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β—衰變理論*

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在湯川秀樹的關于 β- 蛻變理論中，發散積分可以用電荷和質量之重正比去掉，但是在通常的費米理論中是不可能的。從核子之 β- 蛻變，μ-e 蛻變及 μ 介子與核子間作用三個作用常數幾乎彼此相等一點上，我們推出此中間介子的生命期在 10^{-16} 秒左右。因之我們可以把這中間介子看做一個新粒子，這個新粒子因爲他的非常短的生命期而來被實驗發現。所有在可觀察的現象中仍存在發散困難的介子理論本文未予討論，在所討論到的介子理論中，如果這中間介子是僞矢介子並且與核子及 μ 介子等的作用是僞矢耦合，結果較好。

1. 提　綱

近來發現電動力學中之發散積分，只代表不可觀察的電子的電荷和質量之重正化現象。因之在電動力學中發散積分並不給出任何困難。這種進展對于場論的其他方面如介子理論及 β- 蛻變等也有很大的意義，因爲那些理論都模倣電動力學。除非在那些理論中發散積分也只代表不可觀察的現象，不然模倣便無意義了。因此在可觀察的結果中具有發散困難的其他理論，可以認爲不能採納。這樣使通常介子理論之可能類型大大減少。

在通常的 β- 蛻變理論中，當計算放射更正時，我們可以發現其發散積分困難非常嚴重，絕不能僅用重正化觀念完全去掉的。因之根據上述的討論，可認爲不能採納。

湯川秀樹[1] 曾假設核子和電子—中微子間不直接發生作用，而是經過一中間介子場。根據他的理論，核子首先放出一帶電介子，此介子再蛻變成電子和

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1. H. Yukawa, *Proc. Phys. Math. Soc. Japan* **17** (1935), 48.

微中子。宇宙線中的介子確是不穩定的，並且也發生 β- 蛻變。不過計算出[2]的介子的生命期不對。並且最近實驗證明[3]宇宙線介子（μ 介子）與核子之作用很弱，因此更使問題複雜。

這裏我們指明可以使湯川理論不發生發散困難，也就是說，所有發散積分，都只代表不可觀察的現象。在費米理論中因爲作用函數包含四個場量而使發散積分特別嚴重。這種作用函數與電動力學中只包含三個場量不同。在湯川理論中，引入兩類作用，一是核子與介子間，一是電子——中微子與介子間。每類作用函數都只包含三個場量。根據過去的研究[4]，可以選擇介子和作用函數的類型使湯川理論免除發散困難。並且湯川理論與實驗不符的地方也可以用假設一新的中間介子補救，此中間介子因爲他的極短的生命期而未被發現。

從實驗上大家知道質子——中子場，電子——中微子場，介子——中微子場中任意兩個場的作用强度都幾乎相等，這表明此中間介子與每個場有相同的作用强度。由此我們可以推出如果此中間介子的質量大于 200 電子質量，那麼他的生命期要短于 10^{-16} 秒，在今天的實驗技術可以發現的範圍以外。

本文中只考慮以下四類可以免除發散困難的帶電介子場。(i) 標介子標耦合。(ii) 贋標介子贋標耦合，(iii) 贋標介子贋矢耦合，(iv) 贋矢介子贋矢耦合。在核子之 β- 蛻變中，我們發現如果電子——中微子的能量比介子的質量小得多，那麼我們的結果在第一次近似值上與通常的費米結果幾乎一樣。由于費米結果與實驗甚相符和，我們推定此比值確實很小。這點在 μ 介子之 β- 蛻變中就不同了。那時輕粒子的能量高到 50 兆電子伏以上，除非中間介子非常重，本文理論將給出與費米理論不同的結果。在這點上可以用實驗決定那類理論比較正確。

2. 核子之 β- 蛻變

首先我們寫下認爲可以採納的介子場。爲了簡單起見，我們假設介子核子間的拉氏函數密度 L_1 與介子和電子——中微子間的拉氏函數密度具有同一類型。在前一篇文章中，我們指明只有四類介子場可以免除發散，根據提綱的討論這四類可以採納。

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2. S. Sakata, *Proc. Phys. Math. Soc. Japan* **32** (1941), 191; S. Rosental, *Kgl. Danske Vid. Sels. Math.-fys. Medd.* **18** (1941), No. 7; *Phys. Rev.* **60** (1941), 612; A. H. Bethe & L. W. Nordheim, *Phys. Rev.* **57** (1940), 998; E. C. Nelson, *Phys. Rev.* **60** (1941), 830; L. Rosenfeld, *Nuclear Forces*, p. 330 (North Holland Pub. Co., 1948).

3. M. Conversi, E. Pancini & O. Piccioni, *Phys. Rev.* **71** (1947), 209; C. M. G. Lattes, C. P. S. Occhialini & C. F. Powell, *Nature* **160** (1947), 463; above authors & H. Muirhead, *Nature* **159** (1947), 694.

4. Ning Hu, *Phys. Rev.* **80** (1950), 1109; 胡寧，中國物理學報 **8** (1951), 40.

(a) 標介子標耦合（本文中取 $\hbar=c=1$）:

$$L_1^a = g_a \psi(x)\,\Psi^+(x)\,Q\,\Psi(x) + \text{共軛複數},$$

$$L_2^a = f_a \psi(x)\,\chi^+(x)\,\varphi(x) + \text{共軛複數}, \tag{1a}$$

(b) 贗標介子贗標耦合:

$$L_1^b = g_b \psi\,\Psi^+ \gamma_5\, Q\, \Psi + \text{共軛複數},$$

$$L_2^b = f_b \psi\,\chi^+ \gamma_5\, \varphi + \text{共軛複數}, \tag{1b}$$

(c) 贗標介子贗矢耦合:

$$L_1^c = \frac{g_c}{\kappa}\frac{\partial\psi}{\partial x_\mu}\Psi^+ \gamma_5 \gamma_\mu Q \Psi + \text{共軛複數},$$

$$L_2^c = \frac{f_c}{\kappa}\frac{\partial\psi}{\partial x_\mu}\chi^+ \gamma_5 \gamma_\mu \varphi + \text{共軛複數}, \tag{1c}$$

(d) 贗矢介子贗矢耦合:

$$L_1^d = g_d \psi_\mu \Psi^+ \gamma_5 \gamma_\mu Q \Psi + \text{共軛複數},$$

$$L_2^d = f_d \psi_\mu \chi^+ \gamma_5 \gamma_\mu \varphi + \text{共軛複數}, \tag{1d}$$

κ 是中間介子質量，$\psi(x)$ 是標介子或贗標介子之場算符，$\psi_\mu(x)$ 是贗矢介子之場算符。 $\Psi(x)$, $\chi(x)$ 和 $\varphi(x)$ 分別是核子電子和中微子的場算符。 如果 Ψ^+ 和 Ψ 被 χ^+ 和 φ 所代替，L_1 便變成 L_2。 Q 是把質子變成中子的算符 Q^* 反過來。 g, f 是耦合常數。 在平常攝動計算中首先要把 L_1 和 L_2 用一個切變換變成哈密頓算符。 不過這時會有與費米理論中同樣的核子與電子—中微子間的接觸作用出現。在費米理論中我們已知高次攝動計算會有發散結果。但是在現在的理論中，進一步的考察可以發現這些接觸作用是不存在的。 在高次的攝動計算中，有新的接觸作用抵消上述接觸作用。 這種繞圈子手續是通常的攝動方法之缺點。 在近來發展的費曼[5] 方法中，根本沒有接觸作用出現故我們

5. R. P. Feynman, *Phy. Rev.* **67** (1949), 769.

採取費曼方法。費曼方法在文獻中討論應用得很多，假設讀者都已熟習。利用費曼方法，第二級躍遷矩陣元可以立刻寫下：

$$(\boldsymbol{p}, \boldsymbol{P}_2 \mid H_2^a \mid -\boldsymbol{p}_n, \boldsymbol{P}_1)$$

$$= 4\pi\, g_a f_a\, [\Psi^+(\boldsymbol{P}_2)\, Q\, \Psi(\boldsymbol{P}_1)]\, [\chi^+(\boldsymbol{p})\, \varphi(-\boldsymbol{p}_n)]\, \frac{(2\pi)^4}{2i} D_F(k), \qquad (2a)$$

$$(\boldsymbol{p}, \boldsymbol{P}_2 \mid H_2^b \mid -\boldsymbol{p}_n, \boldsymbol{P}_1)$$

$$= 4\pi\, g_b f_b\, [\Psi^+(\boldsymbol{P}_2)\gamma_5\, Q\, \Psi(\boldsymbol{P}_1)]\, [\chi^+(\boldsymbol{p})\gamma_5\, \varphi(-\boldsymbol{p}_n)]\, \frac{(2\pi)^4}{2i} D_F(k), \qquad (2b)$$

$$(\boldsymbol{p}, \boldsymbol{P}_2 \mid H_2^c \mid -\boldsymbol{p}_n, \boldsymbol{P}_1) = 4\pi \frac{g_c f_c}{\kappa^2} [\Psi^+(\boldsymbol{P}_2)\gamma_5\gamma_\mu k_\mu\, Q\, \Psi(\boldsymbol{P}_1)]$$

$$[\chi^+(\boldsymbol{p})\gamma_5\gamma_\nu k_\nu\, \varphi(-\boldsymbol{p}_n)]\, \frac{(2\pi)^4}{2i} D_F(k), \qquad (2c)$$

$$(\boldsymbol{p}, \boldsymbol{P}_2 \mid H_2^d \mid -\boldsymbol{p}_n \boldsymbol{P}_1)$$

$$= 4\pi\, g_d f_d\, [\Psi^+(\boldsymbol{P}_2)\gamma_5\gamma_\mu\, Q\, \Psi(\boldsymbol{P}_1)]\, [\chi^+(\boldsymbol{p})\gamma_5\gamma_\mu\, \varphi(-\boldsymbol{p}_n)]\, \frac{(2\pi)^4}{2i} D_F(k)$$

$$- 4\pi \frac{g_d f_d}{\kappa^2} [\Psi^+(\boldsymbol{P}_2)\gamma_5\gamma_\mu k_\mu\, Q\, \Psi(\boldsymbol{P}_1)]$$

$$[\chi^+(\boldsymbol{p})\gamma_5\gamma_\nu k_\nu\, \varphi(-\boldsymbol{p}_n)]\, \frac{(2\pi)^4}{2i} D_F(k), \qquad (2d)$$

$\Psi(\boldsymbol{p}) = \int \Psi(x)\, e^{-i\boldsymbol{P}\cdot x}\, d^3x$， $\chi(\boldsymbol{p}) = \int \chi(x)\, e^{-i} \cdot d^3x$ 等等

$$D_F(k) = \frac{-2i}{(2\pi)^4} \frac{1}{k_\mu k_\mu + \kappa^2} \qquad (3)$$

$k_\mu = (k_0, \boldsymbol{k})$， $k_0 = E + E_n = E_0$， $\boldsymbol{k} = \boldsymbol{p} - \boldsymbol{p}_n$， E 和 E_n 是電子和中微子的能量，$\boldsymbol{p}$ 和 $\boldsymbol{p}_n$ 是他們的動量。（$-\boldsymbol{p}_n$ 是反中微子的動量）。$\boldsymbol{P}_1$ 和 $\boldsymbol{P}_2$ 分別是核子在起始和終了狀態的動量。在 (2c) 和 (2d) 的第二項中的 $D_F(k)$ 嚴格的說應該是 $D'_F(k)$，

$$D'_F(k) = \frac{D_F(k)}{1 - \frac{1}{2} D_c(k)\, D_F(k)} \qquad (4)$$

$D_G(k)$ 代表虛介子的放射效應並且與 f, g 同一數量級。用 (4) 代 (3) 的理由見前一篇文章[4]。現在旣然 k_μ 非常小並且 g 和 f 也非常小，因此 $D'_F(k)$ 幾乎等于 $D_F(k)$。

先考慮 (a)。k_0 和 $\boldsymbol{k}$ 平常小于 5 兆電子伏。如果 κ 相當大，譬如大于 200 電子質量，$D_F(k)$ 可代以 $\frac{-2i}{(2\pi)^4}\frac{1}{\kappa^2}$。在坐標空間，我們有:

$$H_2^a = 4\pi \frac{g_a f_a}{\kappa^2} \int \Psi^+(x)\,\Psi(x)\,\chi^+(x)\,\varphi(x)\,d^3x \tag{5}$$

(5) 式正是費米理論中的標形哈密頓函數，只不過用 $\frac{4\pi g_a f_a}{\kappa^2}$ 代替了 g_β。

(b) 和 (c) 曾經奈爾遜[6] 研究過。他證明二者給出相同結果。根據他的計算雖然合于加算夫一鐵勒選擇定則，但是生命期與電子的最大能量 E_0 之 7 方成正比，而實驗結果則與 E_0 之五方成正比。

在 (d) 情形下，我們注意，

$$\begin{aligned}\chi^+(\boldsymbol{p})\,\gamma_5\gamma_\mu k_\mu\,\varphi(-\boldsymbol{p}_n) &= \chi^+(\boldsymbol{p})\,\gamma_5(\gamma_\mu p_\mu - \gamma_\mu p_{n\mu})\,\varphi(-\boldsymbol{p}_n)\\ &= m\,\chi^+(\boldsymbol{p})\,\gamma_5\,\varphi(-\boldsymbol{p}_n)\end{aligned} \tag{6}$$

$p_\mu = (E, \boldsymbol{p})$ 和 $p_{n\mu} = (E_n, \boldsymbol{p}_n)$，

同樣我們得，

$$\begin{aligned}\Psi^+(\boldsymbol{P}_2)\,\gamma_5\gamma_\mu k_\mu\,Q\,\Psi(\boldsymbol{P}_1) &= \Psi^+(\boldsymbol{P}_2)\,\gamma_5(\gamma_\mu P_{2\mu} - \gamma_\mu P_{1\mu})\,Q\,\Psi(\boldsymbol{P}_1)\\ &= -2M\,\Psi^+(\boldsymbol{P}_2)\,\gamma_5\,Q\,\Psi(\boldsymbol{P}_1)\end{aligned} \tag{7}$$

因之，代 $D_F(k)$ 以 $\frac{-2i}{(2\pi)^4}\frac{1}{\kappa^2}$，(2d) 變成

$$\begin{aligned}H_2^d = {}& 4\pi \frac{g_d f_d}{\kappa^2} \int \Psi^+(x)\,\gamma_5\gamma_\mu\,Q\,\Psi(x)\,\chi^+(x)\,\gamma_5\gamma_\mu\,\varphi(x)\,d^3x\\ &- 4\pi \frac{g_d f_d}{\kappa^2}\,\frac{2mM}{\kappa^2} \int \Psi^+(x)\,\gamma_5\,\Psi(x)\,\chi^+(x)\,\gamma_5\,\varphi(x)\,d^3x\end{aligned} \tag{8}$$

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6. E. C. Nelson, *Phys. Rev.* **60** (1941), 830.

$\frac{2mM}{\kappa^2}$ 大約在 ⅕ 左右，(κ 取做 200 m)。 因之 (8) 中第二項之係數小于第一項。 並且第二項在非相對論之極限情形下等于零，所以第二項對容許躍遷無關。(8) 變成，

$$H_2^d = 4\pi \frac{g_d f_d}{\kappa^2} \int \Psi^+ \gamma_5 \gamma_\mu \Psi \chi^+ \gamma_5 \gamma_\mu \varphi \, d^3 x \tag{9}$$

這正是費米理論中贋矢形哈密頓函數。 其選擇定則是加莫夫—鐵勒型。

因之如果 κ 相當大，我們的理論與通常費米關于 β- 蛻變理論結果完全相同。 但是費米理論中之發散困難在現在的理論中不復存在。 因爲 (5) 式和 (8) 式所代表的接觸作用在現在的理論中僅是取每種近似法的結果。

既然加莫夫—鐵勒選擇定則比較與實驗符合，此中間介子場當是贋矢介子贋矢耦合。 以下我們估計此中間介子之生命期。 簡單計算結果，不同類型的中間介子的生命期是:

$$\tau_a = \frac{\hbar}{\kappa f_a^2 c} \tag{10a}$$

$$\tau_b = \frac{\hbar}{\kappa f_b^2 c} \tag{10b}$$

$$\tau_c = \frac{\kappa}{m} \frac{\hbar}{\kappa f_c^2 c} \tag{10c}$$

$$\tau_d = \frac{3}{2} \frac{\hbar}{\kappa f_d^2 c} \tag{10d}$$

我們考慮 (d)。 g_d, f_d 和費米作用常數 g_β 有下列關係，

$$g_\beta = 4\pi \frac{g_d f_d}{\kappa^2} \frac{\hbar^2}{c^2} \tag{11}$$

由實驗結果[7] g_β 大約是 2×10^{-49} 爾格 · 厘米^3。 又由實驗結果，核子場，電子—中微子場，μ 介子—中微子場，三者之中任意兩種之間的作用强度都幾乎相等。 由現在的理論觀點看起來，只有此中間介子與三種場的耦合常數幾乎相等才行。 因之我們有 $g_d = f_d$，

7. G. Gamow & C. L. Critchfield, Theory of Atoms, Nucleus and Nuclear Energy Sources, p. 127 (Clarendon Press, Oxford, 1949).

$$g_d^2 = f_d^2 = 1.1 \times 10^{-29} \left(\frac{\kappa}{m}\right)^2 \text{ 爾格·厘米} \tag{12}$$

代入 (10d)，得

$$\tau_d = 5 \times 10^{-9} \left(\frac{m}{\kappa}\right)^3 \text{ 秒} \tag{13}$$

設 $\kappa = 200\,m$, $\tau_d = 6 \times 10^{-16}$ 秒。 由下節我們將指出 κ 必須大于 $200\,m$，因此 τ_d 更小于 6×10^{-16} 秒。 就我們所知，尚無一種實驗能測量具有如此短的生命期之介子。

3. μ 介子蛻變:

在上節我們曾假設介子場和核子場及介子場和電子—中微子場間有相同的作用函數和作用常數。 結果以贗矢介子贗矢耦合與實驗最相接近。 本節中，我們假設 μ 介子蛻變成電子—中微子也經過相同的中間贗矢介子，並且具有同樣的作用函數和作用常數。 正如前文所說，這點相當于在費米理論中各場間有相同的作用常數。 μ 介子蛻變曾經季姆諾，海勒，饒[8] 研究過，他們模倣費米關于 -β 蛻變的理論。 他們指出蛻變產物是兩個中微子還是一個中微子和一個反中微子對結果大有影響。 在本文所應用的理論中，這兩種情形相當于以下兩類拉氏函數。

(a) "反對稱電荷交換" 理論

$$L_2 = f_d \psi \chi^+ \gamma_5 \gamma_\mu \varphi + \text{共軛複數}, \qquad L_3 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \phi + \text{共軛複數} \tag{14}$$

(b) "簡單電荷交換" 理論

$$L_2 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \chi + \text{共軛複數}, \qquad L_3 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \phi + \text{共軛複數} \tag{15}$$

ϕ 是 μ 介子之場算符，χ, ψ, φ 和上節有同樣意義。 在計算相當于上節 (2d) 之矩陣元時，情形 (a) 有兩個中微子發射算符，情形 (b) 有一個中微子發射算符和一個反中微子發射算符 (或者，一個中微子發射算符和一個吸收算符) 按照包里不相容原理。 在第一種情形應當使兩個中微子發射算符反對稱。 因之 μ 介子之躍遷幾率如下:

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8. J. Tiomno, J. A. Wheeler & R. R. Rau, *Rev. Mod. Phys.* **21** (1949), 144.

$$d\,w_a = 2\pi\,\frac{4\,\pi^2\,p\,dp}{(2\,\pi)^3}\,\frac{1}{d\,E_o}\int \tfrac{1}{2}\,\Sigma_{\text{自旋}}\,|\,(\boldsymbol{p}_1,\ \boldsymbol{p}_2\,|\,H_2\,|\,\boldsymbol{p},\ \boldsymbol{p}_m)$$

$$-\,(\boldsymbol{p}_2,\ \boldsymbol{p}_1\,|\,H_2\,|\,\boldsymbol{p},\ \boldsymbol{p}_m)\,|^2\,\frac{1}{(2\,\pi)^3}\,d^3\,p_2 \tag{16}$$

積分取在總能量等于 E_o 時。

$$(\boldsymbol{p}_1,\ \boldsymbol{p}_2\,|\,H_2\,|\,\boldsymbol{p},\ \boldsymbol{p}_m)$$

$$= 4\pi\,f_d^2\,\varphi^+(\boldsymbol{p}_1)\gamma_5\,\gamma_\mu\,\phi(\boldsymbol{p}_m)\,\varphi^+(\boldsymbol{p}_2)\,\gamma_5\,\gamma_\mu\,\chi(-\boldsymbol{p})\,\frac{1}{(p_1-p_m)_\mu\,(p_1-p_m)_\mu+\kappa^2} \tag{17}$$

情形（b）之躍遷幾率是：

$$d\,w_b = 2\pi\,\frac{4\,\pi^2\,p\,dp}{(2\,\pi)^3}\,\frac{1}{d\,E_o}\int \Sigma_{\text{自旋}}\,|\,(\boldsymbol{p}_1,\ \boldsymbol{p}\,|\,H_2\,|\,\boldsymbol{p}_2,\ \boldsymbol{p}_m)\,|^2\,\frac{1}{(2\,\pi)^3}\,d^3\,p_2 \tag{18}$$

$$(\boldsymbol{p}_1,\ \boldsymbol{p}\,|\,H_2\,|\,\boldsymbol{p}_2,\ \boldsymbol{p}_m)$$

$$= 4\pi\,f_d^2\,\varphi^+(\boldsymbol{p}_1)\gamma_5\,\gamma_\mu\,\phi\,(\boldsymbol{p}_m)\chi^+(\boldsymbol{p})\gamma_5\,\gamma_\mu\,\varphi\,(-\boldsymbol{p}_2)\,\frac{1}{(p_1-p_m)_\mu\,(p_1-p_m)_\mu+\kappa^2} \tag{19}$$

在（17）和（19）中相當于（2d）的第二項被忽略，因爲它具有很小的因子 $\frac{m\,\mu}{\kappa^2}$。$\boldsymbol{p}_1$，$\boldsymbol{p}_2$，$\boldsymbol{p}$，$\boldsymbol{p}_m$ 分別是兩個中微子，電子，和 μ 介子之動量。（a）和（b）的不同僅僅是因爲在（a）裏面兩個中微子發射算符需要反對稱化，這種情形在核子之 β- 蜕變中當然不會發生。假設蜕變之 μ 介子靜止，我們有：（情形（b））

$$\Sigma_{\text{自旋}}\,|\,(\boldsymbol{p}_1,\ \boldsymbol{p}\,|\,H_2\,|\,\boldsymbol{p}_2,\ \boldsymbol{p}_m)\,|^2$$

$$= 16\,\pi^2\,f_d^2\left\{2-\cos\,\theta_{12}-\frac{p}{E}\,\cos\,\theta_2\right\}\frac{1}{(\kappa^2-\mu^2-2\,\mu\,p)^2} \tag{20}$$

μ 是 μ 介子質量，θ_1 是 $\boldsymbol{p}_1$ 與 $\boldsymbol{p}$ 間的角度，θ_2 是 $\boldsymbol{p}_2$ 與 $\boldsymbol{p}$ 間的角度，θ_{12} 是 $\boldsymbol{p}_1$ 與 $\boldsymbol{p}_2$ 間的角度。由能量和動量不滅定律，

$$\begin{aligned} E_0 = \mu = E + p_1 + p_2 \\ \boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p} = 0 \end{aligned} \tag{21}$$

我們有，

$$p_2 = \frac{(\mu-E)^2-p^2}{\mu-E+p\,\cos\,\theta_1},\qquad p_1 = \frac{(\mu-E)^2+p^2+2\,p\,(\mu-E)\,\cos\,\theta_1}{\mu-E+p\,\cos\,\theta_1},$$

$$\frac{d\,p_2}{d\,E} = \frac{(\mu-E)^2+p^2+2\,p\,(\mu-E)\,\cos\,\theta_1}{2\,(\mu-E+p\,\cos\,\theta_1)^2} \tag{22}$$

$$1-\cos\,\theta_{12} = \frac{2\,(\mu-E+p\,\cos\,\theta_1)^2}{(\mu-E)^2+p^2+2\,p\,(\mu-E)\,\cos\,\theta_1}$$

代入 (18)，得，

$$d w_b = 16\pi^2 f_d^4 \frac{p^2\,dp}{4(2\pi)^3}\left\{\frac{2\kappa^2-\mu^2-m^2}{E\,p\,\mu^2}\ \ln\ \frac{\kappa^2-\mu E+\mu p}{\kappa^2-\mu E-\mu p}\right.$$
$$\left.-\frac{4}{\mu E}+\left(\frac{2\kappa^2}{\mu E}+4\right)\frac{(\mu-E)^2-p^2}{(\kappa^2-\mu E)^2-\mu^2 p^2}\right\} \tag{23}$$

當 $\kappa >> \mu$，(23) 變成[9]

$$d w_b = 16\pi^2 f_d^4 \frac{p^2\,dp}{4(2\pi)^3\kappa^4}\left(6(\mu-E)^2-6p^2+\frac{4}{3}p^2\frac{\mu}{E}\right) \tag{24}$$

在情形 (a) 中。 利用 (22) 得，

$$\tfrac{1}{2}\Sigma_{\text{自旋}}\left|(\boldsymbol{p}_1,\boldsymbol{p}_2\,|\,H_2\,|\,\boldsymbol{p},\boldsymbol{p}_m)-(\boldsymbol{p}_2,\boldsymbol{p}_1\,|\,H_2\,|\,\boldsymbol{p},\boldsymbol{p}_m)\right|^2$$
$$=16\pi^2 f_d^4\left\{\left(1-\tfrac{1}{2}\cos\theta_{12}-\frac{p}{2E}\cos\theta_1\right)\frac{1}{(\kappa^2-\mu^2+2\mu p_2)^2}\right.$$
$$+\left(1-\tfrac{1}{2}\cos\theta_{12}-\frac{p}{2E}\cos\theta_2\right)\frac{1}{(\kappa^2-\mu^2+2\mu p_1)^2} \tag{25}$$
$$\left.+\left(1+\frac{m}{2E}\right)(1-\cos\theta_{12})\frac{1}{(\kappa^2-\mu^2+2\mu p_1)(\kappa^2-\mu^2+2\mu p_2)}\right\}$$

代入 (16)，

$$d w_a = 16\pi^2 f_d^4 \frac{p^2\,dp}{4(2\pi)^3}\left\{\left[\frac{2\kappa^2-\mu^2-m^2}{E\,p\,\mu^2}\right.\right.$$
$$\left.+\left(2+\frac{m}{E}\right)\frac{(\mu-E)^2-p^2}{\mu p(\kappa^2-\mu E)}\right]\ \ln\ \frac{\kappa^2-\mu E+\mu p}{\kappa^2-\mu E-\mu p} \tag{26}$$
$$\left.-\frac{4}{\mu E}+\left(\frac{2\kappa^2}{\mu E}+4\right)\frac{(\mu-E)^2-p^2}{(\kappa^2-\mu E)^2-\mu^2p^2}\right\}$$

(23) 和 (26) 所代表的躍遷幾率畫在第 (1)，(2) 圖中。 我們先考慮 $\kappa > \mu$ 情形。 從圖上可以看出電子分佈曲線之最高點當 κ 減小時向高能方向移動。 季姆諾，海勒等之結果相當于 $\kappa = \infty$ 時。 我們的結果與他們的相差頗小，甚至當 κ 小到 $400\,m$ 時還是如此。 但是精密的實驗結果是可能區別的。

現在考慮 $\kappa<\mu$ 情形。 從 (20) 和 (25) 我們看出 $p=\dfrac{\mu^2-\kappa^2}{2\mu}$ 時 dw 變成無窮。 既然 p 一定大于零，這種情形只有在 $\kappa<\mu$ 時才發生。 當 p 具有此數值時，中間狀態之能量與起始狀態之能量相等，其情形與原子物理學中之螢光共振現象相似。 正如外斯可普夫在螢光共振現象的理論一樣，現在也需

9. 式子 (24) 應當和註 8 文中的 (17)式一樣，想是該文作者不小心弄錯了。

要考慮放射阻尼效應才能免除無窮。不過以下的討論可以證明 $\kappa<\mu$ 不合于事實。根據不滅原理，如果 $\kappa<\mu$，μ 介子蜕變成 κ 介子和中微子是可能的。既然 f_d 頗小，這種情形之可能性，將更大于 $\mu-e$ 蜕變而進一步由 κ 介子蜕變出來的電子能量將不會是連續的，這點與實驗不合。因此 $\kappa<\mu$ 不合于事實。

4. 討 論:

量子場論中所發生的發散困難是由于對粒子在極高能量時彼此間的作用一種不正確的描寫。在電動力學中，較低能量時之相互作用是用實驗決定的。既然我們還不能在很高能量下做實驗，我們必須推廣我們在低能所得的結果到高能部分去。有發散困難出現表明此種推廣是錯誤的，在高能的情形下必須更正相互作用才能使積分收斂。從這點看起來，重正化觀念之應用能够去掉發散部分是非常值得注意的事。這表示研究比較低能時的現象確實可以不受高能時的知識的影響。因此我們只在低能做實驗不能決定電子之"構造"。反過來，只描寫電子在低能下的現象也不需要知道他的構造。這點正像量子力學可以與古典力學分開一樣，譬如在剛體力學中，我們不需要任何量子效應的知識。

很難令人相信只有在光子和電子的情形下才這樣。因此在介子理論中只挑選那些用質量和電荷的重正化觀念可以免除發散困難的理論也許是應該的。在本文中我們只討論四類 β- 蜕變。當然這並不是說我們已經考慮了所有可以採納的情形。有時一些更複雜的重正化方法也許可以使旁的理論也免除發散。這點可以前元[4] 贗標介子贗矢耦合爲例，那裏電動力學中通用的重正化方法已不足去掉發散，一種補充辦法必須引入。

最近薩骨恩，加德諾，和胡巴的關于 $\mu-e$ 蜕變的實驗結果[10] 說張量型的費米作用函數比較與實驗結果接近。不過他們的度量粗糙，特別在高能部分，因之還不能算作定論。如果他們的結論是對的，中間介子場將是矢介子張量耦合。既然我們尚未找出一種重正化辦法把這種類型的發散去掉，這種理論並未具有比相當費米理論更好的優點。不過當然比費米型理論更易于找到重正化方法。

本文的一些結論並非只限于本文所考慮的情形。我們不難看出無論什麼型的中間介子場也無論什麼型的耦合，以下的二結論總是正確的:

(i) 中間介子之生命期極短。

(ii) $\mu-e$ 蜕變中電子之能譜的最高點比相當的費米作用向高能方向移動。

第二點結論可與實驗比較。

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10. R. Sagune, W. L. Gardner & H. W. Hubbard, *Phys. Rev.* **82** (1951), 557.

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ON THE THEORY OF BETA-DISINTEGRATION

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Abstract

It is pointed out that the divergence in Yukawa's theory of β-disintegration can be removed by charge and mass renormalizations, whereas in conventional Fermi's theory this is not the case. From the known interaction constants for nuclear β-decay, for μ—e decay and for the interaction of μ-mesons with nucleons, it is found that the life time of the intermediary meson should be of the order 10^{-16} sec. We can therefore consider this intermediary meson as a new particle which has not been observed experimentally owing to its extremely short life time. Best agreement with experiment is obtained if the intermdiary meson is pseudo-vector in nature and under pseudo-vector interaction with the Fermi fields, those types of interactions being excluded which lead to divergent results for observable effects.

1. Introduction

It has been pointed out recently that the divergent integrals in quantum electrodynamics represent only unobservable charge and mass renormalizations of the electron. Consequently the presence of divergence will not present any difficulty in actual calculation, since we shall always obtain convergent results for observable effects. This new development is of great importance in other types of field theories, such as the meson theory and the theory of β-disintegration, of which the formulation has followed closely the analogy with quantum electrodynamics. Now the formulation of these theories can no longer be justified on the ground of analogy unless divergent results in these theories can also be shown to represent only unobservable effects. Those theories which lead to divergent observable results must therefore be considered as un-acceptable. This greatly reduces the ambiguity in meson theory since we can thus exclude certain type of interactions which lead to divergent results for observable effects.

It will be more urgent to consider the theory of β-disintegration since the theory has been established more firmly on experimental basis than the meson theory. However, when one tries to calculate the radiative correction to the process of β-disintegration due to emission and re-absorption of an electron-neutrino pair, one finds immediately that the divergence cannot be removed by charge and mass renormalizations as in the case of quantum electrodynamics. According to the criteria mentioned above, the theory of β-disintegration in the present form must be considered as un-acceptable.

As an alternative to the conventional theories of β-disintegration of the Fermi type, Yukawa[1] proposed that nucleons and electron-neutrino pairs do not interact directly, but through the intermediary of a meson field. According to Yukawa's scheme, the nucleon first emits a charged meson, which then decays into an electron and a neutrino. Yukawa's theory was supported by the experimental fact that mesons observed in cosmic rays are unstable and undergo β-disintegration. Quantitative calculations,[2] however, do not lead to the correct value for the life time of the meson. The situation are complicated still further by recent experiments[3] which show that the cosmic ray mesons (μ-mesons) do not interact strongly with the nucleons. The interaction constant determined from these experiments turns out to be only a negligible fraction of that required by nuclear forces.

It will be pointed out here that a theory of Yukawa type may be made divergence-free in the sense that the divergent integrals can be removed by charge and mass renormalizations. The strong divergence in Fermi's theory arises from the fact that the interaction function contains the field quantities to the fourth order. This type of interaction is essentially different from that in electrodynamics which contains field quantities only to the third order. In Yukawa's theory, two types of interactions are introduced, namely, the interaction between nucleons and mesons and the interaction between mesons and electron-neutrino pairs. Each type of interaction function contains field quantities only to the third power as in the case of electrodynamics. From our investigations on meson theory[4], we know that it is possible to choose the types of meson field and interaction function to make the Yukawa theory divergent free after charge and mass renormalizations. Furthermore, the descrepancy of Yukawa's theory with experiment will disappear if we assume that the intermediary meson field is an entirely new type of particle which has not been observed experimentally owing to its extremely short life time.

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1. H. Yukawa, *Proc. Phys. Math. Soc. Japan* **17** (1935), 48.
2. S. Sakata, *ibid,* **23** (1941), 191.
   S. Rosental, *Kgl. Danske Vid. Sels. Math.-fys. Medd.* **18** (1941), No. 7; *Phys. Rev.* **60** (1941), 612;
   A. H. Bethe and L. W. Nordheim, *Phys. Rev.* **57** (1940), 998;
   E. C. Nelson, *Phys. Rev.* **60** (1941), 830;
   See also: L. Rosenfeld, *Nuclear Forces,* p. 330 (North Holland Pub. Co., 1948).
3. M. Conversi, E. Pancini and O. Piccioni, *Phys. Rev.* **71** (1947), 209;
   C. M. G. Lattes, G. P. S. Occhialini and C. F. Powell, *Nature* **160** (1947), 463; above authors and H. Muirhead, *Nature* **159** (1947), 694.
4. Ning Hu, *Phys. Rev.* **80** (1950), 1109; *Chinese J. Phys.* **8** (1951), 40.

It is well known experimentally that the strength of interactions between any two of the proton-neutron field, the μ-meson-neutrino field and the electron-neutrino field are the same. This can be so only when the intermediary meson field interacts with each of these three fields also with the same strength of interaction. We shall find from this result and from the observed life time of the neutron that if the mass of the intermediary meson is arround or greater than 200 times the electron mass, then the life time of the intermediary meson is around or less than 10^{-16} second, in agreement with our assumption that the life time of the new meson is extremely short.

Only the following four types of charged meson fields have been made divergence-free by using proper programs of renormalizations. They are (i) scalar mesons with scalar coupling, (ii) pseudo-scalar mesons with pseudo-scalar coupling, (iii) pseudo-scalar mesons with pseudo-vector coupling and (iv) pseudo-vector mesons with pseudo-vector coupling. Therefore in order to make the theory of β-disintegration divergence-free, the intermediary meson field must be one of the above four types. In the case of nucleons decaying into electron-neutrino pairs, we find in the following calculations that if the ratio of the total energy of the electron-neutrino pair to the mass of the intermediary meson is negligibly small, then our result is identical with that obtained in Fermi's theory in the lowest order of approximation, as has been shown by many authors before. The excellent agreement of Fermi's theory with experiment shows that this ratio must indeed be very small. The situation will be different, however, in the case of μ-meson decaying into an electron-neutrino pair, where the total energy of the light particles is as large as 50 Mev. Unless the intermediary meson is extremely heavy, the result of the present theory will be different from that predicted by a theory of the Fermi type. This thus offers a ground for comparing the merits of these two types of theories with experiment.

2. β-DISINTEGRATION OF NUCLEONS.

We shall first list all acceptable types of intermediary meson fields. For simplicity we assume that the interaction Lagrangian density L_1 between mesons and nucleons is of the same type as the interaction Lagrangian density L_2 between mesons and electron-neutrino pairs. In a previous paper[4] it was shown that there are altogether four types of meson fields which lead to convergent results for all observable effects and are therefore acceptable according to the criteria stated in the Introduction. These fields are (a) *scalar mesons with scalar coupling* (the unit $\hbar = c = 1$ will be used in the present paper):

$$L_1^a = g_a \psi(x) \Psi^+(x) Q \Psi(x) + \text{Compl. Conj.},$$

$$L_2^a = f_a \psi(x) \chi^+(x) \varphi(x) + \text{Compl. Conj.} \tag{1a}$$

(b) *pseudo-scalar mesons with pseudo-scalar coupling:*

$$L_1^b = g_b \Psi^+ \gamma_5 Q \Psi + \text{Compl. Conj.},$$

$$L_2^b = f_b \psi \chi^+ \gamma_5 \varphi + \text{Compl. Conj.} \tag{1b}$$

(c) *pseudo-scalar mesons with pseudo-vector coupling:*

$$L_1^c = \frac{g_c}{\kappa} \frac{\partial \psi}{\partial x_\mu} \Psi^+ \gamma_5 \gamma_\mu Q \Psi + \text{Compl. Conj.},$$

$$L_2^c = \frac{f_c}{\kappa} \frac{\partial \psi}{\partial x_\mu} \chi^+ \gamma_5 \gamma_\mu \varphi + \text{Compl. Conj.} \tag{1c}$$

and (d) *pseudo-vector mesons with pseudo-vector coupling*

$$L_1^d = g_d \psi_\mu \Psi^+ \gamma_5 \gamma_\mu Q \Psi + \text{Compl. Conj.},$$

$$L_2^d = f_d \psi_\mu \chi^+ \gamma_5 \gamma_\mu \varphi + \text{Compl. Conj.} \tag{1d}$$

Here κ is the rest mass of the intermediary meson, $\psi(x)$ is the field operator for scalar or pseudo-scalar meson field and $\psi_\mu(x)$ is the field operator for the pseudo-vector meson field. $\Psi(x)$, $\chi(x)$ and $\varphi(x)$ are respectively the field operators for the nucleons, electrons and neutrinos. It is seen from the above expressions that L_1 reduces to L_2 if Ψ^+ and Ψ are resplaced by χ^+ and φ. This is due to our simplifying assumption that L_1 and L_2 are functions of the same type. Q is an operator converting a field operator for a proton into that of a neutron, or vice versa. g and f are coupling constants. In conventional perturbation calculations, one has to transform L_1 and L_2 first into interaction Hamiltonians by using a contact transformation. However, in the process of transformation, contact interactions between nucleons and electron-neutrino pairs of the same type as those in Fermi's theory will appear. We know already from investigations on Fermi's theory that these contact interactions lead to divergent results for observable effects in higher approximations. A closer examination will show, however, that these contact interactions are not really there at all. In the evaluation of the second order compound matrix elements, a new contact interaction will be obtained which just cancels the contact interaction introduced before. This round-about way of introducing and then removing a contact interaction is one of the defects of the conventional perturbation method. If on the other hand, one follows the new formulation recently devised by Feynman[5], no contact interaction will appear in the calculation. Since in our calculation it is necessary to be very sure that the contact interaction do not appear in the result, we shall follow Feynman's formulation. This formulation is now well explained in literature, we shall assume that the reader is familiar with it.

Using Feynman's method, the second order transition matrix elements representing β-disintegration of a nucleon can be written down immediately. We have for the case (a)-(d):

$$(\boldsymbol{p},\ \boldsymbol{P}_2 \mid H_2^a \mid -\boldsymbol{p}_n,\ \boldsymbol{P}_1)$$
$$= 4\pi g_a f_a \left[\Psi^+(\boldsymbol{P}_2)\, Q\, \Psi(\boldsymbol{P}_1)\right]\left[\chi^+(\boldsymbol{p})\, \varphi(-\boldsymbol{p}_n)\right]\cdot \frac{(2\pi)^4}{2i} D_F(k) \qquad (2a)$$

$$(\boldsymbol{p},\ \boldsymbol{P}_2 \mid H_2^b \mid -\boldsymbol{p}_n,\ \boldsymbol{P}_1)$$
$$= 4\pi g_b f_b \left[\Psi^+(\boldsymbol{P}_2)\, \gamma_5\, Q\, \Psi(\boldsymbol{P}_1)\right]\left[\chi^+(\boldsymbol{p})\, \gamma_5\, \varphi(-\boldsymbol{p}_n)\right]\cdot \frac{(2\pi)^4}{2i} D_F(k) \qquad (2b)$$

$$(\boldsymbol{p},\ \boldsymbol{P}_2 \mid H_2^c \mid -\boldsymbol{p}_n,\ \boldsymbol{P}_1) = 4\pi \frac{g_c f_c}{\kappa^2}\left[\Psi^+(\boldsymbol{P}_2)\, \gamma_5 \gamma_\mu k_\mu\, Q\, \Psi(\boldsymbol{P}_1)\right]$$
$$\left[\chi^+(\boldsymbol{p})\, \gamma_5 \gamma_\nu k_\nu\, \varphi(-\boldsymbol{p}_n)\right]\cdot \frac{(2\pi)^4}{2i} D_F(k) \qquad (2c)$$

$$(\boldsymbol{p},\ \boldsymbol{P}_2 \mid H_2^d \mid -\boldsymbol{p}_n,\ \boldsymbol{P}_1)$$
$$= 4\pi g_d f_d \left[\Psi^+(\boldsymbol{P}_2)\, \gamma_5 \gamma_\mu\, Q\, \Psi(\boldsymbol{P}_1)\right]\left[\chi^+(\boldsymbol{p})\, \gamma_5 \gamma_\mu\, \varphi(-\boldsymbol{p}_n)\right]\cdot \frac{(2\pi)^4}{2i} D_F(k)$$
$$- 4\pi \frac{g_d f_d}{\kappa^2}\left[\Psi^+(\boldsymbol{P}_2)\, \gamma_5 \gamma_\mu k_\mu\, Q\, \Psi(\boldsymbol{P}_1)\right]$$
$$\left[\chi^+(\boldsymbol{p})\, \gamma_5 \gamma_\nu k_\nu\, \varphi(-\boldsymbol{p}_n)\right]\cdot \frac{(2\pi)^4}{2i} D_F(k) \qquad (2d)$$

where $\Psi(\boldsymbol{P}) = \int \Psi(x)\, e^{-i\boldsymbol{P}\cdot x}\, d^3x, \quad \chi(\boldsymbol{p}) = \int \chi(x)\, e^{-i\boldsymbol{p}\cdot x}\, d^3x$, etc.,

$$D_F(k) = -\frac{2i}{(2\pi)^4}\frac{1}{k_\mu k_\mu + \kappa^2} \qquad (3)$$

and $k_\mu = (k_0,\ \boldsymbol{k})$, $k_0 = E + E_n = E_0$; $\boldsymbol{k} = \boldsymbol{p} - \boldsymbol{p}_n$, E and E_n being the energies and $\boldsymbol{p}$ and $\boldsymbol{p}_n$ the momenta of the electron and neutrino ($-\boldsymbol{p}_n$

5. R. P. Feynman, *Phys. Rev.* **67** (1949), 769.

being the momentum of the anti-neutrino). $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are the momenta of the nucleon in initial and final states. Strictly speaking, the function $D_F(k)$ in (2c) and in the second term of (2d) should be replaced by $D_F(k)$ defined by

$$D'_F(k) = \frac{D_F(k)}{1 - \frac{1}{2} D_C(k) D_F(k)} \tag{4}$$

where $D_C(k)$ represents radiation effect of the order g and f to the virtual meson. The argument for using (4) instead of (3) can be found in a previous paper[4]. In our case since the values of k_μ involved are small and also since g and f are extremely small constants, the second term in the denomenator of (4) may be neglected. Thus (4) reduces again to (3).

We shall first consider the case (a). k_0 and $\boldsymbol{k}$ are usually not greater than 5 Mev. If the value of κ is sufficiently large, say 200 m, being the electron mass, we may simply replace $D_F(k)$ by $-2i/(2\pi)^4\kappa^2$. Summing over all values of $\boldsymbol{k}$, $\boldsymbol{p}_n$ and $\boldsymbol{P}_1$ we obtain

$$H_2^a = \iiint (\boldsymbol{p}, \boldsymbol{P}_2 \mid H_2^a \mid \boldsymbol{p}_n \boldsymbol{P}_1) \frac{d^3 k}{(2\pi)^3} \frac{d^2 p_n}{(2\pi)^3} \frac{d^3 P_1}{(2\pi)^3}$$

$$= 4\pi \frac{g_a f_a}{\kappa^2} \int \Psi^+(x) \Psi(x) \chi^+(x) \varphi(x) d^3 x \tag{5}$$

which is just the scalar form of interaction Hamiltonian in Fermi's theory if we replace $4\pi g_a f_a/\kappa^2$ by g_β the interaction constant for β-decay in Fermi's theory.

The cases (b) and (c) have been investigated by Nelson[6]. He has shown that these two cases are in fact equivalent to each other. According to his calculation Gamov-Teller selection rule is valid, i.e., $\Delta J=0\pm 1$ (no $J=0\rightarrow J=0$), with no parity change; but the theoretical life time is proportional to the seventh power of maximum energy E_0 in disagreement with the experimental fifth power dependence.

Turning to case (d) we notice that

$$\chi^+(\boldsymbol{p}) \gamma_5 \gamma_\mu \varphi(-\boldsymbol{p}_n) = \chi^+(\boldsymbol{p}) \gamma_5 (\gamma_\mu p_\mu - \gamma_\mu p_{n\mu}) \varphi(-\boldsymbol{p}_n)$$

$$= m \chi^+(\boldsymbol{p}) \gamma_5 \varphi(-\boldsymbol{p}_n) \tag{6}$$

where $p_\mu=(E, \boldsymbol{p})$ and $p_{n\mu}=(E_n, \boldsymbol{p}_n)$ (6) follows immediately from the following two relations

6. E. C. Nelson, *Phys. Rev.* **60** (1941), 830.

$$\chi^{+}(\boldsymbol{p})\,\gamma_5\gamma_\mu = m\,\chi^{+}(\boldsymbol{p})\,, \qquad \gamma_\mu p_{n\mu}\,\varphi\,(-\boldsymbol{p}_n) = 0$$

Similarly we obtain

$$\begin{aligned}\Psi^{+}(\boldsymbol{P}_2)\,\gamma_5\gamma_\mu k_\mu Q\,\Psi(\boldsymbol{P}_1) &= \Psi^{+}(\boldsymbol{P}_2)\,\gamma_5\,(\gamma_\mu P_{2\mu} - \gamma_\mu P_{1\mu})\,Q\,\Psi(P_1)\\ &= -2M\,\Psi^{+}(\boldsymbol{P}_2)\,\gamma_5\,Q\,\Psi(\boldsymbol{P}_1) \qquad (7)\end{aligned}$$

on account of the following two equations

$$\Psi^{+}(\boldsymbol{P}_2)\,\gamma_\mu P_{2\mu} = M\,\Psi^{+}(\boldsymbol{P}_2)$$

$$\gamma_\mu P_{1\mu}\Psi(\boldsymbol{P}_1) = -M\,\Psi(\boldsymbol{P}_1)$$

Therefore (2d) becomes, on replacing $D_F(k)$ by $-2i/(2\pi)^4\kappa^2$,

$$\begin{aligned}H_2^d &= 4\pi\frac{g_d f_d}{\kappa^2}\int \Psi^{+}(x)\,\gamma_5\gamma_\mu Q\,\Psi(x)\,\chi^{+}(x)\,\gamma_5\gamma_\mu\,\varphi(x)\,d^3x\\ &\quad - 4\pi\,\frac{2mM}{\kappa^2}\,\frac{g_d f_d}{\kappa^2}\int \Psi^{+}(x)\,\gamma_5\,\Psi(x)\,\chi^{+}(x)\,\gamma_5\,\varphi(x)\,d^3x \qquad (8)\end{aligned}$$

$2mM/\kappa^2$ is of the order 1/5 if we use $\kappa = 200m$. Thus the coefficient of the second term in (8) is smaller than that of the first term. Furthermore, the second term vanishes in the non-relativistic limit, hence it has no contribution to the allowed transitions. On neglecting relativistic terms, (8) becomes

$$H_2^d = 4\pi\frac{f_d g_d}{\kappa^2}\int \Psi^{+}\gamma_5\gamma_\mu Q\,\Psi\,\chi^{+}\gamma_5\gamma_\mu\,\varphi\,d^3x \qquad (9)$$

This is just the pseudo-vector interaction in Fermi's theory of β-disintegration, and gives rise to Gamow-Teller selection rule.

The above investigation shows that if κ is sufficiently large our results for β-disintegration is the same as those obtained by the conventional theory of the Fermi type. The divergence difficulty of the conventional theory will not appear in the present theory since the contact interactions (5), (8) and (9) appear in our theory only as an approximation.

Since the experimental evidence favours Gamow-Teller selection rule, it follows from our result that the best choice of the intermediary meson field is the pseudo-vector meson with pseudo-vector couping. In the following we shall proceed to estimate the life time of the intermediary meson. It has been shown by Sakata by straight-forward calculations that the values of the life-time of the different types of mesons considered in the present paper are given by

$$\tau_a = \frac{\hbar^2}{\kappa f_a^2 c} \tag{10a}$$

$$\tau_b = \frac{\hbar^2}{\kappa f_b^2 c} \tag{10b}$$

$$\tau_c = \frac{\kappa}{m} \frac{\hbar^2}{\kappa f_c^2 c} \tag{10c}$$

$$\tau_d = \frac{3}{2} \frac{\hbar^2}{\kappa f_d^2 c} \tag{10d}$$

We shall consider the case (d). g_d and f_d are connected with Fermi interaction constant g_β by the following relation

$$g_\beta = 4\pi \frac{g_d f_d}{c^2} \frac{\hbar^2}{\kappa^2} \tag{11}$$

The value of g_β has been determined from experiment[7] to be $\approx 2\times10^{-49}$ erg cm^3. It is known experimentally that the coupling constants between any two of the following three fields, namely, the nucleon field, the electron-neutrino field and the μ-meson-neutrino field, are nearly the same. From the stand-point of the present theory, this can only be so if the interaction constants of the intermediary meson with any of the three fields mentioned above are also nearly the same. We thus obtain, on putting $g_d = f_d$,

$$g_d^2 = f_d^2 = 1.1 \times 10^{-29} (\kappa/m)^2 \quad \text{erg-cm} \tag{12}$$

Inserting this in (10d), we obtain

$$\tau_d = 5 \times 10^{-9} (m/\kappa)^3 \quad \text{sec.}. \tag{13}$$

For $\kappa = 200$ m, we have $\tau_d = 6\times10^{-16}$ sec. From the investigation in the next section we shall see that the value of κ should be greater than 200 m. This

7. G. Gamow and C. L. Critchfield, *Theory of Atoms, Nucleus and Nuclear Energy Sources*, p. 127, (Clarendon Press, Oxford, 1949).

means that τ_d should be smaller than 6×10^{-16} sec. As far as we know, there is yet no device to detect the existence of a particle with such a short life-time.

3. The decay of μ-meson into electron and neutrinos.

In the last section we have assumed that the interaction function and the coupling constant between the meson field and the nucleon field are the same as those between the meson field and the electron-neutrino field. We have concluded that the case of pseudo-vector meson field with pseudo-vector coupling is in best agreement with experiment. In this section we shall assume that the decay of μ-meson into electron and neutrinos is due to the same intermediary pseudo-vector meson with the same interaction function and coupling constant. As we have mentioned before, this will lead to a universal interaction among the Fermi fields as has been established by experiment. This problem has been treated by Tiomno, Wheeler and Rau[8] by analogy with Fermi's theory of β-disintegration. These authors have pointed out that different results will be obtained depending on whether two neutrinos or one neutrino and one anti-neutrino are resulted from the disintegration. In the present theory this corresponds to the following two types of interaction Lagrangians:

(a) *"Anti-symmetrical charge exchange" theory*

$$L_2 = f_d \psi \chi^+ \gamma_5 \gamma_\mu \varphi + \text{Compl. Conj.}, \quad L_3 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \phi + \text{Compl. Conj.} \quad (14)$$

(b) *"Simple charge exchange" theory*

$$L_2 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \chi + \text{Compl. Conj.}, \quad L_3 = f_d \psi \varphi^+ \gamma_5 \gamma_\mu \phi + \text{Compl. Conj.} \quad (15)$$

where ϕ is the field operator for the μ-meson, ψ, φ and χ have the same meaning as before. In evaluating the compound matrix element corresponding to (2d), we shall obtain two neutrino emission operators for the case (a) and one neutrino-emission operator and one anti-neutrino-emission operator (or in other words, one neutrino-emission operator and one neutrino-absorption operator) for the case (b). In the first case the two neutrino-emission operators should be anti-symmetrized in accordance with the Pauli principle. The transition probability for the decay of μ-meson is thus given by the following expression:

8. J. Tiomno, J. A. Wheeler and R. R. Rau, *Rev. Mod. Phys.* **21** (1949), 144.

$$d w_a = 2\pi \frac{4\pi^2 p\, dp}{(2\pi)^3} \frac{1}{dE_0}$$

$$\int \tfrac{1}{2} \Sigma_{\text{Spin}} \,|\, (\boldsymbol{p}_1, \boldsymbol{p}_2 \,|\, H_2 \,|\, \boldsymbol{p}, \boldsymbol{p}_m) - (\boldsymbol{p}_2, \boldsymbol{p}_1 \,|\, H_2 \,|\, \boldsymbol{p}, \boldsymbol{p}_m) \,|^2 \frac{d^3 p_2}{(2\pi)^2} \tag{16}$$

where the integration is taken over the energy shell between E_0 and E_0+dE_0, and

$$(\boldsymbol{p}_1, \boldsymbol{p}_2 \,|\, H_2 \,|\, \boldsymbol{p}, \boldsymbol{p}_m) = 4\pi f_d^2\, \varphi^+ (\gamma_5 \gamma_\mu \phi(\boldsymbol{p}_m)\, \varphi^+ (\boldsymbol{p}_2) \gamma_5 \gamma_\mu \chi(-p)$$

$$\frac{1}{(p_1 - p_m)_\mu (p_1 - p_m)_\mu + \kappa^2} \tag{17}$$

In the (b) the same transition probability is given by

$$d w_b = 2\pi \frac{4\pi^2 p\, dp}{(2\pi)^3} \frac{1}{dE_0} \int \Sigma_{\text{Spin}} \,|\, (\boldsymbol{p}_1, \boldsymbol{p} \,|\, H_2 \,|\, \boldsymbol{p}_2 \boldsymbol{p}_m) \,|^2 \frac{d^3 p_2}{(2\pi)^3} \tag{18}$$

where

$$(\boldsymbol{p}_1 \boldsymbol{p} \,|\, H_2 \,|\, \boldsymbol{p}_2 \boldsymbol{p}_m) = 4\pi f_d^2\, \varphi^+ (\boldsymbol{p}_1) \gamma_5 \gamma_\mu \phi(\boldsymbol{p}_m)\, \chi^+ (\boldsymbol{p}) \gamma_5 \gamma_\mu \varphi(-\boldsymbol{p}_2)$$

$$\frac{1}{(p_1 - p_m)_\mu (p_1 - p_m)_\mu + \kappa^2} \tag{19}$$

In (17) and (19) the relativistic terms corresponding to the second term of (2d) have been neglected since they contain the factor $m\mu/\kappa^2$ which is very small. p_1, p_2, p and p_m are respectively the momenta of the two neutrinos, the electron and the μ-meson. The difference in cases (a) and (b) arises only from the fact that two neutrino-emission operators have to be anti-symmetrized in case (a). The choice of L_2 in (14) and (15) will not make any difference in the case of β-disintegration of nucleons since no anti-symmetrization is needed. In the following we shall use the coordinate system in which the μ-meson is at rest. By carrying out the spin summations for the Fermi particles, we obtain for the case (b)

$$\Sigma_{\text{Spin}} \,|\, (\boldsymbol{p}_1 \boldsymbol{p} \,|\, H_2 \,|\, \boldsymbol{p}_2 \boldsymbol{p}_m) \,|^2$$

$$= 16\pi^2 f_d^4 \left\{ 2 - \cos\theta_{12} - \frac{p}{E} \cos\theta_2 \right\} \frac{1}{(\kappa^2 - \mu^2 - 2\mu p)^2} \tag{20}$$

where μ is the mass of the μ-meson, θ_1 is the angle between p_1 and p, θ_2 is the angle between p and p_2, and θ_{12} is the angle between p_1 and p_2. From conservation laws

$$\begin{aligned} E_0 = \mu = E + p_1 + p_2 \\ \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p} = 0 \end{aligned} \tag{21}$$

we obtain

$$p_2 = \frac{(\mu - E)^2 - p^2}{\mu - E + p \cos\theta_1}, \qquad p_1 = \frac{(\mu-E)^2 + p^2 + 2p\cos\theta_1(\mu-E)}{\mu - E + p\cos\theta_1},$$

$$\frac{dp_2}{dE_0} = \frac{(\mu-E)^2 + p^2 + 2p\cos\theta_1(\mu-E)}{2(\mu E + p\cos\theta_1)^2} \tag{22}$$

$$1 - \cos\theta_{12} = \frac{2(\mu-E+p\cos\theta_1)^2}{(\mu-E)^2 + p^2 + 2p(\mu-E)\cos\theta_1}$$

Inserting these results in (18), we get

$$d w_b = 16\pi^2 f_d^4 \frac{p^2\, dp}{4(2\pi)^3} \left\{ \frac{2\kappa^2 - \mu^2 - m^2}{E p \mu^2} \ln \frac{\kappa^2 - \mu E + \mu p}{\kappa^2 - \mu E + \mu p} - \frac{4}{\mu E} + \left(\frac{2\kappa^2}{\mu E} + 4\right) \frac{(\mu E)^2 - p^2}{(\kappa^2 - \mu E)^2 - \mu^2 p^2} \right\} \tag{23}$$

When $\kappa >> \mu$, (23) reduces to[9]

$$d w_b = 16\pi^2 f_d^2 \frac{p^2\, dp}{4(2\pi)^3 \kappa^4} \left[6(\mu E)^3 - 6p^2 + \frac{4}{3} p^2 \frac{\mu}{E} \right] \tag{24}$$

We turn next to the case (a). After summation over the spins of the Fermi particles and using the relations (22), we obtain

$$\tfrac{1}{2}\Sigma_{\text{Spin}} \left| (\mathbf{p}_1, \mathbf{p}_2 | H_2 | \mathbf{p}, \mathbf{p}_m) - (\mathbf{p}_2, \mathbf{p}_1 | H_2 | \mathbf{p}, \mathbf{p}_m) \right|^2$$

$$= 16\pi^2 f_d^2 \left\{ 1 - \tfrac{1}{2}\cos\theta_{12} - \frac{p}{2E}\cos\theta_1 \right) \frac{1}{(\kappa^2 - \mu^2 + 2\mu p_2)^2}$$

9. (24) should be identical with the expression (17) in the paper of Tiomno, Wheeler and Rau (Ref. 8). However we find that this is not the case. We believe that this is due to a slip in calculation on the part of these authors.

$$+ \left(1 - \tfrac{1}{2}\cos\theta_{12} - \frac{p}{2E}\cos\theta_2\right)\frac{1}{(\kappa^2 - \mu^2 + 2\mu p_1)^2}$$

$$+ \left(1 + \frac{m}{2E}\right)(1 - \cos\theta_{12})\frac{1}{(\kappa^2 - \mu^2 + 2\mu p_1)(\kappa^2 - \mu^2 + 2\mu p_2)}\Big\} \quad (25)$$

Inserting this in (16) we have

$$d\,w_a = 16\,\pi^2 f_d^4 \frac{p^2\,dp}{4\,(2\,\pi)^3}$$

$$\Big\{\Big[\frac{2\,\kappa^2 - \mu^2 - m^2}{E\,p\,\mu^2} + \left(2 + \frac{m}{E}\right)\frac{(\mu - E)^2 - p^2}{\mu\,p\,(\kappa^2 - \mu\,E)}\Big]\ln\frac{\kappa^2 - \mu\,E + \mu\,p}{\kappa^2 - \mu\,E - \mu\,p}$$

$$- \frac{4}{\mu\,E} + \left(\frac{2\,\kappa^2}{\mu\,E} + 4\right)\frac{(\mu - E)^2 - p^2}{(\kappa^2 - \mu\,E)^2 - \mu^2\,p^2}\Big\} \quad (26)$$

The transition probabilities (23) and (26) are plotted in Figs. 1 and 2. For a reason to be explained in a moment, we have restricted ourselves to the case $\kappa < \mu$. It is seen that the maximum of the distribution curves is shifted to the high energy region as κ decreases. The results of Tiomno, Wheeler and Rau corresponds to the limiting case $\kappa = \infty$. The deviation of our curves from those of Tiomno, Wheeler and Rau is very slight even

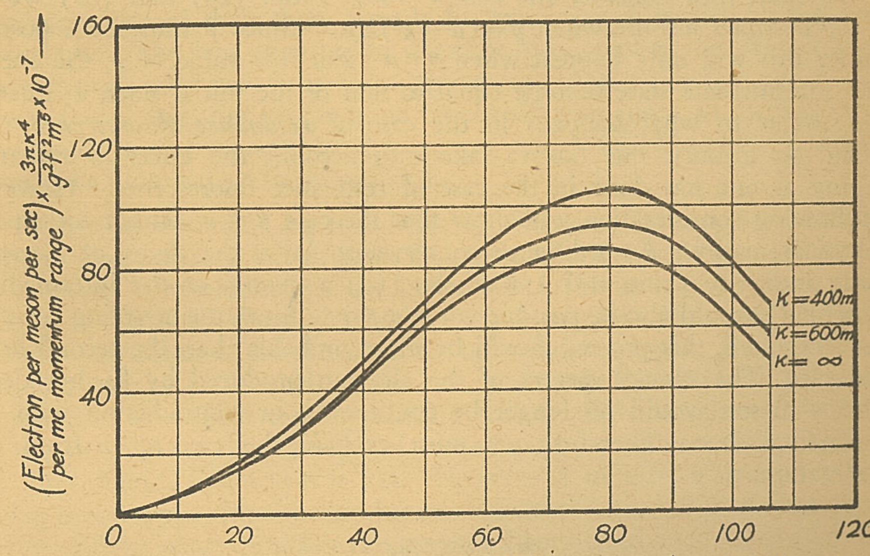

Fig. 1 Simple Charge Exchange Theory

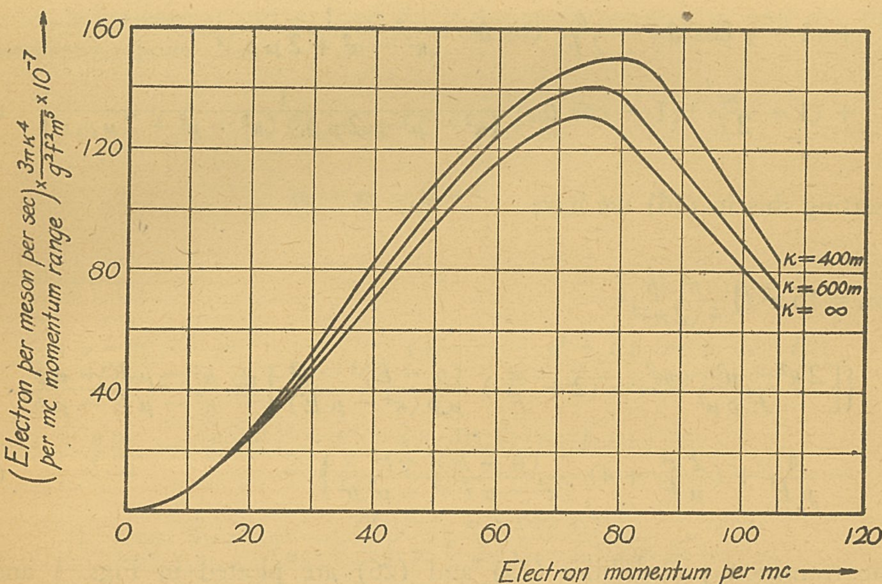

Fig. 2 Antisymmetric Charge Exchange Theory

when κ is as small as 400 m. Nevertheless it is still possible to detect the difference by accurate measurements.

We shall now consider the case $\kappa < \mu$. From (20) and (25) we see that dw becomes infinite when $p = (\mu^2 - \kappa^2)2/\mu$. Since p must be a positive quantity this will only happen when $\kappa < \mu$. For this value of p, the energy of the intermediate state become equal to that of the initial state, a situation quite similar to what happens in the case of *resonance fluorescence*. To remove the infinity, one has to take into account the effect of radiation damping as one has done in the case of resonance fluorescence. However, the following consideration will show that the case $\kappa < \mu$ cannot correspond to physical reality. According to conservation laws, the decay of μ-meson directly into a neutrino and a κ-meson (i.e., a quanta of the intermediary meson field) would also be possible when $\kappa < \mu$. Since the coupling constant f_d is very small, this process should be more probable than the second order μ-e decay. The energy spectra of the electron produced by further decay of the κ-meson would no longer be continuous, in contradiction with the experimental fact. Therefore we must exclude the case $\kappa < \mu$ from our consideration.

4. Discussions

The divergence difficulty in quantum field theory is due to the incorrect description of interaction at extremely high energies. In the case of electrodynamics, the interaction at relatively low energies is determined from experiment. Since we cannot yet perform experiments at extremely high energies we must extrapolate our interaction function to the high energy region. The occurence of divergence shows merely that the extrapolation is not correct, and modifications must be introduced at very high energies to make the integrals convergent. From this point of view, the success in removing the divergence by charge and mass renormalizations is of great significance. It means that the phenomena at relatively low energies are really separated from those at extremely high energies. We cannot determine the "structure" of the electron, which is synonymous with the behavior of the electron at extremely high energies, by performing experiments at low energies. Likewise the description of the phenomena at low energies will not need the knowledge of the structure of the electron. The situation is quite similar to the separation between quantum mechanics and classical mechanics. The mechanics of a rigid body, for instance, was developed without any knowledge of the quantum effect.

It will be difficult to believe that this separation is only accidental between electrons and photons, and not generally true among other types of fundamental particles. It is therefore a sound philosophy in dealing with the general problems of fundamental interactions, to accept only those theories which can be free from divergent difficulties after charge and mass renormalizations. In the present paper we have considered four types of β-disintegrations. This does not imply, however, that we have already considered all acceptable cases. Sometimes a program of renormalization much more complicated than those already known to us is needed to remove the divergence. The best example to illustrate this point is the case of pseudoscalar mesons with pseudo-vector coupling treated in a previous paper[4], where it was found that the program of renormalization used in electrodynamics is not sufficient to remove the divergences, and a supplementary program of renormalization has to be introduced.

Recent observations on μ-e decay by Sagune, Gardner and Hubbard[10] indicate that a theory which leads to a tensor-form Fermi interaction might be in better agreement with the experiment. Their measurements are too rough, especially in the high energy region, to make the assertion definite. If their conclusion is correct, a vector intermediary meson field with tensor coupling would be needed. Since we have not found yet a program of renormalization which can successfully remove the divergence for this type of meson field, the advantage of introducing an intermediary field over the direct Fermi interaction is lost. However it should be stressed here that owing to the types of interaction functions involved, successful renormalization would still be more probable in present theory than in a theory of Fermi type.

10. R. Sagune, W. L. Gardner and H. W. Hubbard, *Phys. Rev.* **82** (1951), 557.

The general results obtained in the present paper are by no means peculiar only to the cases we have considered. It will not be difficult to see that for any type of intermediary meson field and for any type of interaction function, the following two conclusions should be of general validity: (i) The life time of the intermediary meson should be extremely short. (ii) The energy spectra of the electron resulted from μ-e decay should be shifted slightly to the high energy side from a curve obtained from the corresponding Fermi interaction. Thus apart from the consideration whether an acceptable field theory should be divergence-free after charge and mass renormalizations, the second conclusion may be used as a direct check of the present theory with the experiment.