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高自旋场对静态球对称黑洞熵的贡献

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利用改进后的 brick-wall 模型,研究具有高自旋的引力场对静态球对称黑洞熵的贡献.结果表明:在静态球对称黑洞中,自旋为 2 的引力场的量子熵仍与视界面积成正比.当选择与标量场相同的截断因子时,其量子熵为标量场的两倍,为 Dirac 场的 4/7.

关键词:黑洞熵,brick-wall 模型,自旋场,Teukolsky 型主方程
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1.引 言

自从 1985 年 't Hooft 建立 brick-wall 模型[1] 以来,Bekenstein-Hawking 熵的量子修正在各种几何中被广泛地研究[2—4],但这些研究很少涉及高自旋的情况,1995 年 Solodukshin 通过路径积分方法,研究了标量场对史瓦西黑洞熵的量子修正[5],最近文献分别讨论了自旋场对 Barrioda-Vilenkin 黑洞熵的量子修正[6—8].为了使结论更具有普遍性,本文利用 Teukolsky 型主方程,进一步研究高自旋场对静态球对称黑洞熵的贡献.

2.Teukolsky 型主方程

在静态球对称时空中,采用超前 Enddington 坐标,四维不变线元可表示为

$$\mathrm{d}S^2 = f(r)(r-r_+)\mathrm{d}V^2 - 2\mathrm{d}V\mathrm{d}r - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \tag{1}$$

式中 r_+ 为黑洞的外视界位置.建立零标架,并求出逆变形式

$$\begin{aligned}
l^\mu &= (0,1,0,0),\\
n^\mu &= \left(-1,-\frac{1}{2}f(r)(r-r_+),0,0\right),\\
m^\mu &= \frac{1}{\sqrt{2}r}(0,0,1,\mathrm{i}/\sin\theta),\\
\bar{m}^\mu &= \frac{1}{\sqrt{2}r}(0,0,1,-\mathrm{i}/\sin\theta),
\end{aligned}\tag{2}$$

容易验证,所建标架满足度规条件,零矢条件和伪正交关系.

$$\begin{aligned}
g_{\mu\nu} &= l_\mu n_\nu + n_\mu l_\nu - m_\mu\bar{m}_\nu - \bar{m}_\mu m_\nu,\\
l^\mu l_\mu &= n^\mu n_\mu = m^\mu m_\mu = \bar{m}^\mu\bar{m}_\mu = 0,\\
l^\mu m_\mu &= l^\mu\bar{m}_\mu = n^\mu m_\mu = n^\mu\bar{m}_\mu = 0,\\
l^\mu n_\mu &= -m^\mu\bar{m}_\mu = 1.
\end{aligned}\tag{3}$$

利用 $\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}(g_{\alpha\mu,\nu}+g_{\alpha\nu,\mu}-g_{\mu\nu,\alpha})$ 可求出(1)式度规所对应的不为零的联络,再利用(2)式标架,并由

$$\begin{aligned}
\varepsilon &= \frac{1}{2}(l_{\mu;\nu}n^\mu l^\nu - m_{\mu;\nu}\bar{m}^\mu l^\nu),\\
\alpha &= \frac{1}{2}(l_{\mu;\nu}n^\mu\bar{m}^\nu - m_{\mu;\nu}\bar{m}^\mu\bar{m}^\nu),\\
\gamma &= \frac{1}{2}(l_{\mu;\nu}n^\mu n^\nu - m_{\mu;\nu}\bar{m}^\mu n^\nu),\\
\beta &= \frac{1}{2}(l_{\mu;\nu}n^\mu m^\nu - m_{\mu;\nu}\bar{m}^\mu m^\nu),\\
\rho &= l_{\mu;\nu}m^\mu\bar{m}^\nu, \qquad \pi = -n_{\mu;\nu}\bar{m}^\mu l^\nu,\\
\mu &= m_{\mu;\nu}\bar{m}^\mu m^\nu, \qquad \tau = l_{\mu;\nu}m^\mu n^\nu
\end{aligned}\tag{4}$$

和

$$\begin{aligned}
\psi_0 &= -C_{\mu\nu\rho\sigma}l^\mu m^\nu l^\rho m^\sigma,\\
\psi_1 &= -C_{\mu\nu\rho\sigma}l^\mu n^\nu l^\rho m^\sigma,\\
\psi_2 &= -\frac{1}{2}C_{\mu\nu\rho\sigma}(l^\mu n^\nu l^\rho n^\sigma - l^\mu n^\nu m^\rho\bar{m}^\sigma),
\end{aligned}$$

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$$\psi_3 = - C_{\mu\nu\rho\sigma}\bar{m}^\mu n^\nu l^\rho n^\sigma,$$
$$\psi_4 = - C_{\mu\nu\rho\sigma}\bar{m}^\mu n^\nu \bar{m}^\rho n^\sigma, \quad (5)$$

其中 $C_{\mu\nu\rho\sigma}$ 为共形张量,它满足

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{2}(g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\mu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho}) - \frac{1}{6}(g_{\mu\sigma}R_{\nu\rho} - g_{\mu\rho}R_{\nu\sigma})R. \quad (6)$$

可求得不为零的旋系数

$$\rho = -1/r,$$
$$\alpha = -\beta = -\mathrm{ctg}\theta/(2\sqrt{2}r),$$
$$\mu = -f(r)(r - r_+)/(2r), \quad (7)$$
$$\gamma = \frac{1}{4}[f'(r)(r - r_+) + f(r)]$$

和独立的非零 Well 张量

$$\psi_2 = \frac{1}{12}[f''(r)(r - r_+) + 2f'(r)] - \frac{1}{6r}[f'(r)(r - r_+) + f(r)] + \frac{1}{6r^2}f(r)(r - r_+) - \frac{1}{6r^2}. \quad (8)$$

以上计算结果表明 静态球对称时空属于 Pettov D 型.所以引力场方程不但可以线性化[9],而且还可以进一步简化,将(7),(8)式代入用微扰简化后的无源引力场方程

$$[(D - 3\varepsilon + \bar{\varepsilon} - 4\rho - \bar{\rho})(\Delta - 4\gamma + \mu) - (\delta + \bar{\pi} - \bar{\alpha} - 3\beta - 4\tau) \times (\bar{\delta} + \pi - 4\alpha) - 3\psi_0]\psi_0^B = 0, \quad (9)$$

$$[(\Delta + 3\gamma - \bar{\gamma} + 4\mu - \bar{\mu})(D + 4\varepsilon - \rho) - (\bar{\delta} - \bar{\tau} + \bar{\beta} + 3\alpha + 4\pi) \times (\delta - \tau + 4\beta) - 3\psi_2]\psi_4^B = 0. \quad (10)$$

(9)式是描述自旋为 2 的场方程,(10)式是描述自旋为 -2 的场方程.(9)和(10)式中微分算子为

$$D = l^\mu\partial_\mu = \frac{\partial}{\partial r},$$
$$\Delta = n^\mu\partial_\mu = -\frac{\partial}{\partial t} - \frac{1}{2}f(r)(r - r_+)\frac{\partial}{\partial r}, \quad (11)$$
$$\delta = m^\mu\partial_\mu = \frac{1}{\sqrt{2}r}\frac{\partial}{\partial\theta} + \frac{1}{\sqrt{2}r}\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi},$$
$$\bar{\delta} = \bar{m}^\mu\partial_\mu = \frac{1}{\sqrt{2}r}\frac{\partial}{\partial\theta} - \frac{1}{\sqrt{2}r}\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}.$$

将(11)式代入(9)式,整理可得到

$$\Big\{r^2f(r)(r - r_+)\frac{\partial^2}{\partial r^2} + 2r^2\frac{\partial}{\partial V}\frac{\partial}{\partial r} + 10r\frac{\partial}{\partial V} + [3r^2f'(r)(r - r_+) + 3r^2f(r) + 6rf(r)(r - r_+)]\frac{\partial}{\partial r} + \Sigma + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\theta\frac{\partial}{\partial\theta}\Big) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + 4\mathrm{i}\frac{\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - 4\mathrm{ctg}^2\theta + 2\Big\}\psi_0^B = 0, \quad (12)$$

式中

$$\Sigma = \frac{5r^2}{2}[f''(r)(r - r_+) + 2f'(r)] + 10r[f'(r)(r - r_+) + f(r)] + 5f(r)(r - r_+) - 5, \quad (13)$$

分离变量,

令 $$\psi_0^B = \mathrm{e}^{-\mathrm{i}EV}R(r)Y(\theta,\varphi), \quad (14)$$

代入(12)式,可得到径向方程

$$\Big\{r^2f(r)(r - r_+)\frac{\partial^2}{\partial r^2} - 2\mathrm{i}Er^2\frac{\partial}{\partial r} + [3r^2f'(r)(r - r_+) + 3r^2f(r) + 6rf(r)(r - r_+)]\frac{\partial}{\partial r} - 10\mathrm{i}Er + \Sigma - \lambda^2\Big\}R(r) = 0 \quad (15)$$

和横向方程

$$\Big[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\theta\frac{\partial}{\partial\theta}\Big) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + 4\mathrm{i}\frac{\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - 4\mathrm{ctg}^2\theta + 2 + \lambda^2\Big]Y(\theta,\varphi) = 0, \quad (16)$$

式中 λ^2 是分离变量常数.方程(16)表明 Y 是自旋为 2 的加权球谐函数,其分离变量常数应取

$$\lambda = \sqrt{(l-2)(l+2+1)} = \sqrt{(l-2)(l+3)}, \quad (17)$$

式中 $l \geqslant 2$,于是径向方程(15)式可写成

$$\Big\{r^2f(r)(r - r_+)\frac{\partial^2}{\partial r^2} - 2\mathrm{i}Er^2\frac{\partial}{\partial r} + [3r^2f'(r)(r - r_+) + 3r^2f(r) + 6rf(r)(r - r_+)]\frac{\partial}{\partial r} - 10\mathrm{i}Er + \Sigma - (l-2)(l+3)\Big\}R(r) = 0. \quad (18)$$

3. 自旋场对静态球对称黑洞熵的贡献

采用 WKB 近似,令 $R(r) = \mathrm{e}^{\mathrm{i}S(r)}$,可得到自旋场

所对应的径向波数

$$\kappa = \frac{\partial S}{\partial r} = \frac{E \pm \sqrt{E^2 - [(l-2)(l+3) - \Sigma]f(r)(r-r_+)/r^2}}{f(r)(r-r_+)}. \tag{19}$$

根据半经典的量子化条件 $\oint \kappa \mathrm{d}l = 2n\pi$，再利用改进后的 brick-wall 模型，便得到能量 $\leqslant E$ 的量子态数

$$g(E) = \frac{1}{\pi}\int(2l+1)\mathrm{d}l\int_{r_++\varepsilon}^{r_++\varepsilon+h}\{E^2 - [(l-2)(l+3) - \Sigma]f(r)(r-r_+)/r^2)^{1/2} \times (f(r)(r-r_+))^{-1}\}\mathrm{d}r, \tag{20}$$

式中 ε 为截断因子，h 为膜的厚度. 根据正则系综理论可得到自旋为 2 的场所对应的自由能

$$F = -\int_0^\infty \frac{g(E)}{e^{\beta E} - 1}\mathrm{d}E. \tag{21}$$

由于方程(9)和(10)具有对称性，同样的方法还可以求出自旋为 -2 的场所对应的自由能，再利用自由能的可加性，黑洞视界附近总自由能为

$$F = -\frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}E}{e^{\beta E} - 1}\int(2l+1)\mathrm{d}l \times \int_{r_++\varepsilon}^{r_++\varepsilon+h}\{E^2 - [(l-2)(l+3) - \Sigma]f(r)(r-r_+)/r^2)^{1/2} \times (f(r)(r-r_+))^{-1}\}\mathrm{d}r. \tag{22}$$

在典型的球对称黑洞——史瓦西黑洞和 Ressner-Nordström 黑洞中，容易验证 $\Sigma = 0$，因此在该时空中完成对(22)式的积分只考虑主导项时，便得到黑洞视界附近的自由能

$$F = -\frac{4\pi^3}{45\beta^4}\frac{r_+^2}{f^2(r_+)}\frac{h}{\varepsilon(\varepsilon+h)}. \tag{23}$$

再利用熵与自由能的关系，可得到引力自旋引起的黑洞量子熵

$$S = \beta^2\frac{\partial F}{\partial \beta}\bigg|_{\beta=\beta_+} = \frac{16\pi^3}{45\beta_+^3}\frac{r_+^2}{f^2(r_+)}\frac{h}{\varepsilon(\varepsilon+h)}, \tag{24}$$

式中 $f(r_+) = f(r)|_{r=r_+}$，而 β_+ 是视界温度倒数，在静态球对称时空中，容易求出 $\beta_+ = 4\pi f^{-1}(r_+)$，代入上式可得到

$$S = \frac{A_H}{180\beta_+}\frac{h}{\varepsilon(\varepsilon+h)}, \tag{25}$$

式中 $A_H = \iint\left(\begin{vmatrix} g_{22} & g_{23} \\ g_{32} & g_{33}\end{vmatrix}\right)^{1/2}\mathrm{d}\theta\mathrm{d}\varphi = 4\pi r_+^2$ 为黑洞的视界面积，可见高自旋场的黑洞熵仍与黑洞的视界面积成正比.

4. 结果分析

当取截断因子满足[10]

$$\frac{h}{\varepsilon(\varepsilon+h)} = 90\beta_+, \tag{26}$$

则静态球对称黑洞中引力自旋场的熵为

$$S = A_H/2. \tag{27}$$

众所周知，在静态球对称黑洞中标量场的统计熵[11,12]为 $S_G = A_H/4$，而 Dirac 场的黑洞熵[13]为 $S_F = 7A_H/8$，可见当选取相同的截断因子时，静态球对称黑洞中引力场的量子熵为标量场的 2 倍，大于标量场的熵，为 Dirac 场的 4/7 倍，而又小于 Dirac 场的熵.

[1] 't Hooft 1985 *Nucl. Phys.* B **256** 727
[2] Li Z H 1999 *Phys. Rev.* D **62** 024001
[3] Jing J L and Yan M L 2001 *Phys. Rev.* D **63** 084028
[4] Li C A, Meng Q M and Su J Q 2002 *Acta. Phys. Sin.* **51** 1897(in Chinese)[李传安、孟庆苗、苏九清 2002 物理学报 **51** 1897]
[5] Solodukhin S N 1995 *Phys. Rev.* D **51** 609
[6] Li Z H 2000 *Chin. Phys. Lett.* **17** 369
[7] Lu M W and Jing J L 2000 *International of Theoretical Physics* **39** 1331
[8] Li G Q 2003 *Acta Phys. Sin.* **52** 1346(in Chinese)[李固强 2003 物理学报 **52** 1346]
[9] Li Z H and Zhao Z 2000 *Journal of Beijing Normal University* (Natural Science) **36** 205(in Chinese)[黎忠恒、赵 峥 2000 北京师范大学学报(自然科学版) **36** 205]
[10] Zhang J Y and Zhao Z 2002 *Acta Phys. Sin.* **51** 2399(in Chinese)[张靖仪、赵 峥 2002 物理学报 **51** 2399]
[11] He H and Zhao Z 2002 *Acta. Phys. Sin.* **51** 2661(in Chinese)[贺 晗、赵 峥 2002 物理学报 **51** 2661]
[12] Zhu B, Yao G Z and Zhao Z 2002 *Acta Phys. Sin.* **51** 2656(in Chinese)[朱 斌、姚国政、赵 峥 2002 物理学报 **51** 2656]
[13] Meng Q M, Su J Q and Li C A 2003 *Acta Phys. Sin.* **52** 1822(in Chinese)[孟庆苗、苏九清、李传安 2003 物理学报 **52** 1822]

The contribution of the high spinning gravitation field to the static black hole entropy with spherical-symmetry

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Abstract

Using the improved brick-wall model, the writers of the article have made some researches in the contribution made by the high spinning gravitation field to static ball-symmetry black hole entropy. The research shows that in the static ball-symmetry black hole, the quanta entropy of the gravitation field in whitch the spinning is 2 is still in direct proportion to the horizon. When the same truncatin genes as theose in scalar guantity field are selected the quanta entropy is two times in scalar quantity field, and four-sevenths of those in Dirac field.

Keywords: black hole entropy, brick-wall model, spinning field, Teukolsky main equation

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