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照相膠板術確定重原子核本性的方法

黃　長　風

（上海交通大學物理系）

重原子核（$Z \geqslant 2$）在照相膠板中的徑跡常帶有 δ- 線，由 δ- 線密度 n——單位徑跡長度 δ- 線的數目——及徑跡長度 R 兩測數可以推算重原子核的原子序數 Z; 本文闡述作者推算所得之 n 與 Z 及 R 的關係：

$$n = C \cdot Z^{p} \cdot R^{-b}$$

$p = 1.54$, $b = 0.46$, 常數 C 之值視所用照相膠板的性質與計數 δ- 線時所用的條件而變。 如用 Ilford G 5 照相膠板，計數長度超過 3 μ 的 δ- 線，而將 n 表示 100 μ 徑跡長度中所含之 δ- 線，則 C 的值爲 0.45, 上述公式較普通 δ- 線公式爲精確而使用尤爲便利。

重原子核徑跡在其終端呈縮細現象，"縮細徑跡長度" L 與原子核之 Z 有關，按 Bohr 的簡易計算知 $L \simeq \frac{1}{2} Z^2$（L 之值以 μ 表之），此公式所示的值與實驗測得的值不符，如以指數定則表示 L 與 Z 的關係：$L \sim Z^{\alpha}$, 由測得結果知 $\alpha \simeq 1$.

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INVESTIGATION ON HEAVY NUCLEI BY PHOTOGRAPHIC METHOD

By HOANG TCHANG-FONG

Physics Department, Chiao Tung University, Shanghai

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ABSTRACT

An improved δ-ray method for determining the nature of heavy nuclei is obtained by application of a semi-empirical formula giving the δ-ray density per 100 μ of the track in function of the remaining range R; $n=C\ Z^{p}R^{-b}$ with $p=1.54$, $b=0.46$ and C depending on the nature of the emulsion and the conditions of measurement.

Discussions are given on the "thin-down effect" and the relation between the thin-down length and the atomic charge.

INTRODUCTION

Heavy nuclear fragments are occasionally observed in energetic nuclear explosions produced in the photographic emulsion by the passage of cosmic radiation[1, 2, 3]. On the other hand, it has been found[4] that primary cosmic radiation at high altitudes, greater than 20 km. above the sea level, contains heavy nuclei with atomic number up to 26. The characteristic features of the tracks of heavy nuclei observed in the photographic emulsion, in contrast with those of protons, α-particles . . . are: firstly, their heavy ionization is often accompanied by δ-rays constituted by secondary electrons emitted by the particles along their passage through the emulsion; and secondly, there is a "thin-down" of the tracks near the end of their ranges. Figure 1 shows some tracks of heavy nuclei observed in Ilford G5 emulsion exposed to cosmic radiation at 30,000 m. above sea level.

The nature of the heavy nuclei can be easily determined by an improved method of δ-ray counts which will be described in the present paper. The method consists in using a semi-empirical formula giving the density of δ-rays as a function of the remaining range. We have furthermore examined the thin-down lengths of the tracks and have deduced their relation with the atomic number.

1. Occhialini and Powell, *Nature* **159** (1947), 93.
2. Hodgson and Perkins, *Nature* **163** (1949), 439.
3. Bonetti and Dilworth, *Phil. Mag.* **40** (1949), 585.
4. Frier, Lofgren, Ney and Oppenheimer; Bradt and Peters, *Phys. Rev.* **74** (1948), 213.

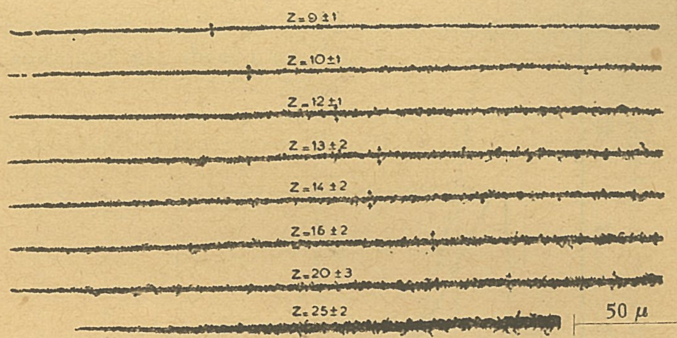

Fig. 1.—Tracks of heavy nuclei observed in Ilford G5 emulsions.

PRINCIPLE OF THE δ-RAY METHOD

The method of δ-ray counts has been introduced by Frier *et al.*[5] and Bradt *et al.*[6]. When a particle of charge Ze and velocity βc passes through the emulsion, it produces δ-rays along its track, the number of δ-rays dn per unit length of track with energies in the interval W, $W+dW$ is given by the following relation due to Mott ;

$$dn = \frac{2\pi N Z^2 e^4}{m_e c^2 \beta^2} \frac{dW}{W^2}, \tag{1}$$

where N is number of electrons per cm^3 of emulsion, and m_e designates the mass of the electron. The total number of δ-rays per unit length of the track having energies greater than W_0 is obtained by integrating this relation from W_0 to W_{max}, which is the maximum energy that the incident particle can transfer to an electron, and is given by Bhabha's formula :

$$W_{max} = 2 m_e c^2 \beta^2. \tag{2}$$

Hence

$$n = 2\pi N \left(\frac{e^2}{m_e c^2}\right)^2 \left(\frac{m_e c^2}{W_0} - \frac{1}{2\beta^2}\right) \frac{Z^2}{\beta^2}. \tag{3}$$

The variations of n with β are shown in Fig. 2 (curve plotted with $Z=10$, $W_0 = 20$ kev.), the scale of n being in arbitrary units.

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5. Frier, Lofgren and Oppenheimer, *Phys. Rev.* **74** (1948), 1818.
6. Bradt and Peters, *Phys. Rev.* **74** (1948), 1828.

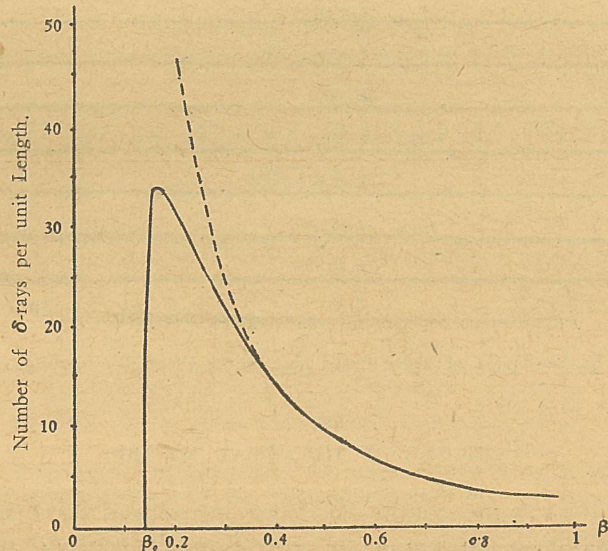

Fig. 2.—Density of δ-rays as a function of β.

We see that when the velocity of the particle decreases, the density of δ-rays increases to a maximum for $\beta_1^2 = W_0/m_e c^2$ and then decreases rapidly to zero (cut-off of δ-rays) for $\beta_0 = \beta_1/\sqrt{2}$.

If we choose the lower limit W_0 of the δ-ray energy small enough, we can neglect the second term in the bracket of the relation (3) for the values of β superior to a certain limit which does not differ much from β_0; in this case we may write:

$$n = 2\pi N \left(\frac{e^2}{m_e c^2} \right)^2 \frac{m_e c^2}{W_0} \frac{Z^2}{\beta^2}$$

$$= \frac{Z^2}{K \beta^2}, \qquad (4)$$

where K designates a constant. The corresponding curve of n is represented in Fig. 2 by the dotted line. The difference between the two curves is practically inappreciable ($<2\%$) for $\beta \geqslant 0.35$, to which corresponds, in case of $Z = 10$, a range in emulsion about 1850 microns.

We have to relate the quantity β to the remaining range R. If M is the mass of the particle, it is well known that the velocity $\beta.c$ of the particle depends only on the reduced range Z^2R/Mc^2 provided that the particle loses its energy only through collision. Theoretical values of β as a function

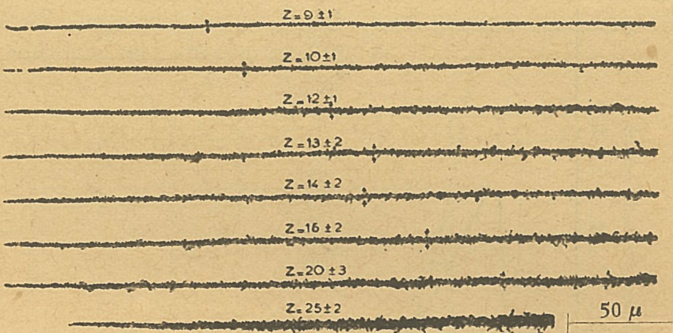

Fig. 1.—Tracks of heavy nuclei observed in Ilford G5 emulsions.

Principle of the δ-ray method

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5. Frier, Lofgren and Oppenheimer, *Phys. Rev.* **74** (1948), 1818.
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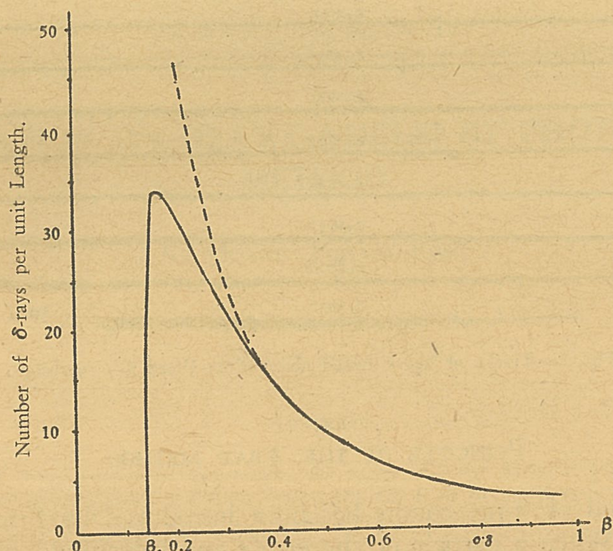

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We have to relate the quantity β to the remaining range R. If M is the mass of the particle, it is well known that the velocity $\beta . c$ of the particle depends only on the reduced range $Z^2 R/Mc^2$ provided that the particle loses its energy only through collision. Theoretical values of β as a function of the reduced range can be deduced from computations by Smith[7]. The results thus obtained are shown in Fig. 3.

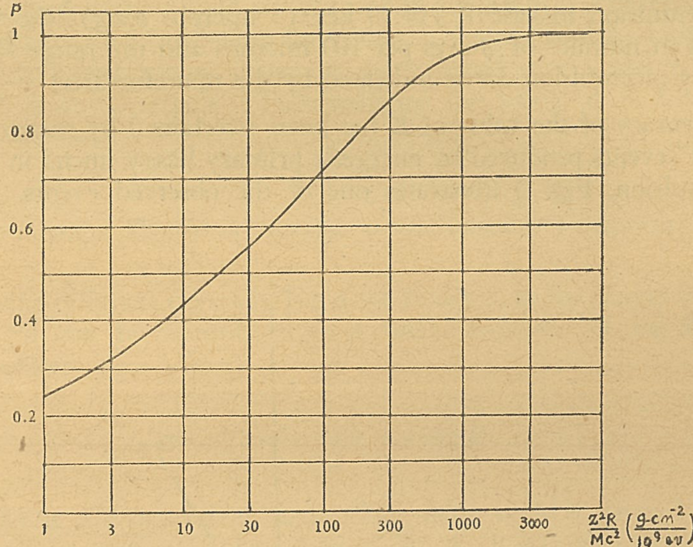

Fig. 3.—β as a function of reduced range (after Smith)

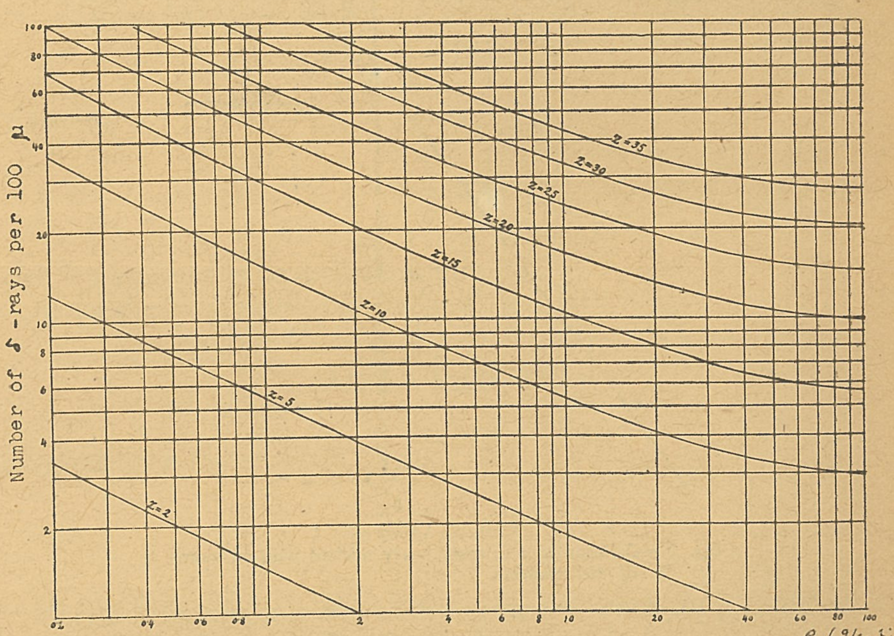

Fig. 4.—Density of δ-rays per 100μ as function of remaining range.

7. Smith, *Phys. Rev.* **72** (1947), 32.

It is now possible to compute the values of n in terms of the remaining range R of the track. Results obtained with the approximate formula (4) are shown in Fig. 4. The value of K is 45 as determined experimentally for Ilford G5 emulsion in case $W_0 = 20$ kev (3 microns long); the value of n is expressed in number of δ-rays per 100 microns and the range R in g-cm^{-2} with density of emulsion 3.9 as indicated by the manufacturer.

The accuracy of the value of K has been ascertained by means of several nuclear "jet" events produced by energetic primary heavy nuclei in the photographic emulsion. Fig. 5 illustrates one of the observed events. From the

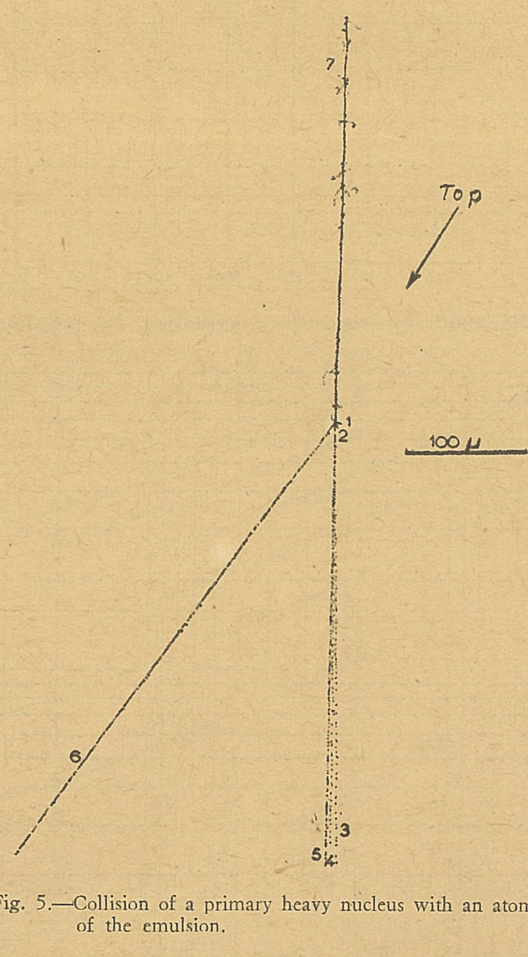

Fig. 5.—Collision of a primary heavy nucleus with an atom of the emulsion.

peculiar feature of asymmetry of the phenomena it is possible to recognize those particles belonging to the incident particle: they are collimated in a narrow cone with axis in the direction of the incident particle. In the present case the value of Z for the incident particle "7" is $Z=4$, a value deduced from the δ-ray counts. After collision with an atom of the emulsion, it disintegrates into three fragments: tracks 3, 4 & 5. The prongs 1, 2 and 6 are due to fragments ejected by the collided nucleus. The charges of the particles (relativistic) producing the tracks 3, 4 and 5 can be estimated from their respective ionizations. Thus we have found that 3 and 4 are protons and that 5 is an α-particle. This confirms the value of Z determined by the δ-ray method.

Approximate Relation Giving n as Explicit Function of R.

From the different curves of Fig 4 we notice that, for ranges within 0.3-20 g-cm^{-2}, $\log n$ can be assimilated to a linear function of $\log R$. We can therefore write:

$$\log n = a(Z) - b \log R, \tag{5}$$

where a(Z) is a Z-dependent term. The value of b is given by the mean slope of straight portions of the curves in Fig. 4. It has been found:

Curve for:	Z=5	10	15	20	25
Slope:	0.46	0.45	0.45	0.47	0.46

We assume:

$$b = 0.46,$$

which is considered to be accurate within 2%.

As for the form of the term a(Z), we set:

$$a(Z) = \log(C Z^p). \tag{6}$$

This gives for n the relation:

$$n = C Z^p R^{-b}. \tag{7}$$

Once b is known, it is easy to determine the values for p and C which fit most closely the curves of Fig. 4. We have found:

$$p = 1.54, \qquad C = 0.45.$$

It is to be noted that the values of b and p are independent of K used to plot the curves of Fig. 4. Only the coefficient C depends on K which, in turn, depends on the composition of the emulsion and the minimum energy W_0 of the δ-rays counted in the measurement of n.

To check the validity of the relation:

$$n = 0.45\, Z^{1.54}\, R^{-0.46}$$

giving directly the density of δ-rays as a function of R, we have considered a certain number of tracks of primary heavy nuclei ending in the stack of plates so that their ranges can be estimated. The following example is taken from a track of 10 g-cm^{-2}, the atomic number measured is $Z = 21 \pm 1$. We have measured the density of δ-rays at every 1.25 g-cm^{-2} from the end and compared the experimental data with the computed values of n. The results are given in the table:

R(g-cm^2)		1.25	2.5	3.75	5.0	6.25	7.5	8.75	10
Measured $n/_{100\mu}$		42.5±2.1	30.2±2.1	25.4±2.8	23.8±3.1	22.4±1.4	21.2±2.1	18.7±1.6	17.5±1.4
Computed N	$Z=20$	39.4	29.8	24.7	21.6	19.4	17.8	16.6	15.6
	$Z=21$	41.5	32.1	26.6	23.4	21.1	19.4	18.1	16.9
	$Z=22$	44.5	34.5	28.6	25.1	22.6	20.8	19.4	18.2

The deviations indicated in the table are due to fluctuations of measurement. The range R is determined with an uncertainty of the order of 0.5 g-cm^{-2}, which does not change much the computed values of n. We see that the agreement between the observed value of n and that deduced from the approximate fomula (7) is quite satisfactory.

Application

The atomic number Z of a heavy nucleus can be determined in the following way. Suppose that its track ends in the stack of plates, its range is known, we measure the δ-ray density n per 100 microns at the point of range R, then using the relation (7) we deduce:

$$Z = (2.22\, n\, R^{0.46})^{1/1.54}.$$

The value of R must be large enough in order to be sure that the β at the point where δ-rays are counted is greater than 0.35 which corresponds to $Z^2R/Mc^2 = 4$ g-cm^{-2}/10^9 e.v. This condition is satisfied if

$$R \geq \frac{7.2}{Z} \text{ g-cm}^{-2},$$

In case the particle traverses the whole stack, the remaining range in not known. However, if the observed portion of the track is long enough to get a significant variation of the δ-ray density $\Delta n/\Delta R$ at its extremities, the method can still be applied with some alteration. Let AB be the length of the observed track, n_1 and n_2 ($<n_1$) be the δ-ray densities at A and B. Assuming R_1 and R_2 ($> R_1$) to be the remaining ranges of A and B, we may write:

$$\Delta R_1 = R_2 - R_1,$$

$$\Delta n_1 = n_2 - n_1.$$

From the relation (7) we deduce:

$$\frac{\Delta n_1}{n_1} = -0.46 \frac{\Delta R_1}{R_1},$$

so that

$$R_1 = -0.46 \frac{\Delta R_1}{\frac{\Delta n_1}{n_1}} = -0.46\, n_1 \frac{\Delta R_1}{n_1 - n_2},$$

where $\Delta R_1 =$ AB is the observed track length (in g-cm^{-2}). Knowing R_1 and n_1 we are led to the preceding case.

Thin-down Effect

In the preceding considerations we have admitted that the effective charge of the particle is equal to Ze, Z being the atomic number of the particle; in other words, we have admitted that the particle is constituted of a nucleus stripped off its orbital electrons. Experimentally, it proves to be the case if the velocity of the particle exceeds 20 times that of the electron of the hydrogen atom, the latter being equal to: $2\pi e^2/h = 10^8$ cm/sec. When the particle slows down and acquires a velocity $\beta.c$ inferior to this limit 2×10^9 cm/sec, the particle begins to pick up its orbital electrons and consequently its effective charge decreases. The ionization, which is proportional to the square of the effective charge, gradually decreases more and more: the track is "thinned down" near the end of its range.

We define the thin-down length L as the remaining range corresponding to the beginning of the tapering of the track. Its value depends on the nature of the particle and has been calculated by Frier et al.[5] using Bohr atom model. The results obtained by these authors are reproduced in Fig. 8. Actually we notice that the relationship $L(Z)$ can be represented closely by:

$$L \simeq \tfrac{1}{2} Z^2 \quad \text{in microns.} \tag{8}$$

Experimentally it is rather difficult to define the thin-down length, and an accurate determination of L is not possible. After several trials, the following method seems to be most reliable: The track is carefully drawn with a projection microscope (magnification ~ 2200), then the diameter d of the waspish part of the track is measured along the thinned down part for every 20 microns of range. The plot of d against R allows us to estimate the thin-down length L in conformity with its definition. Fig. 6 shows such a plot for the track of $Z = 13 \pm 1$ of Fig. 1.

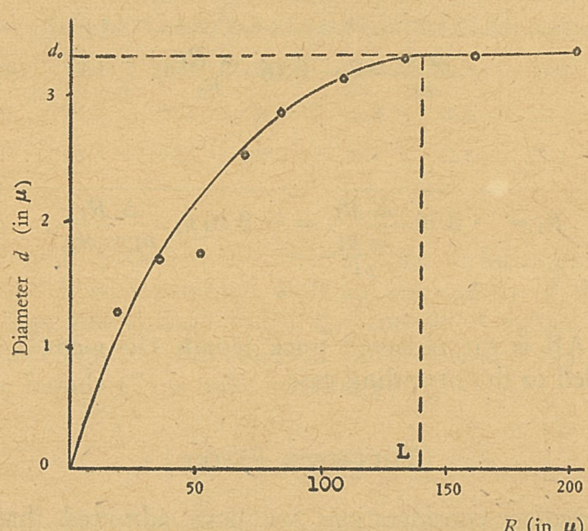

Fig. 6.—Plot of d against R.

From measurements made with tracks of different Z we have found that the cross section of the thin-down portion of the track decreases linearly with the remaining range R. Therefore, if d_0 is the diameter of the track before the thin-down, the plot of $(d/d_0)^2$ against R/L is a straight line inclined at 45° to the axes, the plot remaining the same for any particle whatever may be its Z.. Fig. 7 shows this plot for the particle $Z = 13 \pm 1$ under consideration.

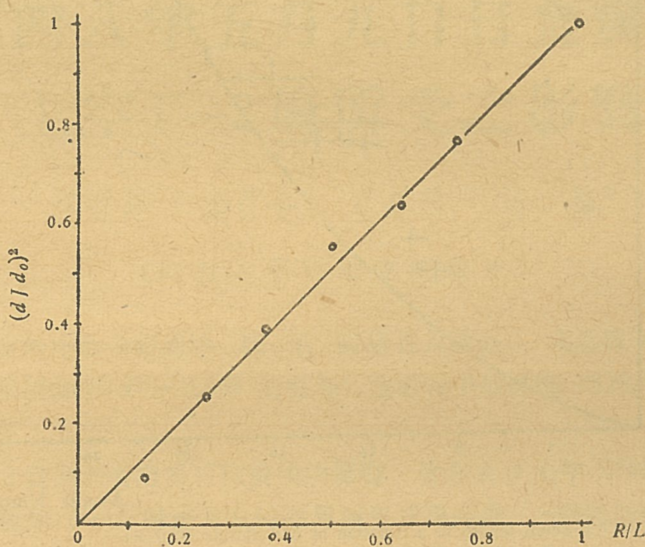

Fig. 7.—Diagram of $(d/d_0)^2$ as a function of R/L.

Since d_o remains constant to a certain extent of $R > L$, its value can be determined with precision. Consequently we can define the thin-down length L by the following relation:

$$\frac{d^2}{R} = \frac{d_0^2}{L}. \tag{9}$$

The accuracy of this method is about 10%.

The results[8] obtained for the eight tracks of Fig. 1 are reported in Fig. 8. The experimental points do not fit the curve of Frier *et al.* For nuclei of moderate Z, the measured L seems to be greater than the predicted lengths. The results seem to indicate that for nuclei of $Z < 20$ the thin-down does begin with the capture of the first orbital electron, but that the capture process probably starts at a velocity greater than the limit assumed in Bohr's theory[9].

8. Hoang Tchang-fong and Morellet, *Comptes Rendus* **231** (1950), 695.
9. Bohr, The Penetration of Atomic Particles through Matter (Copenhagen 1946).

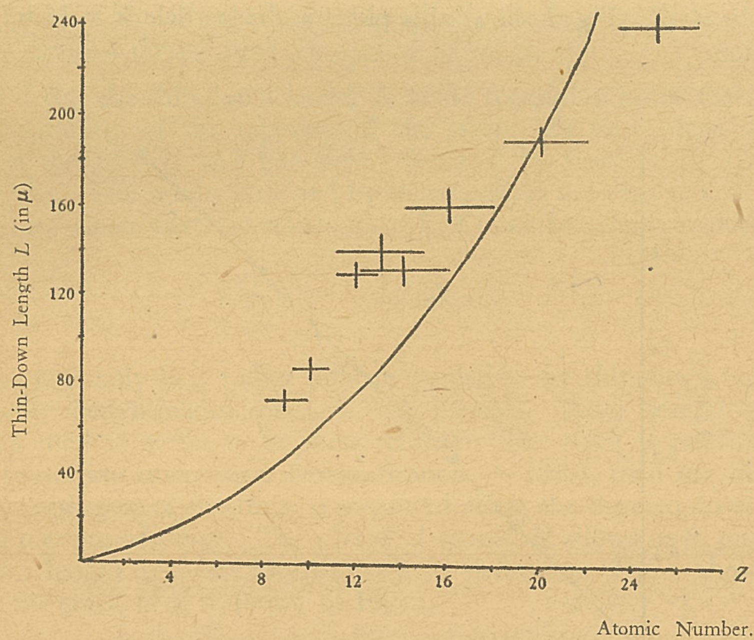

Fig. 8.—Thin-down length as a function of the Atomic Number.
—— Curve after Frier et al.
–+– Experimental Points.

Now, if a power law is assumed to represent approximately the thin-down length: $L \sim Z^{\alpha}$, the value of α deduced from experimental data seems to be near unity. The linear variation of L with Z can be accounted for, if we assume that the electron capture begins at a velocity $2\pi e^2 z/nh$ where ze is the effective charge of the particle and n is the effective total quantum number used in the Thomas-Fermi model and is proportional to $Z^{1/3}$ instead of unity as in the simplified case adopted by Frier *et al.* for their calculations.