諸原子核結合能之問題。初步計算所得結果指明, 核子與中性介子場之作用不含有 τ_3 因子, 又中性介子之質量略大於帶電介子之質量。

在第一部及第二部內已說明由計算所得 ${}^4\text{He}$ 及 ${}^3\text{H}$ 結合能之比約為實驗所得值之二倍。該計算係基於摩勒—羅森費德之原子核位能公式, 應用變分法, 所取之試探函數之形狀非常簡單, 僅含一參變數。上述之差異可能因多種差誤之特殊組成所致, 比方計算 ${}^3\text{H}$ 時所得之結果很壞, 但計算 ${}^4\text{He}$ 時所得之結果較好。然亦可能因摩勒—羅森費德之原子核位能公式尚需加以修改。

摩勒—羅森費德之位能公式所假定之原子核作用力與電荷無關, 似乎有可疑之處, 布拉特* 分析中子質子間之作用力得一結論與此假定相衝突; 最近高能量之散射實驗也好像支持此種見解。因此我們要拋開這個假定, 用下式來替代摩勒—羅森費德之位能:

$$V_{ij} = C_{ij} \{g^2 + f^2(\sigma_i, \sigma_j)\} \frac{e^{-2r_{ij}}}{r_{ij}} + N_{ij} \{g_0^2 + f_0^2(\sigma_i, \sigma_j)\} \frac{e^{-\lambda_0 r_{ij}}}{r_{ij}} \quad (1)$$

其中之 $C_{ij} = \frac{1}{2}(\tau_{i1}\tau_{j1} + \tau_{i2}\tau_{j2})$, $N_{ij} = \tau_{i3}\tau_{j3}$ 或 1 視中性介子場對核子之作用是否含 τ_3 因子而定。常數 g^2, f^2, g_0^2, f_0^2 彼此無關, λ, λ_0 與帶電介子之質量 μ , 中性介子之質量 μ_0 之關係分別為 $\hbar\lambda = \mu c$, $\hbar\lambda_0 = \mu_0 c$ 。如 $N_{ij} = \tau_{i3}\tau_{j3}$, 令 $\mu_0 = \mu$, $g_0^2 = \frac{1}{2}g^2, f_0^2 = \frac{1}{2}f^2$ 所得 (1) 式之特殊形狀即摩勒—羅森費德之位能公式。

* 布拉特 (Blatt), *Phys. Rev.* 74 (1948), 92.

本文將研討 ${}^2\text{H}$ ($I=1$), ${}^3\text{H}$, ${}^4\text{He}$ 之結合能及 ${}^2\text{H}$ ($I=0$) 與 2n 不穩定之條件。這樣可將 (1) 式內之常數初步的確定出來。

今仍取第一部內所引用之波函數。經一些與第一部類似的計算，略去庫倫能，可得下列諸式：

$$\varepsilon = \frac{1}{4} a^2 - s F_2(a) - t \frac{\mu_0}{\mu} F_2\left(\frac{\mu}{\mu_0} a\right) \quad {}^2\text{H} (I=1) \quad (2)$$

$$\varepsilon = \frac{1}{4} a^2 + \eta F_2(a) + \xi \frac{\mu_0}{\mu} F_2\left(\frac{\mu}{\mu_0} a\right) \quad {}^2\text{H} (I=0) \quad (3)$$

$$\varepsilon = \frac{1}{4} a^2 + \xi \mathcal{A} \frac{\mu_0}{\mu} F_2\left(\frac{\mu}{\mu_0} a\right) \quad {}^2n \quad (4)$$

$$\varepsilon = \frac{1}{4} a^2 - p F_3(a) - q \frac{\mu_0}{\mu} F_3\left(\frac{\mu}{\mu_0} a\right) \quad {}^3\text{H} \quad (5)$$

$$\varepsilon = \frac{1}{4} a^2 - p F_4(a) - q \frac{\mu_0}{\mu} F_4\left(\frac{\mu}{\mu_0} a\right) \quad {}^4\text{He} \quad (6)$$

其中之 F_2, F_3, F_4 諸函數已見於第一部，即

$$\left. \begin{aligned} F_2(a) &= \frac{1}{2} \frac{a^3}{(1+a)^2} \\ F_3(a) &= \frac{2}{5\pi} \frac{a^3}{(a^2-1)^3} \left\{ 8a^4 + 9a^2 - 2 - \frac{30a^4}{\sqrt{a^2-1}} \tan^{-1} \sqrt{\frac{a-1}{a+1}} \right\} \\ F_4(a) &= \frac{105}{64} \frac{a^3}{(a+1)^5} \left\{ a^3 + \frac{47}{35} a^2 + \frac{5}{7} a + \frac{1}{7} \right\} \end{aligned} \right\} \quad (7)$$

又視 $N_{ij} = \tau_{i3} \tau_{j3}$ 或 1 引用簡式 $\mathcal{A} = -1$ 或 $+1$ ，則

$$\left. \begin{aligned} s &= \frac{M}{\mu} \frac{g^2 + f^2}{\hbar c}, \quad t = -\frac{M}{\mu} \frac{g_0^2 + f_0^2}{\hbar c} \mathcal{A}, \quad p = \frac{M}{\mu} \frac{g^2 + 3f^2}{3\hbar c}, \\ q &= \frac{M}{\mu} \frac{-(1+2\mathcal{A})g_0^2 + 3f_0^2}{3\hbar c}, \quad \eta = \frac{M}{\mu} \frac{g^2 - 3f^2}{\hbar c} = 3s - 6p, \\ \xi &= \frac{M}{\mu} \frac{g_0^2 - 3f_0^2}{\hbar c} \mathcal{A} = \frac{1+\mathcal{A}}{2} t - \frac{5\mathcal{A}-1}{2} q \end{aligned} \right\} \quad (8)$$

p 及 q 之確定

(5) 及 (6) 兩式內, 設 p 及 q 為某兩數值, 分別求 (5) 式及 (6) 式之極小值則得與 p, q 相當的 ${}^3\text{H}$ 及 ${}^4\text{He}$ 之結合能。反之如 ${}^3\text{H}$ 及 ${}^4\text{He}$ 之結合能為某兩數值, 與其相當的 p 及 q 二值可由下面的方法測定。

今用 a_3, a_4 區別 (5), (6) 兩式內之參變數 a , 此二數彼此無關, 又用 $\varepsilon_3, \varepsilon_4$ 代 (5), (6) 式內之 ε , 分別相當於 ${}^3\text{H}$, ${}^4\text{He}$ 之結合能。因此 (5) 式及 (6) 式可寫成:

$$\varepsilon_3 = G_3(a_3; p, q), \quad \varepsilon_4 = G_4(a_4; p, q) \quad (9)$$

由 (9) 式解出 p 及 q , 稱

$$p = P(a_3, a_4), \quad q = Q(a_3, a_4) \quad (10)$$

再代回原式, 則得 a_3 及 a_4 之恒等式:

$$G_3(a_3; P, Q) - \varepsilon_3 \equiv 0, \quad G_4(a_4; P, Q) - \varepsilon_4 \equiv 0 \quad (11)$$

微分 (11) 式, 所得者仍為 a_3 及 a_4 之恒等式:

$$\left. \begin{aligned} \frac{\partial G_3}{\partial a_3} + \frac{\partial G_3}{\partial p} \frac{\partial P}{\partial a_3} + \frac{\partial G_3}{\partial q} \frac{\partial Q}{\partial a_3} &\equiv 0, & \frac{\partial G_3}{\partial p} \frac{\partial P}{\partial a_4} + \frac{\partial G_3}{\partial q} \frac{\partial Q}{\partial a_4} &\equiv 0 \\ \frac{\partial G_4}{\partial p} \frac{\partial P}{\partial a_3} + \frac{\partial G_4}{\partial q} \frac{\partial Q}{\partial a_3} &\equiv 0, & \frac{\partial G_4}{\partial a_4} + \frac{\partial G_4}{\partial p} \frac{\partial P}{\partial a_4} + \frac{\partial G_4}{\partial q} \frac{\partial Q}{\partial a_4} &\equiv 0 \end{aligned} \right\} \quad (12)$$

從上面恒等式很容易看出, 在某一點 a_3, a_4 , 如 $Q(a_3, a_4)$ 達到其極端值, $P(a_3, a_4)$

也必達到其極端值, 同時 $\frac{\partial G_3}{\partial a_3}$ 及 $\frac{\partial G_4}{\partial a_4}$ 均等於零。故與 $\varepsilon_3, \varepsilon_4$ 相當之 p, q 兩值

可直接得自 (10) 式之極端值, ${}^3\text{H}$ 及 ${}^4\text{He}$ 波函數內之參變數 a_3 及 a_4 也可以於同時確定。

今 (9) 式之形狀為:

$$p = \beta_3(a_3) - q\alpha_3(a_3) = \beta_4(a_4) - q\alpha_4(a_4) \quad (13)$$

其中

$$\beta_A(a) = \frac{1}{4} a^2 - \varepsilon_A, \quad \alpha_A(a) = \frac{\mu_0}{\mu} \frac{F_A\left(\frac{\mu}{\mu_0} a\right)}{F_A(a)}, \quad A = 3 \text{ 或 } 4 \quad (14)$$

在 a_3, a_4 一點, 如

$$q = Q(a_3, a_4) = \frac{\beta_3(a_3) - \beta_4(a_4)}{\alpha_3(a_3) - \alpha_4(a_4)} \quad (15)$$

達到其極端值, 則

$$\frac{\beta'_3(a_3)}{\alpha'_3(a_3)} = \frac{\beta'_4(a_4)}{\alpha'_4(a_4)} = \frac{\beta_3(a_3) - \beta_4(a_4)}{\alpha_3(a_3) - \alpha_4(a_4)} \quad (16)$$

解這些方程式, 用圖解法甚為方便。在 xy 平面上用參變方程式描 C_3 : $x = \alpha_3(a_3)$, $y = \beta_3(a_3)$, 及 C_4 : $x = \alpha_4(a_4)$, $y = \beta_4(a_4)$ 。再畫出二者之共切線。所求之 a_3, a_4 即 C_3, C_4 兩曲線上與切點相當之參變數 a_3, a_4 之值, q 及 p 分別為共切線之斜率及其在 y 軸上之截距。今將 $\mu/\mu_0 < 1$ 時所得之結果列於第 1 表及第 2 表:

第 1 表: $\mu/\mu_0 < 1$ 時 p, q, a_3, a_4 之值。 $\varepsilon_3 = -.37, \varepsilon_4 = -1.27$

$\mu = 286$ 電子質量時此二值相當於 8.4 Mev 及 29.0 Mev

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
p	4.36	4.77	5.42	6.29	8.05	11.56	21.92
q	-10.36	-7.90	-7.18	-7.20	-8.54	-11.76	-21.82
a_3	1.82	1.86	1.88	1.88	1.88	1.88	1.88
a_4	2.73	2.74	2.77	2.84	2.87	2.88	2.92

第 2 表: $\mu/\mu_0 < 1$ 時 p, q, a_3, a_4 之值。 $\varepsilon_3 = -.25, \varepsilon_4 = -.80$

$\mu = 286$ 電子質量時此二值相當於 5.7 Mev 及 18.2 Mev

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
p	7.37	7.94	8.53	9.92	12.73	18.11	36.22
q	-29.40	-20.02	-15.53	-14.53	-16.10	-20.56	-38.25
a_3	1.24	1.28	1.32	1.34	1.36	1.37	1.37
a_4	1.79	1.81	1.90	1.94	1.96	2.01	1.97

上面共選兩組 ε_3 及 ε_4 , 於第 2 表內, 對於結合能計算各方面之差誤略作猜想, 已加算在 ε_3 及 ε_4 中。

$\mu/\mu_0 > 1$ 時, 所得之結果列於第 3 表。

第 3 表: $\mu/\mu_0 > 1$ 時, p, q, a_3, a_4 之值。 $\varepsilon_3 = -0.37, \varepsilon_4 = -1.27$

μ/μ_0	2	2.4	5	10
p	-0.96	-0.47	+0.36	+0.70
q	1.85	1.42	0.74	0.52
a_3	1.8	1.9	1.8	1.9
a_4	3.0	3.1	3.1	3.2

因 p 必為正數, 故僅 $\mu/\mu_0 > 3.3$ 時方具有物理意義, 此區域內 p 及 q 之符號相同。這裏求得的 a_3 及 a_4 的誤差較大, 約為 0.1。

s 及 t 之確定

設 (2) 式於 $a = a_2$ 時有一極小值為 ε_2 , 則

$$t = \frac{\beta'_2(a_2)}{\alpha'_2(a_2)}, \quad s = \beta_2(a_2) - t \alpha_2(a_2) \quad (17)$$

其中之 $\beta_2(a_2)$ 及 $\alpha_2(a_2)$ 即 $A=2$ 時的 (14) 式。在 $\mu/\mu_0 < 1$ 時, 我們曾用 (17) 式計算與數個 a_2 相當的 s 及 t 之值, 在 $\mu/\mu_0 > 1$ 時我們在 xy 平面上描參變曲線 $C_2: x = \alpha_2(a_2), y = \beta_2(a_2)$ 。 C_2 上任一切線之斜率及其在 y 軸上之截距即分別為該處 t 及 s 之值, a_2 即切點參變數之值。

2n 不穩定之條件及 $\mu/\mu_0 > 1$ 之不可能

兩中子不能形成一穩定之原子核 2n , 故 $a > 0$ 時 (4) 式不能有極小值。經

過些數學上之考慮, 可知 $\xi \mathcal{A}$ 必須大於某臨界值 c_0 , 在 $\xi \mathcal{A} = c_0$ 時下式 (參閱 (4), (7) 兩式)

$$\varepsilon = \frac{1}{4} a^2 + c_0 \frac{1}{2} \frac{x^2 a^3}{(1 + xa)^3}, \quad x = \frac{\mu}{\mu_0} \quad (18)$$

無極小值而有一轉折點, 故於該點, $a = a_0$,

$$\left. \begin{aligned} \frac{d\varepsilon}{da} &= \frac{1}{2} a + c_0 \frac{1}{2} \frac{x^2 a^2 (xa + 3)}{(1 + xa)^3} = 0 \\ \frac{d^2\varepsilon}{da^2} &= \frac{1}{2} + c_0 \frac{1}{2} \frac{6x^2 a}{(1 + xa)^4} = 0 \end{aligned} \right\} \quad (19)$$

因此 $xa_0 = -2 + \sqrt{7}$, $c_0 = -\frac{1}{x} \frac{(1 + xa_0)^4}{6xa_0} = -\frac{1.8934}{x}$ (20)

故 3n 不穩定之條件為一不等式:

$$\xi \mathcal{A} \geq -1.8934 \mu_0/\mu \quad (21)$$

下面我們證明 $\mu/\mu_0 > 1$ 時此條件不能滿足。假定 $\mathcal{A} = 1$, 由 (8) 式可得

$$\xi = -q - 2 \frac{M f_0^2}{\mu \hbar c} \leq -q \quad (22)$$

參閱第 3 表可知此與上條件相衝突:

例如取	$\mu/\mu_0 =$	3.3	5	10
由 (22) 式得	$\xi \leq$	-1.04	-0.74	-0.52
但由上條件 (21) 式需要	$\xi \geq$	-0.58	-0.38	-0.19

假定 $\mathcal{A} = -1$, 由 (8) 式知 s 必界於 p 與 $3p$ 二值之間, 利用前所描 C_2 可求 t 之上下限, 結果也和 (21) 式條件互相衝突:

例如取	$\mu/\mu_0 =$	3.3	5	10
由第 3 表	$s =$	0	.36 — 1.08	.70 — 2.10
由 $C_2(0=1)\text{H}^2$	$t =$	1.2	.87 — .60	.60 — .14
故由 (8) 式知	$\xi =$	2.6	1.09 — 1.90	1.32 — 2.70
但由上條件 (21) 式需要	$\xi \leq$	.58	.38	.19

在 $\lambda = -1$ 及 $\lambda = +1$ 兩種情形下衝突都很嚴重, 令我們相信由繪圖方法所得之結果雖不甚準確仍可得一結論即 $\mu, \mu_0 > 1$ 為不可能者。

${}^2\text{H}(I=0)$ 不穩定之條件及對 (1) 式內諸常數之初步確定

今用與上面對 ${}^2\text{n}$ 所採用相同之方法來求 ${}^2\text{H}(I=0)$ 不穩定之條件。設 (3) 式之 η 為某定值 η_0 時, ξ_0 之臨界值與 (3) 式之轉折點相當, 其時 $a = a_0$ 。因此 ξ_0 與 η_0 之關係為下面二式:

$$\left. \begin{aligned} \frac{d\varepsilon}{da} &= \frac{1}{2}a + \eta_0 \frac{1}{2} \frac{a^2(a+3)}{(1+a)^3} + \xi_0 \frac{1}{2} \frac{x^2 a^2 (x+3)}{(1+xa)^3} = 0 \\ \frac{d^2\varepsilon}{da^2} &= \frac{1}{2} + \eta_0 \frac{1}{2} \frac{6a}{(1+a)^4} + \xi_0 \frac{1}{2} \frac{6x^2 a}{(1+xa)^4} = 0 \end{aligned} \right\} \quad x = \mu/\mu_0 \quad (23)$$

由 (23) 式解出 ξ_0 及 η_0 , 則得參變方程式: (省去記號 $_0$)

$$\left. \begin{aligned} \xi &= \xi(a) = \frac{1}{6} \frac{(a+2)^2 - 7}{(x-1)(xa+a+4)} \frac{(1+xa)^4}{x^2 a^2} \\ \eta &= \eta(a) = \frac{1}{6} \frac{(xa+2)^2 - 7}{(x-1)(xa+a+4)} \frac{(1+a)^4}{a^2} \end{aligned} \right\} \quad (24)$$

用參變曲線 Γ 表 ξ, η 之關係, 則 ${}^2\text{H}(I=0)$ 不穩定之條件為: 由 (8) 式求得之 ξ, η 須位於 Γ 線之上方。

當 $\mu/\mu_0 < 1$ 時，計算之結果 q 爲負數，可知 $\Delta = -1$ 爲不可能者，故 $\Delta = 1$ 。因此 2n 不穩定之條件爲 $\xi \geq -1.8934 \mu_0/\mu$ 。

綜合以上之計算，利用 ${}^2\text{H}$ ($I=1$), ${}^3\text{H}$, ${}^4\text{He}$ 之結合能及 2n , ${}^2\text{H}$ ($I=0$) 之不穩定條件，(1) 式中之常數大致確定如下表：

第 4 表： $\mu/\mu_0 < 1$ 時常數之值， $\varepsilon_2 = -.085$, $\varepsilon_3 = -.25$, $\varepsilon_4 = -.80$

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
$\xi \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} -6.31 \\ -2.31 \end{matrix}$	$\begin{matrix} -4.73 \\ -3.12 \end{matrix}$	$\begin{matrix} -3.79 \\ -2.80 \end{matrix}$	$\begin{matrix} -3.16 \\ -2.33 \end{matrix}$	$\begin{matrix} -2.70 \\ -1.98 \end{matrix}$	$\begin{matrix} -2.37 \\ -1.71 \end{matrix}$	$\begin{matrix} -2.10 \\ -1.57 \end{matrix}$
$\eta \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} .54 \\ -1.48 \end{matrix}$	$\begin{matrix} .50 \\ -.80 \end{matrix}$	$\begin{matrix} .68 \\ -.49 \end{matrix}$	$\begin{matrix} .80 \\ -.48 \end{matrix}$	$\begin{matrix} .90 \\ -.48 \end{matrix}$	$\begin{matrix} .98 \\ -.51 \end{matrix}$	$\begin{matrix} .90 \\ -.51 \end{matrix}$
$t \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} -65.11 \\ -61.11 \end{matrix}$	$\begin{matrix} -44.73 \\ -43.12 \end{matrix}$	$\begin{matrix} -34.85 \\ -33.86 \end{matrix}$	$\begin{matrix} -32.21 \\ -31.42 \end{matrix}$	$\begin{matrix} -34.90 \\ -34.18 \end{matrix}$	$\begin{matrix} -43.49 \\ -42.83 \end{matrix}$	$\begin{matrix} -78.60 \\ -78.07 \end{matrix}$
$s \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} 14.92 \\ 14.25 \end{matrix}$	$\begin{matrix} 16.04 \\ 15.61 \end{matrix}$	$\begin{matrix} 17.29 \\ 16.90 \end{matrix}$	$\begin{matrix} 20.11 \\ 19.68 \end{matrix}$	$\begin{matrix} 25.76 \\ 25.30 \end{matrix}$	$\begin{matrix} 36.55 \\ 36.05 \end{matrix}$	$\begin{matrix} 72.74 \\ 72.27 \end{matrix}$
a_2	.61 ₅	.63 ₅	.65 ₅	.68 ₅	.69 ₅	.70 ₅	.70 ₅
$\frac{M}{\mu} \frac{g^2}{\hbar c} \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} 11.33 \\ 10.32 \end{matrix}$	$\begin{matrix} 12.16 \\ 11.56 \end{matrix}$	$\begin{matrix} 13.14 \\ 12.55 \end{matrix}$	$\begin{matrix} 15.28 \\ 14.64 \end{matrix}$	$\begin{matrix} 19.55 \\ 18.86 \end{matrix}$	$\begin{matrix} 27.66 \\ 26.91 \end{matrix}$	$\begin{matrix} 54.78 \\ 54.08 \end{matrix}$
$\frac{M}{\mu} \frac{f^2}{\hbar c} \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} 3.59 \\ 3.93 \end{matrix}$	$\begin{matrix} 3.89 \\ 4.10 \end{matrix}$	$\begin{matrix} 4.17 \\ 4.35 \end{matrix}$	$\begin{matrix} 4.84 \\ 5.04 \end{matrix}$	$\begin{matrix} 6.22 \\ 6.45 \end{matrix}$	$\begin{matrix} 8.89 \\ 9.14 \end{matrix}$	$\begin{matrix} 17.96 \\ 18.20 \end{matrix}$
$\frac{M}{\mu} \frac{g_0^2}{\hbar c} \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} 47.26 \\ 45.26 \end{matrix}$	$\begin{matrix} 32.40 \\ 31.59 \end{matrix}$	$\begin{matrix} 25.19 \\ 24.70 \end{matrix}$	$\begin{matrix} 23.38 \\ 22.96 \end{matrix}$	$\begin{matrix} 25.50 \\ 25.14 \end{matrix}$	$\begin{matrix} 32.03 \\ 31.70 \end{matrix}$	$\begin{matrix} 58.43 \\ 58.16 \end{matrix}$
$\frac{M}{\mu} \frac{f_0^2}{\hbar c} \begin{cases} n \\ \text{H} \end{cases}$	$\begin{matrix} 17.86 \\ 15.86 \end{matrix}$	$\begin{matrix} 12.38 \\ 11.57 \end{matrix}$	$\begin{matrix} 9.66 \\ 9.17 \end{matrix}$	$\begin{matrix} 8.85 \\ 8.43 \end{matrix}$	$\begin{matrix} 9.40 \\ 9.04 \end{matrix}$	$\begin{matrix} 11.47 \\ 11.14 \end{matrix}$	$\begin{matrix} 20.18 \\ 19.91 \end{matrix}$

註：表內 n , H 分別表示因 2n , ${}^2\text{H}$ ($I=0$) 之不穩定條件之限制所得諸常數之極限

第 5 表: $\mu/\mu_0 < 1$ 時常數之值。 $\varepsilon_2 = -0.98$, $\varepsilon_3 = -0.37$, $\varepsilon_4 = -1.27$

μ/μ_0	.6	.7	.8	.9
ξ	-3.16	-2.70	-2.37	-2.10
η	0	0	0	0
t	-17.56	-19.78	-25.89	-45.74
s	12.58	16.10	23.12	45.84
$\frac{M g^2}{\mu \hbar c}$	9.44	12.08	17.34	32.88
$\frac{M f^2}{\mu \hbar c}$	3.15	4.05	5.78	10.96
$\frac{M g_0^2}{\mu \hbar c}$	12.38	14.16	18.82	33.78
$\frac{M f_0^2}{\mu \hbar c}$	5.18	5.62	7.07	11.96

註: 當 $\mu/\mu_0 = .3, .4, .5$ 時, 上述條件不能滿足, $\mu/\mu_0 = .9$ 時也幾乎不能滿足。

庫倫位能之改正仍可得自第一部之 (24) 式, 即 ^3H 與 ^3He 能量之差為 $\Delta\varepsilon_3 = 0.0159 a_3$ 。今用第 1 表內 a_3 之值, 則 $\Delta\varepsilon_3 = -0.030$, 即 0.68 Mev, 可與實驗之結果 0.74 Mev 相比較。

這裏所得 g^2, f^2, g_0^2, f_0^2 之值較摩勒—羅森費德之位能公式中的常數大很多。我們很容易想到一個解釋, 即 (2), (5), (6) 三式內 q 及 t 值均為負數, 亦即 ^2H , ^3H , ^4He 諸穩定原子核因 (1) 式之第二部所生核子間之作用並非互相吸引而為互相排斥, 這樣使 a_2, a_3 及 a_4 之值均小於第一部第二部內所得者, 也就是增加了核子間的距離。為避免因核子間距離增加而使原子核作用力減弱, 強度常數 g^2, f^2, g_0^2, f_0^2 應該變大。以上僅僅是最初步計算之結果。

ON THE BINDING ENERGIES OF THE ATOMIC NUCLEI

 ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$

Part III

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The binding energies of the atomic nuclei ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$ are examined here with a nuclear potential based on the meson theory where the hypothesis of charge independence is given up. Preliminary calculation indicates that the interaction of nucleons with the neutral meson field does not contain the factor τ_3 and that the mass of the neutral meson is somewhat larger than that of the charged meson.

It is shown in the previous papers, Part I and Part II, that the calculated value for the ratio of the binding energy of ${}^4\text{He}$ to that of ${}^3\text{H}$ turns out to be more than twice the experimental value. The calculation was based on the Møller-Rosenfeld nuclear potential and the variation method was used with a one-parameter family of trial functions of a particularly simple form. The above discrepancy could have been originated from a peculiar combination of errors, say that the calculation yielded poor results for ${}^3\text{H}$ but good results for ${}^4\text{He}$. It is not unlikely, however, that the above indicates that the Møller-Rosenfeld nuclear potential has to be modified.

It seems to us that the Møller-Rosenfeld potential contains a doubtful hypothesis, namely the charge independence of nuclear force. Blatt¹, in his analysis of the neutron-proton force, has reached a conclusion in contradiction to this hypothesis; and recent experiments of scattering at high energies seem to further substantiate this view. In what follows we shall therefore give up this hypothesis and replace the Møller-Rosenfeld potential by the following:

1 Blatt, *Phys. Rev.* **74** (1948), 92.

$$V_{ij} = C_{ij} \{g^2 + f^2(\sigma_i, \sigma_j)\} \frac{e^{-\lambda r_{ij}}}{r_{ij}} + N_{ij} \{g_0^2 + f_0^2(\sigma_i, \sigma_j)\} \frac{e^{-\lambda_0 r_{ij}}}{r_{ij}} \quad (1)$$

Here $C_{ij} = \frac{1}{2}(\tau_{i1}\tau_{j1} + \tau_{i2}\tau_{j2})$ and $N_{ij} = \tau_{i3}\tau_{j3}$ or 1 according to whether the interaction of the neutral meson field with a nucleon contains the factor τ_3 or not. The constants g^2, f^2, g_0^2, f_0^2 are all independent, while λ and λ_0 are related to the masses μ of the charged meson and μ_0 of the neutral meson by $\hbar\lambda = \mu c$, $\hbar\lambda_0 = \mu_0 c$. In the case $N_{ij} = \tau_{i3}\tau_{j3}$ the Moller-Rosenfeld potential is obtained by specializing $\mu_0 = \mu$, $g_0^2 = \frac{1}{2}g^2$ and $f_0^2 = \frac{1}{2}f^2$.

In this paper we shall examine the binding energies of ${}^2\text{H}$ (triplet state), ${}^3\text{H}$ and ${}^4\text{He}$, and the non-binding for ${}^2\text{H}$ (singlet state) and 2n . In this way a partial determination of the constants contained in the potential (1) can be attempted.

(a) We adopt the wave functions used in Part I. After some calculation which is similar to that of Part I we get, neglecting the Coulomb energy, the following expressions to be minimized:

$$\varepsilon = \frac{1}{4}a^2 - sF_2(a) - t\frac{\mu_0}{\mu}F_2\left(\frac{\mu}{\mu_0}a\right) \quad \text{for } {}^2\text{H (triplet state)} \quad (2)$$

$$\varepsilon = \frac{1}{4}a^2 + \eta F_2(a) + \xi\frac{\mu_0}{\mu}F_2\left(\frac{\mu}{\mu_0}a\right) \quad \text{for } {}^2\text{H (singlet state)} \quad (3)$$

$$\varepsilon = \frac{1}{4}a^2 + \xi\frac{\mu_0}{\mu}F_2\left(\frac{\mu}{\mu_0}a\right) \quad \text{for } {}^2n \quad (4)$$

$$\varepsilon = \frac{1}{4}a^2 - pF_3(a) - q\frac{\mu_0}{\mu}F_3\left(\frac{\mu}{\mu_0}a\right) \quad \text{for } {}^3\text{H} \quad (5)$$

$$\varepsilon = \frac{1}{4}a^2 - pF_4(a) - q\frac{\mu_0}{\mu}F_4\left(\frac{\mu}{\mu_0}a\right) \quad \text{for } {}^4\text{He} \quad (6)$$

Here F_2 , F_3 and F_4 are the functions already appeared in Part I, viz.

$$\left. \begin{aligned} F_2(a) &= \frac{1}{2} \frac{a^3}{(1+a)^2}, \\ F_3(a) &= \frac{2}{5\pi} \frac{a^3}{(a^2-1)^3} \left\{ 8a^4 + 9a^2 - 2 - \frac{30a^4}{\sqrt{a^2-1}} \tan^{-1} \sqrt{\frac{a-1}{a+1}} \right\}, \\ F_4(a) &= \frac{105}{64} \frac{a^3}{(a+1)^5} \left\{ a^3 + \frac{47}{35} a^2 + \frac{5}{7} a + \frac{1}{7} \right\}. \end{aligned} \right\} \quad (7)$$

Further we have introduced the abbreviation $\Delta = -1$ or $+1$ according as $N_{ij} = \tau_{i3} \tau_{j3}$ or 1 ,

$$\left. \begin{aligned} s &= \frac{M}{\mu} \frac{g^2 + f^2}{\hbar c}, \quad t = -\frac{M}{\mu} \frac{g_0^2 + f_0^2}{\hbar c} \Delta; \quad p = \frac{M}{\mu} \frac{g^2 + 3f^2}{3\hbar c}, \\ q &= \frac{M}{\mu} \frac{(1+2\Delta)g_0^2 + 3f_0^2}{3\hbar c}; \quad \eta = \frac{M}{\mu} \frac{g^2 - 3f^2}{\hbar c} = 3s - 6p, \\ \xi &= \frac{M}{\mu} \frac{g_0^2 - 3f_0^2}{\hbar c} \Delta = \frac{1+\Delta}{2} t - \frac{5\Delta-1}{2} q. \end{aligned} \right\} \quad (8)$$

The determination of p and q as function of μ/μ_0 .

Consider (5) and (6). For a given pair of values of p and q , we obtain a corresponding pair of values for the binding energies of ${}^3\text{H}$ and ${}^4\text{He}$ by minimizing (5) and (6) independently. Conversely, the pair of values of p and q which corresponds to a given pair of values for the binding energies of ${}^3\text{H}$ and ${}^4\text{He}$ can be determined by making use of the following mathematical consideration.

Let us distinguish the parameter a in (5) and (6) respectively by a_3 and a_4 , these being independent of each other. Let us substitute for the ε in (5) and (6) respectively the values ε_3 and ε_4 which correspond to the given binding energies of ${}^3\text{H}$ and ${}^4\text{He}$. We thus write (5) and (6) in the form

$$\varepsilon_3 = G_3(a_3; p, q), \quad \varepsilon_4 = G_4(a_4; p, q). \quad (9)$$

The solution of these equations for p and q will be denoted by

$$p = P(a_3, a_4), \quad q = Q(a_3, a_4). \quad (10)$$

By inserting the solution back we get the identities in a_3 and a_4 :

$$G_3(a_3; P, Q) - \varepsilon_3 \equiv 0, \quad G_4(a_4; P, Q) - \varepsilon_4 \equiv 0, \quad (11)$$

and, by differentiation, we get further identities in a_3 and a_4

$$\left. \begin{aligned} \frac{\partial G_3}{\partial a_3} + \frac{\partial G_3}{\partial p} \frac{\partial P}{\partial a_3} + \frac{\partial G_3}{\partial q} \frac{\partial Q}{\partial a_3} &\equiv 0, & \frac{\partial G_3}{\partial p} \frac{\partial P}{\partial a_4} + \frac{\partial G_3}{\partial q} \frac{\partial Q}{\partial a_4} &\equiv 0, \\ \frac{\partial G_4}{\partial p} \frac{\partial P}{\partial a_3} + \frac{\partial G_4}{\partial q} \frac{\partial Q}{\partial a_3} &\equiv 0, & \frac{\partial G_4}{\partial a_4} + \frac{\partial G_4}{\partial p} \frac{\partial P}{\partial a_4} + \frac{\partial G_4}{\partial q} \frac{\partial Q}{\partial a_4} &\equiv 0, \end{aligned} \right\} \quad (12)$$

It is easily seen from these identities that at a point a_3, a_4 where $Q(a_3, a_4)$ reaches an extreme value, $P(a_3, a_4)$ also reaches an extreme value and further both $\frac{\partial G_3}{\partial a_3}$ and $\frac{\partial G_4}{\partial a_4}$ vanish under the circumstances. Thus the pair of values of p and q corresponding to ε_3 and ε_4 are simply obtained from the extreme values of (10) and the corresponding values a_3 and a_4 for the parameter a of the wave functions for ${}^3\text{H}$ and ${}^4\text{He}$ are obtained at the same time.

In our case (9) is of the special form

$$p = \beta_3(a_3) - q\alpha_3(a_3) = \beta_4(a_4) - q\alpha_4(a_4) \quad (13)$$

with
$$\beta_A(a) = \frac{1}{4}a^2 - \varepsilon_A, \quad \alpha_A(a) = \frac{\mu_0}{\mu} \frac{F_A\left(\frac{\mu}{\mu_0}a\right)}{F_A(a)}, \quad A = 3 \text{ or } 4. \quad (14)$$

At the point a_3, a_4 where

$$q = Q(a_3, a_4) = \frac{\beta_3(a_3) - \beta_4(a_4)}{\alpha_3(a_3) - \alpha_4(a_4)} \quad (15)$$

reaches its extreme value we have

$$\frac{\beta'_3(a_3)}{\alpha'_3(a_3)} = \frac{\beta'_4(a_4)}{\alpha'_4(a_4)} = \frac{\beta_3(a_3) - \beta_4(a_4)}{\alpha_3(a_3) - \alpha_4(a_4)}. \quad (16)$$

It is convenient to solve these equations graphically by tracing the curves C_3 and C_4 in the x - y plane described respectively by the parametric equations $x = \alpha_3(a_3)$, $y = \beta_3(a_3)$ and $x = \alpha_4(a_4)$, $y = \beta_4(a_4)$, and then drawing the common tangent line. The required values for a_3 and a_4 are obtained from the values of the parameters a_3 and a_4 of the curves C_3 and C_4 at the points of contact, while those for q and p are obtained from the slope and the y -intercept of the common tangent line (see Figs. 1 (a), 1 (b)). Values thus obtained for the case $\mu/\mu_0 < 1$ are summarized in Tables 1 and 2.

TABLE 1

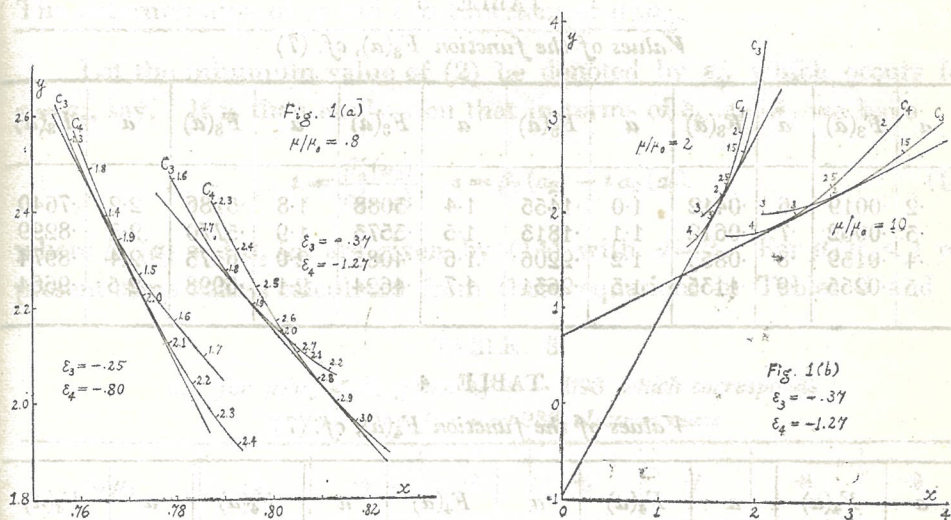
p, q, a_3, a_4 for $\mu/\mu_0 < 1$, given $\varepsilon_3 = -0.37$, $\varepsilon_4 = -1.27$ which correspond to 8.4 Mev and 29.0 Mev for $\mu = 286$ electron mass

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
p	4.36	4.77	5.42	6.29	8.05	11.56	21.92
q	-10.36	-7.90	-7.18	-7.20	-8.54	-11.76	-21.82
a_3	1.82	1.86	1.88	1.88	1.88	1.88	1.88
a_4	2.73	2.74	2.77	2.84	2.87	2.88	2.92

TABLE 2

p, q, a_3, a_4 for $\mu/\mu_0 < 1$, given $\varepsilon_3 = -0.25$, $\varepsilon_4 = -0.80$ which correspond to 5.7 Mev and 18.2 Mev for $\mu = 286$ electron mass

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
p	7.37	7.94	8.53	9.92	12.73	18.11	36.22
q	-29.40	-20.02	-15.53	-14.53	-16.10	-20.56	-38.25
a_3	1.24	1.28	1.32	1.34	1.36	1.37	1.37
a_4	1.79	1.81	1.90	1.94	1.96	2.01	1.97



We have selected two pairs of values for ϵ_3, ϵ_4 , in one of which we have tried by arbitrary guess work to allow for the various sources of error in the calculation of the binding energy. The curves C_3 and C_4 are traced in intervals of 0.1 for the parameters a_3 and a_4 . Because of the opposite signs of p and q , it is necessary to calculate each of the two last terms of (5) and (6) to a high degree of accuracy. In Table 3 and Table 4 we present some values for $F_3(a)$ and $F_4(a)$ which can be interpolated up to the second differences. For the same reason, two figures after the decimal point have to be given for p and q in Tables 1 and 2, although both p and q can be varied simultaneously over a considerable range with little variation in the values for ϵ_3 and ϵ_4 . (For example, for $\mu/\mu_0 = .7$, p can be varied by δp , as large as .6 and q can be varied by δq as large as .9, and by suitably adjusting the ratio $\delta q/\delta p$ we can keep ϵ_3 unchanged and keep the variation $\delta \epsilon_4$ of ϵ_4 as small as .002.)

For $\mu/\mu_0 > 1$ the curves C_3 and C_4 are traced in intervals of 1 for the parameters a_3 and a_4 , and the results are summarized in Table 5, which can be regarded as an extension of Table 1.

TABLE 3

Values of the function $F_3(a)$, cf. (7)

a	$F_3(a)$	a	$F_3(a)$	a	$F_3(a)$	a	$F_3(a)$	a	$F_3(a)$	a	$F_3(a)$
.2	.0019	.6	.0412	1.0	.1455	1.4	.3088	1.8	.5186	2.2	.7640
.3	.0062	.7	.0612	1.1	.1813	1.5	.3573	1.9	.5770	2.3	.8299
.4	.0139	.8	.0853	1.2	.2206	1.6	.4085	2.0	.6375	2.4	.8974
.5	.0255	.9	.1135	1.3	.2631	1.7	.4624	2.1	.6998	2.5	.9664

TABLE 4

Values of the function $F_4(a)$, cf. (7)

a	$F_4(a)$	a	$F_4(a)$	a	$F_4(a)$	a	$F_4(a)$	a	$F_4(a)$
.4	.0138	1.0	.1641	1.6	.4988	2.2	0.9820	2.8	1.5737
.5	.0260	1.1	.2077	1.7	.5702	2.3	1.0740	2.9	1.6807
.6	.0430	1.2	.2564	1.8	.6455	2.4	1.1689	3.0	1.7897
.7	.0652	1.3	.3101	1.9	.7246	2.5	1.2664	3.1	1.9006
.8	.0927	1.4	.3686	2.0	.8071	2.6	1.3665	3.2	2.0135
.9	.1257	1.5	.4315	2.1	.8930	2.7	1.4690	3.3	2.1281

TABLE 5

 p, q, a_3, a_4 for $\mu/\mu_0 > 1$, given $\varepsilon_3 = -.37, \varepsilon_4 = -1.27$

μ/μ_0	2	2.4	5	10
p	-.96	-.47	.36	.70
q	1.85	1.42	.74	.52
a_3	1.8	1.9	1.8	1.9
a_4	3.0	3.1	3.1	3.2

Since p is necessarily positive, only the region $\mu/\mu_0 > 3.3$ could be of physical significance, and in this region p and q are of like sign. The error in a_3 and a_4 is here large, of the order of 0.1.

The determination of s and t as functions of μ/μ_0 .

Let the minimum value of (2) be denoted by ε_2 , which occurs for $a = a_2$, say. It is then easily seen that in terms of ε_2 and a_2 we have

$$t = \frac{\beta_2'(a_2)}{\alpha_2'(a_2)}, \quad s = \beta_2(a_2) - t\alpha_2(a_2), \quad (17)$$

where $\beta_2(a)$ and $\alpha_2(a)$ are given by (14) with $A=2$. For $\mu/\mu_0 < 1$ we present some values calculated from these equations in Tables 6 and 7.

TABLE 6

s, t for $\mu/\mu_0 < 1$, given $\varepsilon_2 = -0.098$ which corresponds
to 2.23 Mev for $\mu = 286$ electron mass

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
$a_2 = .80 \begin{cases} s \\ t \end{cases}$	8.932 -29.88	10.30 -23.66	12.22 -21.67	15.10 -22.23	19.90 -25.50	29.49 -34.02	58.28 -62.01
$a_2 = .85 \begin{cases} s \\ t \end{cases}$	7.86 -24.31	9.03 -19.42	10.66 -17.93	13.11 -18.52	17.20 -21.38	25.38 -28.70	49.89 -52.57
$a_2 = .90 \begin{cases} s \\ t \end{cases}$	6.99 -19.94	8.00 -16.07	9.40 -14.96	11.50 -15.56	15.01 -18.07	22.03 -24.39	43.07 -44.92

TABLE 7

s, t for $\mu/\mu_0 < 1$, given $\varepsilon_2 = -0.085$ which corresponds
to 1.94 Mev for $\mu = 286$ electron mass

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
$a_2 = .65 \begin{cases} s \\ t \end{cases}$	12.82 -52.69	14.90 -40.53	17.81 -36.20	22.18 -36.28	29.46 -40.76	43.95 -53.26	87.71 -95.60
$a_2 = .70 \begin{cases} s \\ t \end{cases}$	10.81 -41.14	12.52 -31.96	14.91 -28.79	18.51 -29.09	24.50 -32.92	36.48 -43.40	72.43 -78.23
$a_2 = .75 \begin{cases} s \\ t \end{cases}$	9.27 -32.64	10.70 -25.60	12.70 -23.26	15.69 -23.68	20.69 -26.99	30.69 -35.80	60.67 -64.92

For $\mu/\mu_0 > 1$, we trace in the x - y plane the curve C_2 described by $x = a_2(a_2)$, $y = \beta_2(a_2)$. The slope and the y -intercept of a tangent line to C_2 then give the values for t and s , a_2 being given by the value of the parameter at the point of contact. Some such curves are shown in figure 2.

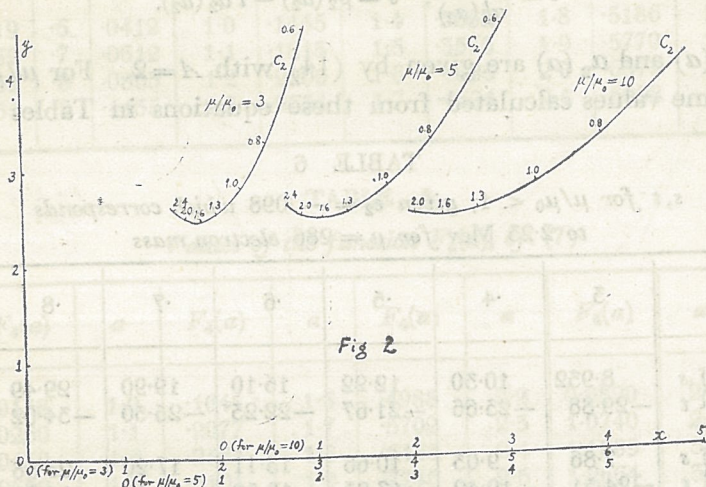


Fig 2

The non-binding of 2n and the exclusion of the case $\mu/\mu_0 > 1$.

That two neutrons do not form a stable nucleus 2n shows that (4) cannot possess any minimum for $a > 0$. By some mathematical consideration familiar in curve tracing, it is seen that $\xi \Delta$ must be more positive than a certain critical value c_0 , say, for which the minimum of the expression (cf. (4) and (7))

$$\varepsilon = \frac{1}{4}a^2 + c_0 \frac{1}{2} \frac{x^2 a^3}{(1+xa)^2}, \quad x = \frac{\mu}{\mu_0} \quad (18)$$

degenerates into a point of inflection. Thus we have

$$\left. \begin{aligned} \frac{d\varepsilon}{da} &= \frac{1}{2}a + c_0 \frac{1}{2} \frac{x^2 a^2 (xa + 3)}{(1+xa)^3} = 0 \\ \frac{d^2\varepsilon}{da^2} &= \frac{1}{2} + c_0 \frac{1}{2} \frac{6x^2 a}{(1+xa)^4} = 0 \end{aligned} \right\} \quad (19)$$

at the point of inflection $a=a_0$, say, whence we get

$$x a_0 = -2 + \sqrt{7} \quad \text{and} \quad c_0 = -\frac{1}{x} \frac{(1 + x a_0)^4}{6 x a_0} = -\frac{1.8934}{x}. \quad (20)$$

The condition for the non-binding of 2n is therefore an inequality

$$\xi \Delta \geq -1.8934 \mu_0/\mu \quad (21)$$

It will be shown that this condition cannot be satisfied in the case $\mu/\mu_0 > 1$. Suppose $\Delta = 1$. We get from (8)

$$\xi = -q - 2 \frac{M}{\mu} \frac{f_0^2}{\hbar c} \leq -q \quad (22)$$

which, by referring to Table 5, contradicts the above condition thus:

For	$\mu/\mu_0 = 3.3$	5	10	
we have	$\xi \leq -1.04$	$-.74$	$-.52$	by (22)
but we demand	$\xi \geq -0.58$	$-.38$	$-.19$	by (21)

Suppose $\Delta = -1$. We see from (8) that s must lie between p and $3p$; with the help of the curves of figure 2 we get the corresponding limits for t and hence, cf. (8), the corresponding limits for ξ . The results again contradict the condition (21) thus:

For	$\mu/\mu_0 = 3.3$	5	10	
we have	$s = 0$	$.36-1.08$	$.70-2.10$	by Table 5
and	$t = 1.2$	$.87-.60$	$.60-.14$	from fig. 2,
so	$\xi = 2.6$	$1.09-1.90$	$1.32-2.70$	by (8).
But we demand	$\xi \leq .58$	$.38$	$.19$	by (21).

The contradiction in both cases with $\Delta = -1$ and with $\Delta = 1$ is so severe that we may trust our graphical treatment as much as to conclude against the case $\mu/\mu_0 > 1$.

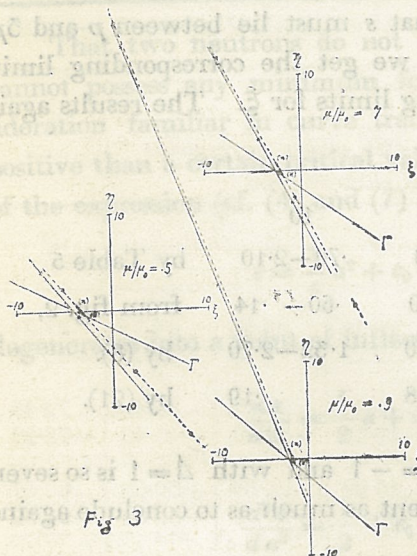
The non-binding of ^2H (singlet state) and the preliminary determination of the constants contained in the potential (1).

By considerations similar to that of the non-binding of 2n we can find the critical value ξ_0 for ξ that corresponds to a given value η_0 for η from the following equations derived from (3):

$$\left. \begin{aligned} \frac{d\varepsilon}{da} &= \frac{1}{2}a + \eta_0 \frac{1}{2} \frac{a^2(a+3)}{(1+a)^3} + \xi_0 \frac{1}{2} \frac{x^2 a^2(xa+3)}{(1+xa)^3} = 0, \\ \frac{d^2\varepsilon}{da^2} &= \frac{1}{2} + \eta_0 \frac{1}{2} \frac{6a}{(1+a)^4} + \xi_0 \frac{1}{2} \frac{6x^2 a}{(1+xa)^4} = 0, \end{aligned} \right\} \quad (x = \mu/\mu_0) \quad (23)$$

at the point of inflection $a=a_0$, say. If we represent the above relation by a curve Γ described by the parametrical equations (the suffix zero being omitted below)

$$\left. \begin{aligned} \xi &= \xi(a) = \frac{1}{6} \frac{(a+2)^2 - 7}{(x-1)(xa+a+4)} \frac{(1+xa)^4}{x^2 a^2}, \\ \eta &= \eta(a) = \frac{1}{6} \frac{(xa+2)^2 - 7}{(x-1)(xa+a+4)} \frac{(1+a)^4}{a^2}, \end{aligned} \right\} \quad (24)$$



which is the solution of (23) for ξ_0 and η_0 , then the condition for the non-binding of ^2H (singlet state) is simply expressed as that the point ξ, η of (8) must lie above Γ . For examples of such curves, see figure 3.

For $\mu/\mu_0 < 1$, the negative sign of q excludes the case of $\Delta = -1$, so we have $\Delta = 1$. The condition for the non-binding of 2n is therefore $\xi \geq -1.8934 \mu_0/\mu$. The points ξ, η as obtained from Tables 1 and 6 are marked by crosses in figure 3 while those obtained from Tables 2 and

7 are marked by circles. The thick portion of the line joining these points gives the range of values for ξ , η compatible with the non-binding of 2n and of 2H (singlet state). This portion is essentially that of a straight line, so any point ξ , η on it can be obtained by linear interpolation from the two end points, distinguished by (n) and (H) in the figures and in Table 8. In Table 9, the two end points practically coincide, as can be seen from the corresponding figures.

TABLE 8

*The constants for $\mu/\mu_0 < 1$, given $\varepsilon_2 = -.085$, $\varepsilon_3 = -.25$
and $\varepsilon_4 = -.80$ (cf. Tables 2 and 7)*

μ/μ_0	.3	.4	.5	.6	.7	.8	.9
$\xi \begin{cases} (n) \\ (H) \end{cases}$	-6.31 -2.31	-4.73 -3.12	-3.79 -2.80	-3.16 -2.33	-2.70 -1.98	-2.37 -1.71	-2.10 -1.57
$\eta \begin{cases} (n) \\ (H) \end{cases}$	.54 -1.48	.50 -.80	.68 .49	.80 -.48	.90 -.48	.98 -.51	.90 -.51
$t \begin{cases} (n) \\ (H) \end{cases}$	-65.11 -61.11	-44.73 -43.12	-34.85 -33.86	-32.21 -31.42	-34.90 -34.18	-43.49 -42.83	-78.60 -78.07
$s \begin{cases} (n) \\ (H) \end{cases}$	14.92 14.25	16.04 15.61	17.29 16.90	20.11 19.68	25.76 25.30	36.55 36.05	72.74 72.27
a_2	.61 ₅	.63 ₅	.65 ₅	.68 ₅	.69 ₅	.70 ₅	.70 ₅
$\frac{M}{\mu} \frac{g^2}{\hbar c} \begin{cases} (n) \\ (H) \end{cases}$	11.33 10.32	12.16 11.56	13.14 12.55	15.28 14.64	19.55 18.86	27.66 26.91	54.78 54.08
$\frac{M}{\mu} \frac{f^2}{\hbar c} \begin{cases} (n) \\ (H) \end{cases}$	3.59 3.93	3.89 4.10	4.17 4.35	4.84 5.04	6.22 6.45	8.89 9.14	17.96 18.20
$\frac{M}{\mu} \frac{g_0^2}{\hbar c} \begin{cases} (n) \\ (H) \end{cases}$	47.26 45.26	32.40 31.59	25.19 24.70	23.38 22.96	25.50 25.14	32.03 31.70	58.43 58.16
$\frac{M}{\mu} \frac{f_0^2}{\hbar c} \begin{cases} (n) \\ (H) \end{cases}$	17.86 15.86	12.38 11.57	9.66 9.17	8.85 8.43	9.40 9.04	11.47 11.14	20.18 19.91

TABLE 9

The constants for $\mu/\mu_0 < 1$, given $\varepsilon_2 = -.098$, $\varepsilon_3 = -.37$ and $\varepsilon_4 = -1.27$
(cf. Tables 1 and 6)

μ/μ_0	.6	.7	.8	.9
ξ	-3.16	-2.70	-2.37	-2.10
η	0	0	0	0
t	-17.56	-19.78	-25.89	-45.74
s	12.58	16.10	23.12	43.84
$\frac{M}{\mu} \frac{g^2}{\hbar c}$	9.44	12.08	17.34	32.88
$\frac{M}{\mu} \frac{f^2}{\hbar c}$	3.15	4.03	5.78	10.96
$\frac{M}{\mu} \frac{g_0^2}{\hbar c}$	12.38	14.16	18.82	33.78
$\frac{M}{\mu} \frac{f_0^2}{\hbar c}$	5.18	5.62	7.07	11.96

Referring to Table 9 it may be mentioned that for $\mu/\mu_0 = .3, .4, .5$, the data are incompatible, and for $\mu/\mu_0 = .9$ the data are almost incompatible, as may be seen from the corresponding figures.

The Coulomb energy correction is still given by the formula (24) of Part I, viz. $\Delta \varepsilon_3 = .0159 a_3$ for ${}^3\text{H}-{}^3\text{He}$. By using the value for a_3 as given in Table I we get $\Delta \varepsilon_3 = .030$, i.e. 0.68 Mev for ${}^3\text{H}-{}^3\text{He}$ in comparison with the experimental value of 0.74 Mev.

The values obtained here for g^2, f^2, g_0^2, f_0^2 are much larger than those of the Møller-Rosenfeld potential. We easily find the explanation for this in the negative signs of q and t , referring to equations (2), (5) and (6). Namely for the stable nuclei ${}^2\text{H}$, ${}^3\text{H}$ and ${}^4\text{He}$, the second part of the potential (1) leads to repulsion instead of attraction between the

nucleons. As a consequence of this the nucleons are kept at larger distances from each other, as shown by the smaller values of a_2 , a_3 and a_4 , in comparison with those of Part I and Part II. In order to compensate for the weakening of the nuclear force with increasing internucleonic distances, the strength constants g^2 , f^2 , g_0^2 , f_0^2 have to be larger. The results are however all preliminary.

如前所見，核子間的距離較大，故核力之強度必須較大，以維持核子之結合。

前次所得之核力強度常數 g^2 , f^2 , g_0^2 , f_0^2 均較前次為大，此係因核子間之距離較大，故核力之強度必須較大，以維持核子之結合。

如前所見，核子間的距離較大，故核力之強度必須較大，以維持核子之結合。

(1)

$$g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$

代入式 (1) 中得

之核力強度常數 g^2 與 f^2 之值如下：

$$(2) \quad g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$

核力強度常數

$$g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$

則，核力強度常數 g^2 與 f^2 之值如下：

$$E(2) = \frac{1}{2} \left(\frac{2b}{1b} \right) + \frac{1}{2} \left(\frac{2b}{1b} \right) = \frac{2b}{1b}$$

5

或 (3) 由

式中之核力強度

$$(4) \quad g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$

如前所見，核子間的距離較大，故核力之強度必須較大，以維持核子之結合。

$$(5) \quad g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$

由式 (1) 中之核力強度

之核力強度常數 g^2 與 f^2 之值如下：

$$(6) \quad g^2 = g_0^2 + f^2 = 1.0 \times 10^{-13} \text{ erg cm}^2$$