FACTORS AFFECTING THE FREQUENCY DEVIATION IN REACTANCE-TUBE FREQUENCY MODULATION CIRCUITS

By Chai Yeh (葉楷) and Y. K. Tz'u (慈雲桂)

Department of Electrical Engineering
and Department of Physics,
National Tsing Hua University, Peiping, China.

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ABSTRACT

In reactance-tube frequency modulation circuits, the fractional frequency deviation is expected to vary with the phase shift constants of the modulator and the L_f/C_f ratio of the tank-circuit of the oscillator. The way in which the frequency deviation is affected by these factors is discussed theoretically, Experimental checks are made, Two types of circuits are studied, the inductive and the capacitive reactance-tube circuits. It is found that within the straight portion of the operating characteristic of the tube, the agreement is fairly satisfactory.

INTRODUCTION

It has been proved by Crosby[1] that the signal-to-noise ratio improvement of a frequency-modulation system over an equivalent amplitude-modulated one is equal to the square root of three times the deviation ratio ($\sqrt{3}\ \Delta f/f_a$) for the fluctuation noise, and twice the deviation ratio ($2\ \Delta f/f_a$) for the impulse noise. Hence, the quality and fidelity of a frequency-modulation system are greatly affected by the frequency deviation of the system. The latter is therefore considered as one of the important factors in designing such a system.

In the reactance-tube frequency modulation circuit, as will be shown later, the frequency deviation is a function of both the phase shift constants of the modulator and the inductance to capacitance ratio of the tank-circuit of the oscillator. It is the aim of this study to investigate the effect of these factors upon the frequency deviation of this kind of modulation circuit.

1. M. G. Crosby, "Frequency Modulation Propagation Characteristics", *Proc. I. R. E.* **24** (1936). 898-913; M. G. Crosby, "Frequency Modulation Noise Characteristics" *Proc. I. R. E.* **25** (1937), 472-514.

Two types of reactance-tube frequency modulation circuits[2] are considered. In Fig. 1a., the radio frequency voltage from the tank-circuit of the oscillator is fed to the control grid of the modulator through the blocking condenser C_b and the resistance R_{ph}. Resistance R_{ph} and the capacitance C_{gk} form a phase shifter which converts the modulator tube 6L7 into a reactance-tube. As R_{ph} is made large compared to the reactance of C_{gk}, the phase of the current which flows through the combination is determined by the resistance and is therefore in phase with the applied voltage. This same current flows through the grid-to-cathode capacitance C_{gk} and the voltage drop across it lags behind the current by nearly 90°. When this lagging voltage is amplified the resulting plate current also lags behind the applied voltage by approximately 90°. Hence, the modulator tube offers an inductive reactance to the oscillating circuit. We refer to this type of circuit as the inductive circuit.

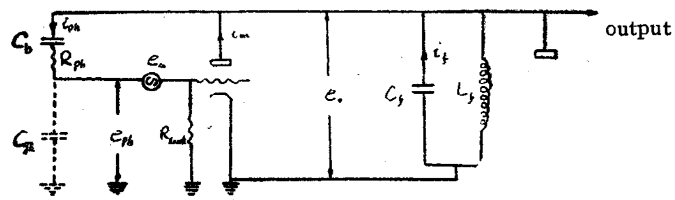
In Fig. 1b., the radio frequency voltage is applied to the control grid of the modulator through a series condenser C_{ph}. C_{ph} and the grid-leak resistance R_{gk} form the phase-shifter. If C_{ph} is small enough to have a reactance which is high compared to the resistance R_{gk}, then the current flowing through the phase shifter is determined by the condenser and therefore leads the applied voltage. This leading current flows through the resistor R_{gk} resulting in a leading voltage drop across the grid-to-cathode terminals (in phase with the current). When this leading grid voltage is amplified, a capacitive reactance appears in the plate circuit since the current flowing in the plate circuit leads the applied voltage. We refer to this type of circuit as the capacitive circuit.

The magnitude of the effective susceptance of the reactance-tube is proportional to the gain of the tube. Thus an increase in the gain of the modulator tube in the inductive circuit (Fig. 1a) decreases the effective inductance in parallel with the tank-circuit of the oscillator. This causes the oscillation frequency to increase. In the capacitive circuit (Fig, 1b) an increase of the gain of the modulator increases the capacitance in parallel with the tank-circuit and thus causes the frequency of the oscillator to decrease.

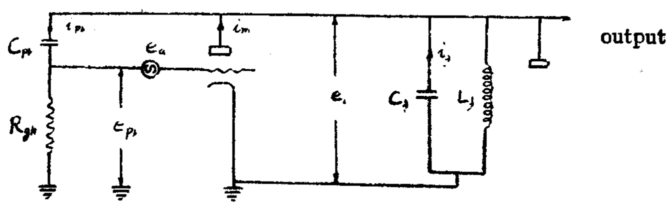
2. M. G. Crosby, "Reactance-tube Frequency Modulators." *RCA Rev.* **5** (1940), 89-96; Charles Travis, "Automatic Frequency Control," *Proc. I. R. E.* **23** (1935), 1125-1141; D. E. Foster and S. W. Seeley, "Automatic Tuning, Simplified Circuits, and Design Practice." *Proc. I. R. E.* **25** (1937), 298-313; C. F. Sheaffer, "Frequency Modulations" *Proc. I.R.E.* **28** (1940), 66-67; John R Carson, "Notes on the Theory of Modulation," *Proc. I. R. E.* **10** (1922), 57-64; Balth Van Der Pol, "Frequency Modulation," *Proc. I. R. E.* **18** (1930), 1194-1205; Hans Roder, "Amplitude, Phase and Frequency Modulation," *Proc. I. R. E.* **19** (1931), 2145-2176.

THEORY

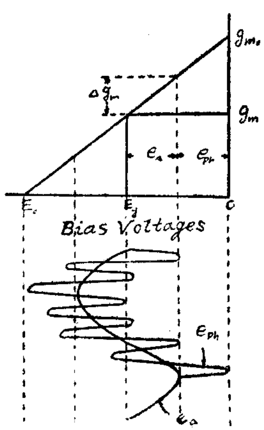
The equations expressing the frequency deviation in both types of **the** reactance-tube frequency-modulation circuit may be derived in terms of the **tube** and circuit constants as follows. In either circuit, it is assumed that the tube **has** a linear transconductance characteristic from cutoff to zero bias, (Fig. **1c).**



(a) The inductive circuit.



(b) The capacitive circuit.



Circuit Constants:

- g_m=transconductance at operation point.
- g_{m_0}=transconductance at zero bias.
- e_{ph}=modulator grid leak radio frequency voltage.
- e_a=modulator grid leak audio frequency voltage.
- e_0=peak oscillator tank voltage.
- C_f=oscillator tank fixed capacitance.
- L_f=oscillator tank fixed inductance.
- X_0=reactance per leg of oscillator tank.

(c) The ideal transconductance curve.

Fig. 1 Fundamental circuit of reactance-tube modulator.

Further, let us assume that the phase shift network always supplies the grid with a voltage that is nearly 90° out of phase during the audio cycle of the modulation and that the in-phase component of the modulator load to the oscillator may be neglected. Under the quiescent carrier condition the radio-frequency peak modulation plate current is[3]

$$i_m = e_{ph}\, g_m \qquad (1)$$

and the radio-frequency peak oscillator tank current is

$$i_f = e_0 / X_0 = e_0\, \omega_0\, C_f. \qquad (2)$$

The fraction of the tank current flowing through the modulator is

$$\left(\frac{i_m}{i_f}\right)_0 = \frac{e_{ph}\; g_m}{e_0\, \omega_0\, C_f}. \qquad (3)$$

At peak of modulation (peak $e_a = e_{ph}$ in Fig. 1c) g_m becomes 50 per cent greater, and the fraction of the tank current flowing through the modulator is given by

$$\left(\frac{i_m}{i_f}\right)_{100} = \frac{3}{2}\, \frac{e_{ph}\, g_m}{e_0\, \omega_0\, C_f}. \qquad (4)$$

The fractional change of current from zero to full modulation is the difference between equations (4) and (3), *i. e.*,

$$\left(\frac{i_m}{i_f}\right)_{100} - \left(\frac{i_m}{i_f}\right)_0 = \frac{1}{2}\, \frac{e_{ph}\, g_m}{e_0\, \omega_0\, C_f}. \qquad (5)$$

Under the quiescent carrier condition the frequency of the oscillator is approximately given by

$$f_0 = \frac{1}{2\pi\sqrt{C_f\, L_f}}. \qquad (6)$$

In the process of modulation, as described above, either C_f or L_f is changed with the modulating voltage. Expressing C_f or L_f in terms of i_f from equation (2), equation (6) may be rewritten as

$$f = \frac{\sqrt{2\pi\, f_0\, e_0}}{2\pi\sqrt{i_f\, L_f}}, \quad \text{or} \quad f = \frac{\sqrt{2\pi\, f_0\, i_f}}{2\pi\sqrt{e_0\, C_f}}. \qquad (7)$$

3. E. S. Winlund, "Drift Analysis of the Crosby Frequency-modulated Transmitter Circuit." *Proc. I. R. E.* **29** (1941), 390-398.

Differentiating (7) and dividing through by equation (7) we have

$$\frac{df}{f} = \pm \frac{1}{2} \frac{di_f}{i_f} \tag{8}$$

in which the positive and negative signs stand for inductive and capacitive circuits respectively; However, because only the magnitude is concerned, both signs may be dropped. Combining equations (5) and (8) we have

$$\frac{\Delta f}{f} = \frac{1}{2} \frac{\Delta i_f}{i_f} = \frac{e_{ph}\, g_m}{4 e_0\, \omega_0\, C_f}, \tag{9}$$

that is, the resultant fractional frequency change is one-half the resultant fractional change of current.

Now, in Fig. 1a,

$$e_{ph} = i_{ph}\, X_{gk} = i_{ph}/\omega C_{gk} \tag{10}$$

and

$$i_{ph} = \frac{e_0}{R_{ph} + \dfrac{R_{leak}/\omega\, C_{gk}}{R_{leak} + 1/\omega C_{gk}}}. \tag{11}$$

As the second term in the denominator of (11) is negligibly small in comparison with R_{ph}, we may write

$$i_{ph} = \frac{e_0}{R_{ph}}. \tag{11a}$$

Substituting into (10):

$$e_{ph} = \frac{e_0}{R_{ph}\, \omega\, C_{gk}}.$$

Then equation (9) becomes, after replacing f by f_0,

$$\frac{\Delta f}{f_0} = \frac{e_0}{R_{ph}\, \omega\, C_{gk}} \cdot \frac{g_m}{4\, e_0\, \omega_0\, C_f} = \frac{g_m}{4\omega_0\, C_{gk}\, R_{ph}} \cdot \sqrt{\frac{L_f}{C_f}}. \tag{12}$$

In Fig. 1b,

$$e_{ph} = i_{ph}\, R_{gk}. \tag{13}$$

And

$$i_{ph}=\frac{e_0}{\dfrac{1}{\omega C_{ph}}+\dfrac{R_{gk}/\omega C_{gk}}{R_{gk}+1/\omega C_{gk}}}. \tag{13a}$$

As the second term in the denominator of (13a) is negligibly small in comparison with $1/\omega C_{ph}$, we may write

$$i_{ph}=e_0\,\omega\,C_{ph}. \tag{14}$$

Substituting in (13):

$$e_{ph}=e_0\,\omega\,C_{ph}\,R_{gk},$$

which gives, replacing f by f_0,

$$\frac{\Delta f}{f_0}=\tfrac{1}{2}\,\omega_0\,C_{ph}\,R_{gk}\,g_m\,\sqrt{\frac{L_f}{C_f}}, \tag{15}$$

provided $f \doteq f_0$.

Equations (12) and (15) express the way in which the fractional frequency deviations in the two types of reactance-tube frequency modulation circuits vary with the various factors involved. We shall now proceed to verify these equations experimentally.

EXPERIMENTAL RESULTS AND DISCUSSION

Equations (12) and (15) indicate that the fractional frequency deviation of a reactance-tube F-M circuit is a function of g_m, $\sqrt{L_f/C_f}$ and the phase shift network constants. In both types of circuit, $\Delta f/f_0$ is directly proportional to g_m and $\sqrt{L_f/C_f}$. In the inductive circuit, $\Delta f/f_0$ is inversely proportional to the phase shift constants C_{gk} and R_{ph}. In the capacitive circuit, it is directly proportional to the phase shift C_{ph} and R_{gk}. In order to test the accuracy of the theory, a series of experiments were carried out for both types of circuits. In such case, the influencing factors were varied one at a time, while keeping the rest of them unchanged.

The frequency deviation was measured by a heterodyne frequency meter[4].

4. M. G. Crosby, "A Method of Measuring Frequency Deviation" *RCA Rev.* **4** (1940), 473-477.

The plate to control grid transconductance (g_m, $=200\ \mu$ mhos) at the operating point of the reactance-tube 6L7 was measured by a well-known circuit[5].

The results of measurement on both types of circuits are plotted on Figs. 2 and 3 respectively.

Figs. 2a, 2b, and 2c are the results obtained with the capacitive circuit. In these figures the fractional frequency deviation is plotted against the various factors such as C_{ph}, R_{gk} and L_f/C_f respectively.

In each figure, a theoretical curve (calculated by equation (15)) is plotted with it.

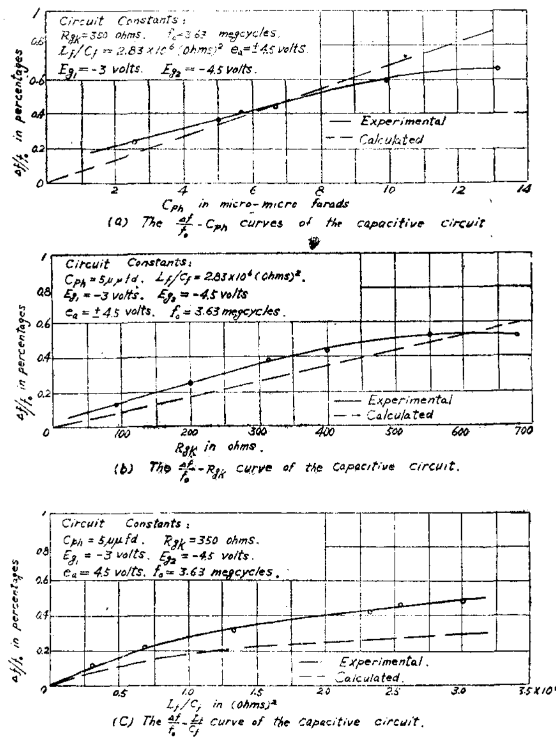


Fig. 2 The experimental and calculated curves of the capacitive circuit.

It is seen that the theoretical and experimental results check with each other fairly well. In Figs. 2a and 2b, the experimental curves for larger values of C_{ph} and R_{gk} respectively bend downward to some extent, and deviate from

5. See for example August Hund, "High Frequency Measurements," pp. 353-360.

the straight line relationship predicated by the theory. However, it must not be forgotten that in deriving equation (13a) it was assumed that $\frac{1}{\omega C_{ph}} \gg \frac{R_{gk}/\omega C_{gk}}{R_{gh}+1/\omega C_{gk}}$. As C_{ph} becomes larger, the above assumption is no longer valid. Similar reasoning for the larger values of R_{gk} explains the bending of the experimental curve indicated in Fig. 2b. With regard to the discrepancy between the theoretical and experimental curves in Fig. 2c, we presume that the experimental inaccuracy in the determination of the constants g_m, R_{gk} and C_{ph} etc., accounts for the actual difference.

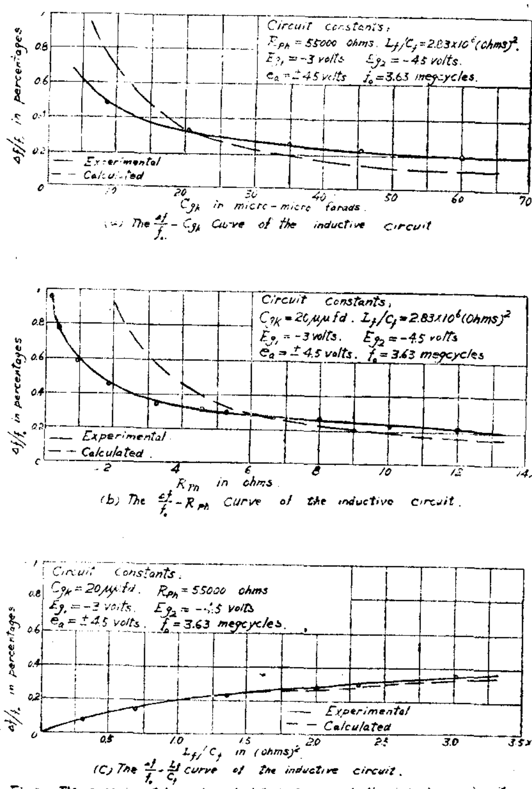


Fig 3 The experimental and Calculated Curves of the inductive circuit.

Figures 3a, 3b and 3c are the results obtained with the inductive circuit. The theoretical and experimental results also check with each other very well.

In Figs. 3a and 3b, a considerable variance appears for small values of C_{gk} and R_{ph}. This is because of the fact that for small C_{gk} and R_{ph}, the term $\frac{R_{leak}/\omega C_{gk}}{R_{leak}+1/\omega C_{gk}}$ is no longer negligible in comparison with R_{ph}. During the experiment, it was observed that when the L_f/C_f ratio of the tank-circuit exceeded a certain value in the respective circuits, unstable oscillation resulted. These limiting values were 4.6 and 4.7 for the inductive and capacitive circuits respectively.

The approximation $(f=f_0)$ used in deriving equation (15) gives rise to an error of less than one per cent and is therefore tolerable.

These equations have been checked at several other carrier frequencies, giving similar results. Only one such set is presented here.

CONCLUSIONS

From the equations (12) and (15) as well as the experimental curves in Figs. 2 to 3., it can be concluded that the frequency deviation in both types of F-M circuits is proportional to the transconductance of the reactance tube, and the square root of *L-C* ratios of the oscillating tank-circuit. Besides, in the inductive circuit the frequency deviation is inversely proportional to the phase-shifting resistance R_{ph} and the grid-to-cathode capacitance C_{gk}. In the capacitive circuit, however, the frequency deviation is directly proportional to the phase shift constants C_{ph} and R_{gk}. From equation (15), it is seen that $\Delta f/f_0$ in the capacitive circuit is proportional to frequency. Therefore this type of circuit will give larger deviation at higher frequency. It is also shown that modulator tubes having smaller inter-electrode capacitance C_{gk} (grid-to-cathode), and higher and linear transconductance characteristic are more suitable for this purpose.

The above work was done in Kunming in 1943-1945, when we were cut off from the outside world academically. It was found after the work was ready for publication that in the book "Frequency Modulation" by A. Hund, pp. 162-174, published in 1942, a similar but different analysis of these circuits had been made. However, we differ from him in our starting point and in results and our deductions were further substantiated with experiment.