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准坐标下完整力学系统的非 Noether 守恒量——Hojman 定理的推广*

乔永芬[1)2)3)] 赵淑红[3)] 李仁杰[2)]

1)(浙江工程学院机械工程与自动化系,杭州 310033)

2)(莱阳农学院基础部,莱阳 265204)

3)(东北农业大学工程学院,哈尔滨 150030)

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利用时间不变的无限小变换下的 Lie 对称性,研究准坐标下完整力学系统的一类新守恒量.建立系统的运动微分方程,给出无限小变换下的 Lie 对称性确定方程.将 Hojman 定理推广,并举例说明结果的应用.

关键词:准坐标,完整力学系统,Lie 对称性,非 Noether 守恒量,Hojman 定理

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1. 引 言

动力学系统的守恒量不仅具有数学重要性,而且表现为深刻的物理规律,它已成为近代分析力学的一个重要研究方向,寻求守恒量的主要方法有 Noether 理论、Lie 对称性和积分因子理论等[1—12].

1992 年,Hojman 既不用 Lagrange 函数,也不用 Hamilton 函数,而由运动微分方程出发导出了一类新的守恒量[13].Pillay 和 Leach 证明了利用 Hojman 守恒律对所有 Noether 对称性都是平凡的[14],即对 Noether 对称性而言得不到有意义的守恒量.因此,为用 Hojman 方法导出有意义的非平凡守恒量,需要的对称性不是 Noether 的,这种守恒量称为非 Noether 守恒量.

文献[15—17]分别利用 Lie 对称性和形式不变性给出广义坐标下完整系统的非 Noether 守恒量.由于准坐标的含义比广义坐标更广泛,用其表示的运动微分方程更具有普遍性.本文试图利用时间不变的无限小变换下的 Lie 对称性建立准坐标下完整力学系统的非 Noether 守恒量.

2. 系统的运动微分方程

假设力学系统的位形由 n 个广义坐标 $q_1, q_2, \cdots, q_n$ 来确定.选准速度为广义速度的线性式,即

$$\omega_s = \sum_{r=1}^{n} a_{sr}\dot{q}_r \qquad (s = 1,2,\cdots,n), \tag{1}$$

其中系数 a_{sr} 仅依赖于广义坐标 q.设由(1)式可解出广义速度

$$\dot{q}_s = \sum_{r=1}^{n} b_{sr}\omega_r, \tag{2}$$

其中

$$b_{sr}a_{rk} = \delta_{sk}, \qquad \delta_{sk} = \begin{cases} 1, s = k, \\ 0, s \neq k. \end{cases}$$

定义对准坐标的偏导数为

$$\frac{\partial}{\partial \pi_s} = \sum_{r=1}^{n} b_{sr}\frac{\partial}{\partial q_r}. \tag{3}$$

令 $\widetilde{L}$ 为用准速度表示的 Lagrange 函数,它与广义坐标下的 Lagrange 函数有如下关系:

$$\widetilde{L}(q_s, \omega_s, t) = L\Big[q_s, \Big(\sum_{r=1}^{n} b_{sr}\omega_r\Big), t\Big]. \tag{4}$$

准速度表示的非势广义力为

$$\widetilde{Q}_s = \sum_{k=1}^{n} b_{kr}Q_k, \tag{5}$$

于是系统的运动微分方程有形式[3]

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\widetilde{L}}{\partial\omega_s} - \frac{\partial\widetilde{L}}{\partial\pi_s} + \sum_{l=1}^{n}\sum_{k=1}^{n}\gamma_{ks}^{l}\frac{\partial\widetilde{L}}{\partial\omega_l}\omega_k = \widetilde{Q}_s \qquad (s = 1,2,\cdots,n), \tag{6}$$

其中

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$$\gamma^{l}_{ks} = \sum_{m=1}^{n}\sum_{r=1}^{n}\left(\frac{\partial a_{lm}}{\partial q_r} - \frac{\partial a_{lr}}{\partial q_m}\right) b_{rk} b_{ms}. \tag{7}$$

设系统为非奇异的，即

$$\det\left[\frac{\partial^2 \tilde{L}}{\partial \omega_k \partial \omega_s}\right] \neq 0, \tag{8}$$

则由方程(6)得到准坐标下力学系统运动方程的显式

$$\dot{\omega}_s = \alpha_s(q,\omega,t) \qquad (s = 1,2,\cdots,n). \tag{9}$$

3. 无限小变换与 Lie 对称性的确定方程

选时间不变的无限小变换

$$t^* = t,\quad \pi_s^*(t^*) = \pi_s(t) + \Delta\pi_s \qquad (s = 1,2,\cdots,n) \tag{10}$$

或其展开式

$$t^* = t,\quad \pi_s^*(t^*) = \pi_s(t) + \varepsilon\xi_s(q,\omega,t) \qquad (s = 1,2,\cdots,n), \tag{11}$$

其中 ε 为无限小参数，ξ_s 为无限小生成元. 注意到这里的 π_s，π_s^* 只是一种记号，表示变换前后的准坐标，而 $\Delta\pi_s$ 有意义. 由 Lie 对称性理论知[3]，在无限小变换下的 Lie 对称性的确定方程表示为

$$\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) = \frac{\partial\alpha_k}{\partial\pi_k}\xi_k + \frac{\partial\alpha_s}{\partial\omega_k}\left(\frac{\bar{d}}{dt}\xi_k - \gamma^k_{rt}\omega_r\xi_t\right) + \frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t), \tag{12}$$

其中

$$\frac{\bar{d}}{dt} = \frac{\partial}{\partial t} + \omega_k\frac{\partial}{\partial\pi_k} + \alpha_k\frac{\partial}{\partial\omega_k}. \tag{13}$$

如果无限小变换(11)的生成元 ξ_s 满足方程(12)，则为 Lie 对称的.

4. Hojman 定理的推广

定理 如果无限小生成元 ξ_s 满足确定方程(12)，以及

$$\frac{\partial}{\partial\pi_s}(\gamma^s_{rt})\omega_r\xi_t = 0, \tag{14}$$

且存在某函数 $\mu = \mu(t,q,\omega)$，使得

$$\frac{\partial\alpha_s}{\partial\omega_s} + \frac{\bar{d}}{dt}\ln\mu = 0, \tag{15}$$

则准坐标下的完整系统有如下 Hojman 守恒量：

$$I_{\mathrm{H}} = \frac{1}{\mu}\frac{\partial}{\partial\pi_s}(\mu\xi_s) + \frac{1}{\mu}\frac{\partial}{\partial\omega_s}\left[\mu\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] = \mathrm{const}. \tag{16}$$

证明 将(16)式按(13)式求对时间的导数，有

$$\frac{\bar{d}}{dt}I_{\mathrm{H}} = \frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial\pi_s}\xi_s\right) + \frac{\bar{d}}{dt}\frac{\partial\xi_s}{\partial\pi_s} + \frac{\bar{d}}{dt}\left[\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right) + \frac{\partial}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right]. \tag{17}$$

注意到关系式

$$\begin{aligned}
&\frac{\bar{d}}{dt}\left[\frac{\partial}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] \\
&= \frac{\partial}{\partial t}\left[\frac{\partial}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] + \omega_k\frac{\partial}{\partial\pi_k}\left[\frac{\partial}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] \\
&\quad + \alpha_k\frac{\partial}{\partial\omega_k}\left[\frac{\partial}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] \\
&= \frac{\partial}{\partial\omega_s}\left[\frac{\partial}{\partial t}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right) + \omega\frac{\partial}{k\partial\pi_k}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right) + \alpha_k\frac{\partial}{\partial\omega_k}\left(\frac{\bar{d}}{dt}\xi_s - \gamma^s_{rt}\omega_r\xi_t\right)\right] \\
&\quad - \frac{\partial\omega_k}{\partial\omega_s}\frac{\partial}{\partial\pi_k}\frac{\bar{d}}{dt}\xi_s + \frac{\partial\omega_k}{\partial\omega_s}\frac{\partial}{\partial\pi_k}(\gamma^s_{rt}\omega_r\xi_t) - \frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}\frac{\bar{d}}{dt}\xi_s + \frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}(\gamma^s_{rt}\omega_r\xi_t) \\
&= \frac{\partial}{\partial\omega_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) - \frac{\partial}{\partial\omega_s}\frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t) - \frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s - \frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}\frac{\bar{d}}{dt}\xi_s \\
&\quad + \frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}(\gamma^s_{rt}\omega_r\xi_t),
\end{aligned} \tag{18}$$

又因

$$\frac{\bar{d}}{dt}\frac{\partial\xi_s}{\partial\pi_s} = \frac{\partial^2\xi_s}{\partial t\partial\pi_s} + \omega_k\frac{\partial^2\xi_s}{\partial\pi_k\partial\pi_s} + \alpha_k\frac{\partial^2\xi_s}{\partial\omega_k\partial\pi_s},$$

$$\frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s = \frac{\partial^2\xi_s}{\partial\pi_s\partial t} + \omega_k\frac{\partial^2\xi_s}{\partial\pi_s\partial\pi_k} + \frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\xi_s}{\partial\omega_k} + \alpha_k\frac{\partial^2\xi_s}{\partial\omega_k\partial\pi_s},$$

于是

$$\frac{\bar{d}}{dt}\frac{\partial\xi_s}{\partial\pi_s} = \frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s - \frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\xi_s}{\partial\omega_k}. \tag{19}$$

将(12)式的等号两端对 ω_s 求偏导，并对 s 求和，得

$$\frac{\partial}{\partial\omega_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) = \frac{\partial}{\partial\omega_s}\left[\frac{\partial\alpha_k}{\partial\pi_k}\xi_k + \frac{\partial\alpha_s}{\partial\omega_k}\left(\frac{\bar{d}}{dt}\xi_k - \gamma^k_{rt}\omega_r\xi_t\right) + \frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t)\right]. \tag{20}$$

将(18),(19)和(20)式代入(17)式,有

$$\begin{aligned}\frac{\bar{d}}{dt}I_{H} &= \frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial\pi_s}\xi_s\right)+\frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s\\
&\quad-\frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\xi_s}{\partial\omega_k}+\frac{\bar{d}}{dt}\left[\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s-\gamma^s_{rt}\omega_r\xi_t\right)\right]\\
&\quad+\frac{\partial}{\partial\omega_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right)-\frac{\partial}{\partial\omega_s}\frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t)\\
&\quad-\frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s-\frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}\frac{\bar{d}}{dt}\xi_s\\
&\quad+\frac{\partial\alpha_k}{\partial\omega_s}\frac{\partial}{\partial\omega_k}(\gamma^s_{rt}\omega_r\xi_t)\\
&=\frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial\pi_s}\xi_s\right)\\
&\quad+\frac{\bar{d}}{dt}\left[\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s-\gamma^s_{rt}\omega_r\xi_t\right)\right]\\
&\quad+\frac{\partial^2\xi_s}{\partial\omega_s\partial\pi_k}\xi_k+\frac{\partial}{\partial\omega_s}\left(\frac{\partial\alpha_s}{\partial\omega_k}\right)\frac{\bar{d}}{dt}\xi_k\\
&\quad-\frac{\partial}{\partial\omega_s}\left(\frac{\partial\alpha_s}{\partial\omega_k}\right)\gamma^k_{rt}\omega_r\xi_t.\end{aligned}\tag{21}$$

考虑到关系式

$$\frac{\partial\alpha_s}{\partial\omega_s}=-\frac{1}{\mu}\frac{\bar{d}}{dt}\mu,\tag{22}$$

$$\frac{\bar{d}}{dt}\left(\frac{\partial\mu}{\partial\pi_s}\right)=\frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\mu-\frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\mu}{\partial\omega_k},\tag{23}$$

将(22)和(23)式代入(21)式,消去$\frac{\partial\alpha_s}{\partial\omega_s}$,经整理后,得

$$\begin{aligned}\frac{\bar{d}}{dt}I_{H} &= -\frac{1}{\mu^2}\frac{\bar{d}}{dt}\mu\frac{\partial\mu}{\partial\pi_s}\xi_s+\frac{1}{\mu}\left(\frac{\partial}{\partial\pi_s}\frac{\bar{d}}{dt}\mu\right.\\
&\quad\left.-\frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\mu}{\partial\omega_k}\right)\xi_s+\frac{1}{\mu}\frac{\partial\mu}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s\\
&\quad-\frac{1}{\mu^2}\frac{\bar{d}}{dt}\mu\frac{\partial\mu}{\partial\omega_s}\left(\frac{\bar{d}}{dt}\xi_s-\gamma^s_{rt}\omega_r\xi_t\right)\\
&\quad+\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s-\gamma^s_{rt}\omega_r\xi_t\right)\\
&\quad+\frac{\partial}{\partial\pi_k}\left(-\frac{1}{\mu}\frac{\bar{d}}{dt}\mu\right)\xi_k\\
&\quad+\frac{\partial}{\partial\omega_k}\left(-\frac{1}{\mu}\frac{\bar{d}}{dt}\mu\right)\frac{\bar{d}}{dt}\xi_k\\
&\quad-\frac{\partial}{\partial\omega_k}\left(-\frac{1}{\mu}\frac{\bar{d}}{dt}\mu\right)\gamma^k_{rt}\omega_r\xi_t\\
&\quad+\frac{1}{\mu}\frac{\bar{d}}{dt}\left(\frac{\partial\mu}{\partial\omega_s}\right)\left(\frac{\bar{d}}{dt}\xi_s-\gamma^s_{rt}\omega_r\xi_t\right)\\
&=-\frac{1}{\mu}\frac{\partial\alpha_k}{\partial\pi_s}\frac{\partial\mu}{\partial\omega_k}\xi_s+\frac{1}{\mu}\frac{\partial\mu}{\partial\pi_s}\frac{\bar{d}}{dt}\xi_s\\
&\quad+\frac{1}{\mu}\frac{\bar{d}}{dt}\left(\frac{\partial\mu}{\partial\omega_s}\right)\frac{\bar{d}}{dt}\xi_s-\frac{1}{\mu}\frac{\bar{d}}{dt}\left(\frac{\partial\mu}{\partial\omega_s}\right)\gamma^s_{rt}\omega_r\xi_t\\
&\quad+\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right)-\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t)\\
&\quad-\frac{1}{\mu}\frac{\partial}{\partial\omega_k}\left(\frac{\bar{d}}{dt}\mu\right)\frac{\bar{d}}{dt}\xi_k\\
&\quad+\frac{1}{\mu}\frac{\partial}{\partial\omega_k}\left(\frac{\bar{d}}{dt}\mu\right)\gamma^k_{rt}\omega_r\xi_t\\
&=\frac{1}{\mu}\left\{\frac{\partial\mu}{\partial\omega_s}\left[\frac{\partial\alpha_k}{\partial\pi_k}\xi_k+\frac{\partial\alpha_s}{\partial\omega_k}\frac{\bar{d}}{dt}\xi_k\right.\right.\\
&\quad\left.\left.-\frac{\partial\alpha_s}{\partial\omega_k}\gamma^k_{rt}\omega_r\xi_t+\frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t)\right]\right\}\\
&\quad-\frac{1}{\mu}\frac{\partial\alpha_k}{\partial\pi_k}\frac{\partial\mu}{\partial\omega_s}\xi_k-\frac{1}{\mu}\frac{\partial\alpha_s}{\partial\omega_k}\frac{\partial\mu}{\partial\omega_s}\frac{\bar{d}}{dt}\xi_k\\
&\quad-\frac{1}{\mu}\frac{\partial\mu}{\partial\omega_s}\frac{\bar{d}}{dt}(\gamma^s_{rt}\omega_r\xi_t)\\
&\quad+\frac{1}{\mu}\frac{\partial\alpha_s}{\partial\omega_k}\frac{\partial\mu}{\partial\omega_s}\gamma^k_{rt}\omega_r\xi_t=0.\end{aligned}\tag{24}$$

证毕.

讨论　当 $\omega_s=\dot{q}_s$ 时,再若分别取 $\mu=(t,q,\dot{q})$ 和 $\mu=\mu(q)$,则上述定理依次与文献[13,15]中的相应定理相一致.这说明本文的结果具有一般性.

5. 举　　例

已知力学系统的 Lagrange 函数为

$$L=\frac{1}{2}(q_1^2\dot{q}_1^2+\dot{q}_2^2),\tag{25}$$

广义非势力为

$$Q_1=q_1\dot{q}_1^2-q_1^2(\dot{q}_1+\dot{q}_2),\quad Q_2=Q_2(t,q_1),\tag{26}$$

试研究系统的非 Noether 守恒量.

取准速度为

$$\omega_1=q_1\dot{q}_1,\quad\omega_2=\dot{q}_2,\tag{27}$$

于是,有

$$\tilde{L}=\frac{1}{2}(\omega_1^2+\omega_2^2),\tag{28}$$

$$\tilde{Q}_1=\frac{\omega_1^2}{q_1^2}-\omega_1-q_1\omega_2,\quad\tilde{Q}_2=Q_2(t,q_1).\tag{29}$$

方程(6)给出

$$\dot{\omega}_1=\frac{\omega_1^2}{q_1^2}-\omega_1-q_1\omega_2,\quad\dot{\omega}_2=Q_2(t,q_1),\tag{30}$$

对照方程(9),有

$$\alpha_1=\frac{\omega_1^2}{q_1^2}-\omega_1-q_1\omega_2,\quad\alpha_2=Q_2(t,q_1).\tag{31}$$

确定方程(12)给出

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_1\right) = -\left(\frac{2\omega_1^2}{q_1^3} + \omega_2\right)\frac{1}{q_1}\xi_1 + \left(\frac{2\omega_1}{q_1^2} - 1\right)\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_1 - q_1\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_2,$$

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_2\right) = -\left(\frac{2\omega_1^2}{q_1^3} + \omega_2\right)\frac{1}{q_1}\xi_1 . \tag{32}$$

方程(32)有解

$$\xi_1 = 0, \quad \xi_2 = 1, \tag{33a}$$

$$\xi_1 = 0, \quad \xi_2 = \frac{1}{2}\left(\frac{\omega_1}{q_1} + q_2 + q_1\right)^2 . \tag{33b}$$

(15)式给出

$$\mu = q_1^{-2}\exp t, \tag{34a}$$

$$\mu = q_1^{-2}\exp t\left(\frac{\omega_1}{q_1} + q_2 + q_1\right). \tag{34b}$$

而守恒量(16)式给出

$$I_{\mathrm{H}} = \frac{\omega_1}{q_1} + q_2 + q_1 = \mathrm{const}. \tag{35}$$

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Non Noether conserved quantity of the holonomic mechanical systems in terms of quasi-coordinates ——An extension of Hojman theorem*

Qiao Yong-Fen[1)2)3)] Zhao Shu-Hong[3)] Li Ren-Jie[2)]

[1)] (*Department of Mechanical Engineering and Automation*, *Zhejiang Institute of Science and Technology*, *Hangzhou* 310027, *China*)

[2)] (*Fundamental Courses Section*, *Laiyang Agricultural College*, *Laiyang* 265204, *China*)

[3)] (*Engineering College*, *Northeast Agricultureal University*, *Harbin* 150030, *China*)

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Abstract

Using the Lie symmetry under infinitesimal transformations in which the time in not variable, a new conserved quantity of holonomic mechanical systems in terms of quasi-coordinates is studied. The differential equations of motion of the systems are established. Determining equations of Lie symmetry under infinitesimal transformations are given. The Hojman theorem is generalized. Finally, an example is given to illustrate the application of the results.

Keywords: quasi-coordinate, holonomic mechanical system, Lie symmetry, non Noether conserved quantity, Hojman theorem

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