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高自旋场对 Vaidya-Bonner 黑洞熵的贡献

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利用改进的 brick-wall 模型推导出自旋为 2 的引力场对动态 Vaidya-Bonner 黑洞熵的贡献, 在自然单位制中其值为 $A/2$ 这表明, 当选择与标量场相同的截断因子时, 其熵为标量场的 2 倍, 为 Dirac 场的 4/7.

关键词: Vaidya-Bonner 黑洞, 黑洞熵, 高自旋场, 薄膜 brick-wall 模型

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1. 引 言

1985 年, 为了解释黑洞熵的统计起源, 't Hooft 提出了 brick-wall 模型[1], 人们利用该模型计算各种静态和稳态黑洞的统计熵, 发现黑洞熵均与视界面积成正比[2—6]. 后来, 赵峥等人将 brick-wall 模型改进为薄膜 brick-wall 模型[7]. 许多学者利用这种改进的 brick-wall 模型对动态黑洞标量场的熵和 Dirac 场的熵进行了研究, 均得到预期的结果[8—13]; 但是目前对动态黑洞高自旋场的熵的研究甚少[14]. 本文利用薄膜 brick-wall 模型研究自旋为 2 的引力场对动态 Vaidya-Bonner 黑洞熵的贡献.

2. Teukolsky 型高自旋场方程

在动态 Vaidya-Bonner 空时中, 采用超前 Eddington 坐标, 空时线元可写作

$$\mathrm{d}s^2 = \left(1 - \frac{2m(v)}{r} + \frac{Q^2(v)}{r^2}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r - r^2\mathrm{d}\Omega^2. \tag{1}$$

由(1)式可得逆变度规张量为

$$\begin{aligned} g^{01} &= g^{10} = -1, \\ g^{11} &= -\frac{\Delta}{r^2}, \\ g^{22} &= -\frac{1}{r^2}, \\ g^{33} &= -\frac{1}{r^2\sin^2\theta}, \end{aligned} \tag{2}$$

式中 $\Delta = r^2 - 2m(v)r + Q^2(v)$. 由零曲面方程

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0, \tag{3}$$

得视界面方程为

$$\Delta - 2r^2\dot{r} = 0, \dot{r} = \mathrm{d}r/\mathrm{d}v. \tag{4}$$

由文献[15]提供的方法得视界处的辐射温度为

$$\begin{aligned} T &= \lim_{r\to r_\mathrm{H}}\frac{g_{00} - 2\dot{r}}{2(r - r_\mathrm{H})(1 - g_{00})}/2\pi\kappa_\mathrm{B} \\ &= \frac{r_\mathrm{H}(1 - 2\dot{r}_\mathrm{H}) - m(v)}{2\pi\kappa_\mathrm{B}r_\mathrm{H}^2(1 - 2\dot{r}_\mathrm{H})}, \end{aligned} \tag{5}$$

建立零标架, 其协变和逆变形式分别为

$$\begin{aligned} l_\mu &= (-1, 0, 0, 0), \\ n_\mu &= (-g_{00}/2, 1, 0, 0), \\ m_\mu &= \frac{1}{\sqrt{2}r}(0, 0, -r^2, -\mathrm{i}r^2\sin\theta), \\ \bar{m}_\mu &= \frac{1}{\sqrt{2}r}(0, 0, -r^2, \mathrm{i}r^2\sin\theta); \\ l^\mu &= (0, 1, 0, 0), \\ n^\mu &= (-1, -g_{00}/2, 0, 0), \\ m^\mu &= \frac{1}{\sqrt{2}r}(0, 0, 1, \mathrm{i}/\sin\theta), \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}r}(0, 0, 1, -\mathrm{i}/\sin\theta). \end{aligned} \tag{6}$$

式中 $g_{00} = \Delta/r^2$. (6)式满足度规条件、零矢条件和伪正交关系[16].

利用 $\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}(g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha})$ 逐一计算 64 个联络, 其中不等于零的有

$$\Gamma^0_{00} = g'_{00}/2,$$

$$\Gamma^1_{00} = -\dot{g}_{00}/2 + g_{00}g'_{00}/2,$$

$$\Gamma^0_{22} = -r,$$
$$\Gamma^1_{22} = -g_{00}r,$$
$$\Gamma^0_{33} = -r\sin^2\theta,$$
$$\Gamma^1_{33} = -g_{00}r\sin^2\theta,$$
$$\Gamma^2_{33} = -\sin\theta\cos\theta,$$
$$\Gamma^1_{10} = \Gamma^1_{01} = -g'_{00}/2,$$
$$\Gamma^2_{12} = \Gamma^2_{21} = 1/r,$$
$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta;$$
$$\dot{g}_{00} = \frac{\partial g_{00}}{\partial v} = \frac{2(m\dot{r} - \dot{m}r)}{r^2} + \frac{2Q(r\dot{Q} - \dot{r}Q)}{r^3},$$
$$g'_{00} = \frac{\partial g_{00}}{\partial r} = \frac{2m(v)}{r^2} - \frac{2Q^2(v)}{r^3}, \tag{7}$$

利用旋系数公式[17]可得不为零的旋系数有

$$\alpha = -\beta = -\frac{1}{2\sqrt{2}r}\cot\theta,$$
$$\gamma = g'_{00}/4 = \frac{m(v)}{2r^2} - \frac{Q^2(v)}{2r^3},$$
$$\rho = -\frac{1}{r},$$
$$\mu = -\frac{g_{00}}{2r} = -\frac{1}{2r} + \frac{m(v)}{r^2} - \frac{Q^2(v)}{2r^3}. \tag{8}$$

微分算子为

$$D = l^\mu\partial^\mu = \frac{\partial}{\partial r},$$
$$\bar{D} = n^\mu\partial^\mu = -\frac{\partial}{\partial v} - \frac{g_{00}}{2}\frac{\partial}{\partial r},$$
$$\delta = m^\mu\partial^\mu = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right),$$
$$\bar{\delta} = \bar{m}\partial^\mu = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right). \tag{9}$$

Weyl 张量的 5 个独立分量是[18]

$$\psi_0 = -c_{1313} = -\bar{c}_{1414} = -c_{\mu\nu\tau\lambda}l^\mu m^\nu l^\tau m^\lambda,$$
$$\psi_1 = -c_{1213} = -c_{1343} = -c_{\mu\nu\tau\lambda}l^\mu n^\nu l^\tau m^\lambda.$$
$$\psi_2 = -\frac{1}{2}(c_{1212} - c_{1234}) = -\frac{1}{2}c_{\mu\nu\tau\lambda}l^\mu n^\nu(l^\tau n^\lambda - m^\tau\bar{m}^\lambda),$$
$$\psi_3 = -c_{1243} = -c_{2434} = -c_{\mu\nu\tau\lambda}\bar{m}^\mu n^\nu l^\tau n^\lambda,$$
$$\psi_4 = -c_{2424} = -c_{2323} = -c_{\mu\nu\tau\lambda}\bar{m}^\mu n^\nu \bar{m}^\tau n^\lambda. \tag{10}$$

其中非零 Weyl 张量仅有

$$\psi_2 = \frac{g'_{00}}{2r} = -\frac{m(v)}{r^3} + \frac{Q^2(v)}{r^4}. \tag{11}$$

Einstein 真空引力场方程的张量形式是

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0. \tag{12}$$

其用微扰简化的零标架形式是

$$[(D - 3\varepsilon + \bar{\varepsilon} - 4\rho - \bar{\rho})(\bar{D} - 4\gamma + \mu) - (\delta + \bar{\pi} - \bar{\alpha} - 3\beta - 4\tau)(\bar{\delta} + \pi - 4\alpha) - 3\psi_2]\Psi^B_0 = 0, \tag{13}$$
$$[(\bar{D} + 3\gamma - \bar{\gamma} + 4\mu - \bar{\mu})(D + 4\varepsilon - \rho) - (\bar{\delta} - \bar{\tau} + \bar{\beta} + 3\alpha + 4\pi)(\delta - \tau + 4\beta) - 3\psi_2]\Psi^B_4 = 0. \tag{14}$$

它们分别表示自旋为 ±2 的引力场方程.把(8),(9)和(11)式代入(13)式,并整理得

$$\Big[\Delta\frac{\partial^2}{\partial r^2} + 2r^2\frac{\partial^2}{\partial r\partial v} + 10r\frac{\partial}{\partial v} + \left(5r - 8m + \frac{3Q^2}{r}\right)\frac{\partial}{\partial r} + \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} - \cot\theta\frac{\partial}{\partial\theta} + \frac{4\mathrm{i}\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - 4\cot^2\theta + 2\Big]\Psi^B_0 = 0, \tag{15}$$

令

$$\Psi^B_0 = \exp[-\mathrm{i}Ev]R(r)Y(\theta,\varphi), \tag{16}$$

代入(15)式,可得径向方程和角向方程分别为

$$\Delta\frac{\partial^2 R}{\partial r^2} + \left(5r - 8m + \frac{3Q^2}{r}\right)\frac{\partial R}{\partial r} - 2\mathrm{i}Er^2\frac{\partial R}{\partial r} - (10\mathrm{i}Er + \lambda^2)R = 0, \tag{17}$$
$$\frac{\partial^2 Y}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\varphi^2} - \cot\theta\frac{\partial Y}{\partial\theta} + \frac{4\mathrm{i}\cos\theta}{\sin^2\theta}\frac{\partial Y}{\partial\varphi} + (2 - 4\cot^2\theta + \lambda^2)Y = 0. \tag{18}$$

这里,

$$\lambda = \sqrt{(l-2)(l+3)},\ l \geqslant 2 \tag{19}$$

是分离变量常数,$Y = Y(\theta,\varphi)$是自旋为 2 的引力子的加权球谐函数.故(17)式应写作

$$\Delta\frac{\partial^2 R}{\partial r^2} + \left(5r - 8m + \frac{3Q^2}{r}\right)\frac{\partial R}{\partial r} - 2\mathrm{i}Er^2\frac{\partial R}{\partial r} - [10\mathrm{i}Er + (l-2)(l+3)]R = 0. \tag{20}$$

3. 高自旋场对动态 Vaidya-Bonner 黑洞熵的贡献

令 $R(r) = \exp[\mathrm{i}s(r)]$,由(20)式可得自旋为 2 的高自旋场对应的径向波数为

$$K_\pm = \frac{\partial s}{\partial r} = \frac{E \pm \sqrt{E^2 - (l-2)(l+3)\Delta/r^4}}{g_{00}}. \tag{21}$$

由 Bohr-Sommerfeld 量子化条件$\oint K\mathrm{d}l = 2n\pi$，并考虑薄膜 brick-wall 模型，可得玻色子的能量$\leqslant E$的量子态数为

$$g(E) = \frac{1}{2\pi}\oint K\mathrm{d}l = \frac{1}{2\pi}\int(2l+1)\,dl\int_{\varepsilon}^{\varepsilon+h}(K_{+}-K_{-})\mathrm{d}r$$

$$= \frac{1}{\pi}\int(2l+1)\mathrm{d}l\int_{\varepsilon}^{\varepsilon+h}\frac{\sqrt{E^2-(l-2)(l+3)g_{00}/r^2}}{g_{00}}\mathrm{d}r, \qquad (22)$$

式中h为薄膜的厚度，ε为薄膜到黑洞外视界的距离．根据薄膜 brick-wall 模型，黑洞的辐射主要存在于黑洞视界附近的一个薄层内，薄层的厚度宏观上是如此地小，完全满足局域平衡工作假设，以致于黑洞的热性质可以当作常数处理；而在微观上又如此地大，以致于可以用热力学统计方法处理视界附近的场．由正则系综理论，玻色场的自由能为

$$F_{\mathrm{B}} = -\int_0^{\infty}\frac{g(E)}{\mathrm{e}^{\beta E}-1}\mathrm{d}E \qquad (23)$$

用类似的方法可得自旋为-2的自旋场对应的自由能．由自由能的可加性，高自旋场的总自由能为

$$F_{\mathrm{B}} = -2\int_0^{\infty}\frac{g(E)}{\mathrm{e}^{\beta E}-1}\mathrm{d}E = -\frac{2}{\pi}\int_0^{\infty}\frac{\mathrm{d}E}{\mathrm{e}^{\beta E}-1}\int(2l+1)\mathrm{d}l$$

$$\times\int_{\varepsilon}^{\varepsilon+h}\frac{\sqrt{E^2-(l-2)(l+3)\Delta/r^4}}{g_{00}}\mathrm{d}r, \qquad (24)$$

完成对上式的积分，其中关于l的积分限为 0 至使被积函数有意义的值，然后忽略高阶无穷小项，可得

$$F = -\frac{4\pi^3}{45\beta^4}\frac{r_{\mathrm{H}}^2}{f^2(r_{\mathrm{H}})}\frac{h}{\varepsilon(\varepsilon+h)}, \qquad (25)$$

式中，$f(r_{\mathrm{H}}) = \frac{1}{r}\Big|_{r=r_{\mathrm{H}}}$，$r_{\mathrm{H}}$是黑洞外视界半径．由自由能与熵的关系$S = \beta^2\frac{\partial F}{\partial\beta}$，得引力场对动态 Vaidya-Bonner 黑洞熵的贡献为

$$S = \beta^2\frac{\partial F}{\partial\beta}\Big|_{\beta=\beta_{\mathrm{H}}} = \frac{16\pi^3}{45}\frac{r_{\mathrm{H}}^2}{f^2(r_{\mathrm{H}})}\frac{h}{\varepsilon(\varepsilon+h)}T_{\mathrm{H}}^3, \qquad (26)$$

式中

$$\beta_{\mathrm{H}} = \frac{1}{T_{\mathrm{H}}} = \frac{2\pi\kappa_{\mathrm{B}}r_{\mathrm{H}}^2(1-2\dot{r}_{\mathrm{H}})}{r_{\mathrm{H}}(1-2\dot{r}_{\mathrm{H}})-m(v)}$$

$$= \frac{2\pi\kappa_{\mathrm{B}}[2m(v)r_{\mathrm{H}}-Q^2(v)]}{r_{\mathrm{H}}(1-2\dot{r}_{\mathrm{H}})-m(v)}, \qquad (27)$$

引入视界面积$A = 4\pi r_{\mathrm{H}}^2$，则(26)式可表示为

$$S = \frac{4\pi^2 A}{45\beta_{\mathrm{H}}^3}\frac{1}{f^2(r_{\mathrm{H}})}\frac{h}{\varepsilon(\varepsilon+h)}$$

$$= \frac{A}{180\beta_{\mathrm{B}}}\frac{h}{\varepsilon(\varepsilon+h)}. \qquad (28)$$

4．结果和讨论

(28)式表明，自旋为 2 的高自旋场对 Vaidya-Bonner 黑洞熵的贡献仍与视界面积成正比．当选取截断因子满足

$$\frac{h}{\varepsilon(\varepsilon+h)} = 90\beta \qquad (29)$$

时，$S|_{s=2} = A/2$．选取上述截断因子时，标量场对 Vaidya-Bonner 黑洞熵的贡献为$S|_{s=0} = A/4$，而 Dirac 场对 Vaidya-Bonner 黑洞熵的贡献[19]为$S|_{s=1/2} = 7A/8$．故自旋为 2 的高自旋场对 Vaidya-Bonner 黑洞熵的贡献为标量场的 2 倍，为 Dirac 场的 4/7 倍．这三种场对 Vaidya-Bonner 黑洞熵的总贡献为

$$S|_{\mathrm{total}} = S|_{s=2} + S|_{s=1/2} + S|_{s=0}$$

$$= 13A/8. \qquad (30)$$

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Contribution of high spin field to the entropy of Vaidya-Bonner black hole

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Abstract

The contribution of gravitational field of spin 2 to the entropy of Vaidya-Bonner black hole is calculated by using thin film brick-wall model proposed by Zhao Zheng et al., which is found to be $A/2$ in nature unit system. It is shown to be just two times the scalar field and 4/7 times the Dirac field when the truncation factor is the same as scalar fields.

Keywords: Vaidya- Bonner black hole, entropy of black hole, high spin fields, thin film brick-wall model

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