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球对称动态黑洞 Dirac 场的统计熵

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利用改进的 brick-wall 模型，计算最一般球对称动态黑洞 Dirac 场的统计熵．结果表明，任一时刻黑洞熵都与黑洞事件视界面积成正比．特别是给出了动比例系数的计算公式，通过计算动比例系数，可直接得出各种球对称动态黑洞 Dirac 场的统计熵．

关键词：事件视界，黑洞熵，Dirac 场，动比例系数

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1. 引 言

目前，大量文献[1—12]对所有黑洞熵的讨论都给出了与视界面积成正比的正确结果．其中用得最多方法是 't Hooft 提出的 brick-wall[1]方法．然而，使用 brick-wall 模型必须先假定黑洞与外界在大尺度范围内存在热平衡，动态黑洞显然不属于这种情况．Li，Gao 和 Zhao 等人针对上述不完美之处，把 brick-wall 模型改进为薄层模型[3—7]．在改进的方案中，只要求在黑洞视界处薄层范围内局部存在热平衡，黑洞的熵被认为是来自视界附近一个薄层的贡献．近来人们利用改进的 brick-wall 膜模型，也得到了令人满意的结果[3—12]．文献[8]已对动态球对称黑洞标量场的熵进行了研究．为使结果更具有普遍性意义，本文利用改进的 brick-wall 膜模型，进一步研究最一般球对称动态黑洞 Dirac 场的统计熵所遵从的规律，给出了动比例系数的计算公式，对于各种具体球对称动态黑洞，当给出黑洞时空线元后，只要计算出动比例系数，就可直接给出黑洞 Dirac 场的统计熵．

2. Dirac 场的经典动量

最一般的球对称动态黑洞时空线元可表示为[13]

$$\mathrm{d}s^2 = A(v,r)\mathrm{d}v^2 - 2B(v,r)\mathrm{d}v\mathrm{d}r - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \tag{1}$$

根据公式 $\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}(g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha})$逐一计算出 64 个联络，其中不等于零的联络为

$$\Gamma^0_{00} = \frac{\dot{B}}{B} + \frac{A'}{2B}, \quad \Gamma^1_{00} = -\frac{\dot{A}}{2B} + \frac{A}{B^2}\left(\dot{B} + \frac{A'}{2}\right),$$
$$\Gamma^1_{11} = \frac{B'}{B}, \quad \Gamma^0_{22} = -\frac{r}{B}, \quad \Gamma^1_{22} = -\frac{A}{B^2}r,$$
$$\Gamma^0_{33} = -\frac{r}{B}\sin^2\theta, \quad \Gamma^1_{33} = -\frac{A}{B^2}r\sin^2\theta,$$
$$\Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^1_{10} = \Gamma^1_{01} = -\frac{A'}{2B},$$
$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}, \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta,$$
$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}, \tag{2}$$

式中 $A' = \frac{\partial A}{\partial r}, B' = \frac{\partial B}{\partial r}, \dot{A} = \frac{\partial A}{\partial v}, \dot{B} = \frac{\partial B}{\partial v}$，构造下面的零标架：

$$l^{\mu} = \frac{1}{B}(0,1,0,0),$$
$$n^{\mu} = (-1, -A/2B, 0, 0),$$
$$m^{\mu} = \frac{1}{\sqrt{2}r}(0,0,1,\mathrm{i}/\sin\theta),$$
$$\bar{m}^{\mu} = \frac{1}{\sqrt{2}r}(0,0,1,-\mathrm{i}/\sin\theta). \tag{3}$$

按照 Newman 和 Penrose 的方法[14]，计算 Ricci 旋系数，得

$$\varepsilon = \pi = \tau = 0, \quad \alpha = -\beta = -\frac{1}{2\sqrt{2}r}\cot\theta,$$
$$\gamma = \frac{1}{2B}\left(\dot{B} + \frac{A'}{2}\right), \quad \rho = -\frac{1}{Br}, \quad \mu = -\frac{A}{2Br}, \tag{4}$$

微分算子为

$$D = l^{\mu}\partial_{\mu} = \frac{1}{B}\frac{\partial}{\partial r},$$

$$\bar{D} = n^{\mu}\partial^{\mu} = -\frac{\partial}{\partial \upsilon} - \frac{A}{2B}\frac{\partial}{\partial r},$$

$$\delta = m^{\mu}\partial^{\mu} = \frac{1}{\sqrt{2}r}\frac{\partial}{\partial \theta} + \frac{1}{\sqrt{2}r}\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi},$$

$$\bar{\delta} = \bar{m}^{\mu}\partial^{\mu} = \frac{1}{\sqrt{2}r}\frac{\partial}{\partial \theta} - \frac{1}{\sqrt{2}r}\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi}. \quad (5)$$

将(4)和(5)式代入旋量粒子的四分量场方程

$$(D + \varepsilon - \rho)\psi_{11} + (\bar{\delta} + \pi - \alpha)\psi_{12} - \frac{\mathrm{i}}{\sqrt{2}}\mu_0\psi_{21} = 0,$$

$$(\bar{D} + \mu - \gamma)\psi_{12} + (\delta + \beta - \tau)\psi_{11} - \frac{\mathrm{i}}{\sqrt{2}}\mu_0\psi_{22} = 0,$$

$$(D + \varepsilon^{*} - \rho^{*})\psi_{22} - (\delta + \pi^{*} - \alpha^{*})\psi_{21} - \frac{\mathrm{i}}{\sqrt{2}}\mu_0\psi_{12} = 0,$$

$$(\bar{D} + \mu^{*} - \gamma^{*})\psi_{21} - (\bar{\delta} + \beta^{*} - \tau^{*})\psi_{22} - \frac{\mathrm{i}}{\sqrt{2}}\mu_0\psi_{11} = 0, \quad (6)$$

式中 μ_0 为粒子的静止质量,采取小质量近似,整理得到

$$\frac{1}{B}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\psi_{11} + \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial \theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi} + \frac{1}{2}\cot\theta\right)\psi_{12} = 0,$$

$$-\left(\frac{\partial}{\partial \upsilon} + \frac{A}{2B}\frac{\partial}{\partial r} + \frac{A}{2Br} + \frac{\dot{B}}{2B} + \frac{A'}{4B}\right)\psi_{12} + \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial \theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi} + \frac{1}{2}\cot\theta\right)\psi_{11} = 0,$$

$$\frac{1}{B}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\psi_{22} - \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial \theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi} + \frac{1}{2}\cot\theta\right)\psi_{21} = 0,$$

$$-\left(\frac{\partial}{\partial \upsilon} + \frac{A}{2B}\frac{\partial}{\partial r} + \frac{A}{2Br} + \frac{\dot{B}}{2B} + \frac{A'}{4B}\right)\psi_{21} - \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial \theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial \varphi} + \frac{1}{2}\cot\theta\right)\psi_{22} = 0. \quad (7)$$

分离变量,令

$$\psi_{11} = r^{-1}\,{}_{-\frac{1}{2}}R_l(\upsilon,r)\,{}_{-\frac{1}{2}}Y_l^m(\theta,\varphi) = r^{-1}R_-\,Y_-,$$

$$\psi_{12} = r^{-2}\,{}_{+\frac{1}{2}}R_l(\upsilon,r)\,{}_{+\frac{1}{2}}Y_l^m(\theta,\varphi) = r^{-2}R_+\,Y_+,$$

$$\psi_{21} = r^{-2}\,{}_{+\frac{1}{2}}R_l(\upsilon,r)\,{}_{-\frac{1}{2}}Y_l^m(\theta,\varphi) = r^{-2}R_+\,Y_-,$$

$$\psi_{22} = r^{-1}\,{}_{-\frac{1}{2}}R_l(\upsilon,r)\,{}_{+\frac{1}{2}}Y_l^m(\theta,\varphi) = r^{-1}R_-\,Y_+, \quad (8)$$

式中 $R_- = {}_{-\frac{1}{2}}R_l(\upsilon,r)$, $R_+ = {}_{+\frac{1}{2}}R_l(\upsilon,r)$, $Y_- = {}_{-\frac{1}{2}}Y_l^m(\theta,\varphi)$, $Y_+ = {}_{+\frac{1}{2}}Y_l^m(\theta,\varphi)$, $\pm\frac{1}{2}$, m, l 分别为自旋量子数,磁量子数和角量子数.将(8)式代入(7)式,整理可得角向方程

$$\left[\frac{\partial^2}{\partial \theta^2} + \cot\theta\frac{\partial}{\partial \theta} + \frac{1}{\sin^2\theta}\left(\frac{1}{4}\cos^2\theta \mp \mathrm{i}\cos\theta\frac{\partial}{\partial \varphi} + \frac{\partial^2}{\partial \varphi^2} - \frac{1}{2}\right) + \lambda^2\right]Y_{\pm} = 0. \quad (9)$$

由文献[15—17]知,方程的解为权是 1/2 的球谐函数,分离变量常数为 $\lambda = \sqrt{(l+s)(l-s+1)}$,这里 l,m 为整数,且满足不等式 $l \geqslant s$,$-l \leqslant m \leqslant l$,整理(7)式可得两个径向方程,其中关于 R_- 的方程为

$$-\frac{Ar^2}{2B^2}\frac{\partial^2 R_-}{\partial r^2} - \frac{r^2}{B}\frac{\partial^2 R_-}{\partial \upsilon \partial r} + \frac{1}{2B^2}\left(r^2\dot{B} - Ar + \frac{Ar^2B'}{B} - \frac{A'r^2}{2}\right)\frac{\partial R_-}{\partial r} + \frac{\lambda^2}{2}R_- = 0. \quad (10)$$

采取 Wengel-Kramers-Brillouin 近似,令 $R_-(\upsilon,r) = \mathrm{e}^{\mathrm{i}[S_-(r) - E\upsilon]}$,并将分离变量常数 $\lambda = \sqrt{(l+s)(l-s+1)}$ 代入,可得到旋量场一个分量所对应的经典动量

$$P = \frac{\partial S_-}{\partial r} = \frac{B}{A}\left[E \pm \sqrt{E^2 - A(l+s)(l-s+1)/r^2}\right]. \quad (11)$$

3. Dirac 场的自由能和熵

根据正则系综理论,系统的自由能 F 可表示为

$$\beta F = \int_0^{\infty}\mathrm{d}\Gamma(E)\ln(1 + \mathrm{e}^{-\beta E}) = -\beta\int_0^{\infty}(\mathrm{e}^{\beta E} + 1)^{-1}\Gamma(E)\,\mathrm{d}E, \quad (12)$$

式中 $\Gamma(E)$ 为系统的能量 $\leqslant E$ 的微观态数,由半径典的索末菲量子理论 $\oint p\,\mathrm{d}l = 2\pi n$,并利用改进的 brick-wall 膜模型,可求得

$$\Gamma(E) = \sum_l (2l+1)n(E) = \frac{1}{\pi}\int_l (2l+1)\,\mathrm{d}l\int_{r_{\mathrm{H}}+\varepsilon}^{r_{\mathrm{H}}+\varepsilon+h}\frac{B}{A}\left[E^2 - A(l+s)(l-s+1)/r^2\right]^{1/2}\mathrm{d}r, \quad (13)$$

式中 r_{H} 为黑洞外视界半径,ε 为紫外截断因子,h 为膜的厚度,令 $A(\upsilon,r) = f(\upsilon,r)(r - r_{\mathrm{H}})$,将(13)式代入(12)式,可得自由能

$$F = -\frac{1}{\pi}\int_0^{\infty}(\mathrm{e}^{\beta E} + 1)^{-1}\mathrm{d}E\int_l (2l+1)\,\mathrm{d}l\int_{r_{\mathrm{H}}+\varepsilon}^{r_{\mathrm{H}}+\varepsilon+h}\frac{B}{A} \times \left[E^2 - A(l+s)(l-s+1)/r^2\right]^{1/2}\mathrm{d}r \approx -\frac{7}{180}\frac{\pi^3 r_{\mathrm{H}}^2}{\beta^4\varepsilon'}\frac{B(\upsilon,r)}{f^2(\upsilon,r)}\bigg|_{r=r_{\mathrm{H}}}, \quad (14)$$

式中 $\varepsilon' = \varepsilon(\varepsilon + h)/h$，积分只保留 $1/\varepsilon$ 主导项，于是可得旋量场一个分量的熵为

$$S_1 = \left(\beta^2 \frac{\partial F}{\partial \beta}\right)\Big|_{\beta=\beta_H} = 7\pi^3 r_H^2 B(\upsilon, r)/45\beta_H^3 \varepsilon' f^2(\upsilon, r)\mid_{r=r_H}, \quad (15)$$

式中 β_H 为视界温度的倒数，对于一般球对称动态黑洞，其事件视界温度为[13]

$$T = \frac{1}{2\pi}\frac{A'_H - 2B'_H \dot{r}_H}{2B_H(1 - 2\dot{r}_H)}, \quad (16)$$

式中下标 H 表示函数在 $r = r_H$ 处取值（下同），将(16)式代入(15)式，根据熵的可加性可求得一般球对称动态黑洞 Dirac 场的统计熵为

$$S_F = 4S_1 = \frac{7\mathscr{A}_H}{2880\pi\varepsilon'}\frac{(A'_H - 2B'_H\dot{r}_H)^3}{B_H^2 f_H^2(1 - 2\dot{r}_H)^3}, \quad (17)$$

式中 $\mathscr{A}_H = \iint \begin{vmatrix} g_{22} & g_{23} \\ g_{32} & g_{33}\end{vmatrix}^{1/2} d\theta d\varphi = 4\pi r_H^2$ 为黑洞事件视界面积，令

$$b = \frac{(A'_H - 2B'_H\dot{r}_H)^3}{B_H^2 f_H^2(1 - 2\dot{r}_H)^3}, \quad (18)$$

则(17)式可变为

$$S_F = \frac{7\mathscr{A}_H}{2880\pi\varepsilon'}b. \quad (19)$$

由(18)式不难看出，$b = b(r_H, \dot{r}_H, \upsilon)$ 随事件视界位置、事件视界速度、Eddington 时间变化，可称为动比例系数，(18)式可作为各种球对称动态黑洞动比例系数的计算公式，对于给定的任一时刻 υ_0，b 为一确定值. 可见，对于一般球对称动态黑洞仍可得到黑洞熵与其事件视界面积成正比的正确结论.

显而易见，当 $\varepsilon' = 0$ 时，熵发散，为避免发散，文献[18]利用量子引力意义上的广义测不准关系解决了这一难题.

对于静态或稳态球对称黑洞，动比例系数 b 为一常数，当调整截断参数，使 $4\varepsilon'/b = 1/90\pi$ 时，由(19)式，可得黑洞熵为

$$S_F = 7\mathscr{A}_H/8 = 7S_{K\text{-}G}/2. \quad (20)$$

这与文献[9]所得结论完全一致，式中 $S_{K\text{-}G}$ 为黑洞标量场的熵.

4. 几种典型的球对称动态黑洞 Dirac 场的统计熵

4.1. 动态 Vaidya 黑洞

动态 Vaidya 黑洞的时空线元为[19]

$$ds^2 = \left(1 - \frac{2m}{r}\right)d\upsilon^2 - 2d\upsilon dr - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (21)$$

式中 $m = m(\upsilon)$，由(21)式可得 $f_H = 1/r_H$，代入(18)式，得

$$b = \frac{1}{r_H(1 - 2\dot{r}_H)^3}. \quad (22)$$

将(22)式代入(19)式，可得动态 Vaidya 黑洞 Dirac 场的统计熵为

$$S_F = \frac{7\mathscr{A}_H}{2880\pi\varepsilon'}\frac{1}{r_H(1 - 2\dot{r}_H)^3}. \quad (23)$$

令 $\dot{r}_H = 0$，当调整截断参数，使 $4\varepsilon'/r_H = 1/90\pi$ 时，可得静态 Schwarzschild 黑洞 Dirac 场的统计熵为 $S_F = 7\mathscr{A}_H/8 = 7S_{K\text{-}G}/2$.

4.2. 球对称带电蒸发黑洞

球对称带电蒸发黑洞的时空线元为[20]

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)d\upsilon^2 - 2d\upsilon dr - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (24)$$

式中 $m = m(\upsilon)$，$Q = Q(\upsilon)$，由(24)式可得 $f_H = (r_H - r_-)/r_H^2$，代入(18)式，可得

$$b = \frac{8(mr_H - Q^2)^3}{r_H^5(r_H - r_-)^2(1 - 2\dot{r}_H)^3}. \quad (25)$$

将(25)式代入(19)式，可得球对称带电蒸发黑洞 Dirac 场的统计熵为

$$S_F = \frac{56\mathscr{A}_H}{2880\pi\varepsilon'}\frac{(mr_H - Q^2)^3}{r_H^5(r_H - r_-)^2(1 - 2\dot{r}_H)^3}. \quad (26)$$

令 $\dot{r}_H = 0$，当调整截断参数，使 $2\varepsilon' r_H^2/\sqrt{m^2 - Q^2} = 1/90\pi$ 时，可得稳态含荷黑洞 Dirac 场的统计熵为 $S_F = 7\mathscr{A}_H/8 = 7S_{K\text{-}G}/2$.

4.3. 广义球对称带电蒸发黑洞

广义球对称带电蒸发黑洞的时空线元为[21]

$$ds^2 = e^{2\psi}\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)d\upsilon^2 - 2e^{\psi}d\upsilon dr - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (27)$$

式中 $m = m(\upsilon, r)$，$Q = Q(\upsilon, r)$，$\psi = \psi(\upsilon, r)$，由(27)式可得 $f_H = e^{2\psi_H}(r_H - r_-)/r_H^2$，代入(18)式，可得

$$b = \frac{8(QQ'r_H - Q^2 + mr_H - m'r_H^2 - r_H^3\dot{r}_H\psi'_H e^{-\psi_H})^3}{r_H^5(r_H - r_-)^2(1 - 2\dot{r}_H)^3}, \quad (28)$$

式中 $Q' = \frac{\partial Q}{\partial r}$，$m' = \frac{\partial m}{\partial r}$，$\psi' = \frac{\partial \psi}{\partial r}$，将(28)式代入(19)式，可得广义球对称带电蒸发黑洞 Diarc 场的统计熵为

$$S_{\mathrm{F}} = \frac{56\mathscr{A}_{\mathrm{H}}}{2880\pi\varepsilon'} \times \frac{(QQ'r_{\mathrm{H}} - Q^2 + mr_{\mathrm{H}} - m'r_{\mathrm{H}}^2 - r_{\mathrm{H}}^3\dot{r}_{\mathrm{H}}\psi'_{\mathrm{H}}\mathrm{e}^{-\psi_{\mathrm{H}}})^3}{r_{\mathrm{H}}^5(r_{\mathrm{H}} - r_-)^2(1 - 2\dot{r}_{\mathrm{H}})^3}. \tag{29}$$

令 $\dot{r}_{\mathrm{H}} = m' = Q' = \psi'_{\mathrm{H}} = 0$，当调整截断参数，使 $2\varepsilon' r_{\mathrm{H}}^2/\sqrt{m^2 - Q^2} = 1/90\pi$时，可得稳态含荷黑洞的 Dirac 场的统计熵为 $S_{\mathrm{F}} = 7\mathscr{A}_{\mathrm{H}}/8 = 7S_{\mathrm{K\text{-}G}}/2$. 令 $Q = 0$，$r_- = 0$，可得一般球对称蒸发黑洞的动比例系数为

$$b = \frac{8(m - m'r_{\mathrm{H}} - \psi'_{\mathrm{H}}r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}\mathrm{e}^{-\psi_{\mathrm{H}}})^3}{r_{\mathrm{H}}^4(1 - 2\dot{r}_{\mathrm{H}})^3}, \tag{30}$$

Dirac 场的统计熵为

$$S_{\mathrm{F}} = \frac{56\mathscr{A}_{\mathrm{H}}}{2880\pi\varepsilon'} \frac{(m - m'r_{\mathrm{H}} - \psi'_{\mathrm{H}}r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}\mathrm{e}^{-\psi_{\mathrm{H}}})^3}{r_{\mathrm{H}}^4(1 - 2\dot{r}_{\mathrm{H}})^3}. \tag{31}$$

综上所述充分表明，对于各种球对称动态黑洞，只要给出时空线元，按照(18)式计算出动比例系数，就可直接给出黑洞 Dirac 场的统计熵，并能过渡到静态或稳态黑洞 Dirac 场的统计熵.

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The statistical entropy of Dirac field in spherically symmetric non-static black hole

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Abstract

In terms of the improved brick-wall model, the statistical entropy of Dirac field in the most general spherically symmetric non-static black hole is calculated. It is shown that the entropy of the black hole is proportional to the area of event horizon at any time. It is noteworthy that the calculating formula for the dynamic proportional coefficient is obtained in this paper. Through calculating these dynamic proportional coefficients, the statistical entropies of the Dirac field in all kinds of the spherically symmetric non-static black hole can be obtained directly.

Keywords: event horizon, entropy of black hole, Dirac field, dynamic proportional coefficient
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