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双曲函数型辅助方程构造具 5 次强非线性项的波方程的新精确孤波解*

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在辅助方程法的基础上利用两种函数变换和一种双曲函数型辅助方程,通过符号计算系统 Mathematica 构造了在力学当中一个重要的模型,有 5 次强非线性项的波方程的新三角函数型和双曲函数型精确孤波解.这种方法寻找其他具 5 次强非线性项的非线性发展方程的新精确解方面具有普遍意义.

关键词:双曲函数型辅助方程,函数变换,具 5 次强非线性项的波方程,精确孤波解

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1. 引 言

构造非线性发展方程的新精确解是在非线性孤立子理论中非常重要的研究课题之一.人们为了得到非线性发展方程的新的精确解提出了许多有效的方法.比如,双曲正切函数法[1],齐次平衡法[2],Jacobi 椭圆函数展开法[3],辅助方程法[4]等.但是利用以上的直接方法[1—3]构造具 5 次强非线性项的非线性发展方程的精确解的文章不多.文献[5—8]用一种三角函数型辅助方程和双曲函数型辅助方程构造了非线性发展方程(组)的精确孤波解.文献[9]用待定函数法得到了具 5 次强非线性项的波方程

$$u_{tt} - ku_{xx} + pu + qu^3 + su^5 = 0 \tag{1}$$

的型如

$$u(x,t) = \pm \left\{\frac{B\,\mathrm{sech}^2 A\xi}{4 + (-2 + H)\mathrm{sech}^2 A\xi}\right\}^{1/2},$$

$$u(x,t) = \pm \{E(1 + \tanh A\xi)\}^{1/2},$$

$$u(x,t) = \pm \{E(1 - \tanh A\xi)\}^{1/2}$$

的精确孤波解.文献[10]利用一种函数变换法和直接积分法相结合的方法得到了方程(1)的型如

$$u(x,t) = \pm \left\{-\frac{M\tanh^2 N\xi}{3 + \tanh^2 N\xi}\right\}^{1/2},$$

$$u(x,t) = \pm \left\{-\frac{M\coth^2 N\xi}{3 + \coth^2 N\xi}\right\}^{1/2},$$

$$u(x,t) = \pm \{P(1 \pm \tanh T\xi)\}^{1/2},$$

$$u(x,t) = \pm \{P(1 \pm \coth T\xi)\}^{1/2}$$

的精确孤波解.本文从文献[5—8]受到启发,利用两种函数变换和一种双曲函数型辅助方程,并通过符号计算系统 Mathematica 很简便地得到了力学中非常重要的模型,具 5 次强非线性项的波方程(1)的新的三角函数型和双曲函数型精确孤波解.这种方法寻找其他具 5 次强非线性项的非线性发展方程的精确解方面具有普遍意义.

2. 方法及应用步骤

假定给定具 5 次强非线性项的非线性发展方程

$$H(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \tag{2}$$

具有行波解 $u(x,t) = u(\xi)$,$\xi = x + \omega t$,并将其代入方程(2)后得常微分方程

$$G(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \cdots) = 0. \tag{3}$$

把方程(3)的解分别取为如下两种形式:

$$u(x,t) = u(\xi) = g_0 + g_1\sinh z(\xi), \tag{4}$$

$$u(x,t) = u(\xi) = f_0 + f_1\cosh z(\xi), \tag{5}$$

其中 g_0, g_1, f_0, f_1 为待定常数.$\sinh z(\xi)$,$\cosh z(\xi)$ 由双曲函数型辅助方程

$$\frac{dz(\xi)}{d\xi} = a + b\sinh^2 z(\xi) \tag{6}$$

确定.我们很容易得到方程(6)的如下解:

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$$\sinh z(\xi)=\left\{\frac{\left(a-\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)^2}{b\left(-2a+b+2\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2>0,\left(-2a+b+2\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)>0\right);\qquad(7)$$

$$\cosh z(\xi)=\left\{\frac{\left(-a+b+\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)^2}{b\left(-2a+b+2\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2>0,\left(-2a+b+2\sqrt{a(-a+b)}\tan\sqrt{a(-a+b)}\xi\right)>0\right);\qquad(8)$$

$$\sinh z(\xi)=\left\{\frac{\left(a-\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)^2}{b\left(-2a+b+2\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2>0,\left(-2a+b+2\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)>0\right);\qquad(9)$$

$$\cosh z(\xi)=\left\{\frac{\left(-a+b+\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)^2}{b\left(-2a+b+2\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2>0,\left(-2a+b+2\sqrt{a(-a+b)}\cot\sqrt{a(-a+b)}\xi\right)>0\right);\qquad(10)$$

$$\sinh z(\xi)=\left\{\frac{\left(a+\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)^2}{b\left(-2a+b-2\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2<0,\left(-2a+b-2\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)>0\right);\qquad(11)$$

$$\cosh z(\xi)=\left\{\frac{\left(a-b+\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)^2}{b\left(-2a+b-2\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)}\right\}^{1/2},$$

$$\left(b>0,ab-a^2<0,\left(-2a+b-2\sqrt{a(a-b)}\coth\sqrt{a(a-b)}\xi\right)>0\right);\qquad(12)$$

$$\sinh z(\xi)=\left\{\frac{\left(a+\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)^2}{b\left(-2a+b-2\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)}\right\}^{1/2},$$

$$\left(b<0,ab-a^2<0,\left(-2a+b-2\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)<0\right);\qquad(13)$$

$$\cosh z(\xi)=\left\{\frac{\left(a-b+\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)^2}{b\left(-2a+b-2\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)}\right\}^{1/2},$$

$$\left(b<0,ab-a^2<0,\left(-2a+b-2\sqrt{a(a-b)}\tanh\sqrt{a(a-b)}\xi\right)<0\right).\qquad(14)$$

将(4),(6)式(或(5),(6)式)代入(3)式,并令 $\sinh^j z(\xi)$,($j=0,1,2,3,4,5$)(或 $\cosh^i z(\xi)\sinh^j z(\xi)$,($i=0,1$;$j=0,1,2$))的系数为零后得到 a,b,g_0,g_1,ω(或 a,b,f_0,f_1,ω)为未知量的非线性代数方程组,用符号计算系统 Mathematica 求出该方程组的解,再把该非线性代数方程组的每一组解分别同(7),(9),(11),(13)式(或(8),(10),(12),(14)式)一起代入(4)式(或(5)式)后就得到具5次强非线性项的非线性发展方程的精确孤波解.

3. 具5次强非线性项的波方程(1)的新精确孤波解

将 $u(x,t)=u(\xi),\xi=x+\omega t$ 代入(1)式后得到常微分方程

$$(\omega^2-k)u''+pu+qu^3+su^5=0.\qquad(15)$$

将(4),(6)式一起代入(15)式并令 $\sinh^j z(\xi)$,($j=0,1,2,3,4,5$)的系数为零后得到非线性代数方程组

$$(p+qg_0^2+sg_0^4)g_0=0,$$
$$(-a^2k-2abk+p+a^2\omega^2+2ab\omega^2)g_1+(3q+5sg_0^2)g_0^2g_1=0,$$
$$(3q+10sg_0^2)g_0g_1^2=0,$$
$$(-4abk-2b^2k+4ab\omega^2+2b^2\omega^2)g_1+(q+10sg_0^2)g_1^3=0,$$
$$sg_0g_1^4=0,$$
$$(-3b^2k+3b^2\omega^2)g_1+sg_1^5=0.$$

用符号计算系统 Mathematica 求出该方程组的解

$$a=\frac{b\Delta}{M},\ g_1=-\frac{\sqrt{N}}{\sqrt{2q}},g_0=0,\omega=\mp\frac{\sqrt{E}}{2\sqrt{3}};\qquad(16)$$

$$a=\frac{b\Delta}{M},\ g_1=\frac{\sqrt{N}}{\sqrt{2q}},g_0=0,\omega=\mp\frac{\sqrt{E}}{2\sqrt{3}};\qquad(17)$$

$$a = \frac{bH}{M},\ g_1 = -\frac{\sqrt{P}}{\sqrt{2q}},\ g_0 = 0,\ \omega = \mp\frac{\sqrt{K}}{2\sqrt{3}};\quad(18)$$

$$a = \frac{bH}{M},\ g_1 = \frac{\sqrt{P}}{\sqrt{2q}},\ g_0 = 0,\ \omega = \mp\frac{\sqrt{K}}{2\sqrt{3}}.\quad(19)$$

将(7)式分别与(16)—(19)式一起代入(4)式后得到具 5 次强非线性项的波方程(1)的如下新的精确孤波解：

$$u_1^{\pm}(x,t) = -\frac{\sqrt{N}}{\sqrt{2q}}\left\{\frac{\left(b\Delta - M\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\tan\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\xi\right)^2}{bM\left(b\Gamma + 2M\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\tan\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\xi\right)}\right\}^{1/2};$$

$$u_2^{\pm}(x,t) = \frac{\sqrt{N}}{\sqrt{2q}}\left\{\frac{\left(b\Delta - M\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\tan\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\xi\right)^2}{bM\left(b\Gamma + 2M\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\tan\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\xi\right)}\right\}^{1/2};$$

其中

$$M\left(b\Gamma + 2M\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\tan\sqrt{\frac{-3b^2\Omega\Delta}{M^2}}\xi\right) > 0,\ b > 0,\ \xi = x \mp \frac{\sqrt{E}}{2\sqrt{3}}t.$$

$$u_3^{\pm}(x,t) = -\frac{\sqrt{P}}{\sqrt{2q}}\left\{\frac{\left(-bH + M\sqrt{\frac{3b^2\Upsilon H}{M^2}}\tan\sqrt{\frac{3b^2\Upsilon H}{M^2}}\xi\right)^2}{bM\left(\Psi b + 2M\sqrt{\frac{3b^2\Upsilon H}{M^2}}\tan\sqrt{\frac{3b^2\Upsilon H}{M^2}}\xi\right)}\right\}^{1/2};$$

$$u_4^{\pm}(x,t) = \frac{\sqrt{P}}{\sqrt{2q}}\left\{\frac{\left(-bH + M\sqrt{\frac{3b^2\Upsilon H}{M^2}}\tan\sqrt{\frac{3b^2\Upsilon H}{M^2}}\xi\right)^2}{bM\left(\Psi b + 2M\sqrt{\frac{3b^2\Upsilon H}{M^2}}\tan\sqrt{\frac{3b^2\Upsilon H}{M^2}}\xi\right)}\right\}^{1/2}.$$

其中

$$M\left(\Psi b + 2M\sqrt{\frac{3b^2\Upsilon H}{M^2}}\tan\sqrt{\frac{3b^2\Upsilon H}{M^2}}\xi\right) > 0,$$

$$b > 0,\ \xi = x \mp \frac{\sqrt{K}}{2\sqrt{3}}t,$$

$$\Delta = -3q^2 + 8ps + 3q\sqrt{q^2 - 4ps},$$

$$N = \frac{q\left(q + 2\sqrt{q^2 - 4ps}\right)}{s} > 0,$$

$$M = 3q^2 - 16ps,$$

$$E = \frac{-5q^2 + 4(3b^2k + 4p)s - 4q\sqrt{q^2 - 4ps}}{b^2 s},$$

$$K = \frac{-5q^2 + 4(3b^2k + 4p)s + 4q\sqrt{q^2 - 4ps}}{b^2 s},$$

$$\Omega = -2q^2 + 8ps + q\sqrt{q^2 - 4ps},$$

$$\Gamma = 9q^2 - 32ps - 6q\sqrt{q^2 - 4ps},\ \Omega\Delta < 0,$$

$$P = \frac{q\left(q - 2\sqrt{q^2 - 4ps}\right)}{s} > 0,$$

$$\Upsilon = 2q^2 - 8ps + q\sqrt{q^2 - 4ps},\ \Upsilon H > 0,$$

$$\Psi = 9q^2 - 32ps + 6q\sqrt{q^2 - 4ps},$$

$$H = -3q^2 + 8ps - 3q\sqrt{q^2 - 4ps},$$

$$q^2 - 4ps > 0.$$

将(5),(6)式一起代入(15)式并令 $\cosh^i z(\xi)\sinh^j z(\xi)$,$(i = 0,1;j = 0,1,2)$的系数为零后得到非线性代数方程组

$$(-3b^2k + 3b^2\omega^2 + sf_1^4)f_1 = 0,$$

$$sf_0 f_1^4 = 0,$$

$$(3q + 10sf_1^2 + 10sf_0^2)f_1^2 f_0 = 0,$$

$$(-4abk + 4ab\omega^2 + qf_1^2 + 2sf_1^4 + 10sf_1^2 f_0^2)f_1 = 0,$$

$$(p + 3qf_1^2 + 5sf_1^4 + qf_0^2 + 10sf_1^2 f_0^2 + sf_0^4)f_0 = 0,$$

$$(-a^2k + p + a^2\omega^2)f_1 + (q + sf_1^2)f_1^3 + (3q + 10sf_1^2 + 5sf_0^2)f_1 f_0^2 = 0.$$

用符号计算系统 Mathematica 求出该方程组的如下解：

$$a=-\frac{3\sqrt{\Delta'(q^2-4ps)}}{2H'\sqrt{M'}},\ b=-\frac{\sqrt{\Delta'}}{2\sqrt{M'}},\ f_0=0,\ f_1=\mp\frac{\sqrt{N'}}{\sqrt{2}};\tag{20}$$

$$a=\frac{3\sqrt{\Delta'(q^2-4ps)}}{2H'\sqrt{M'}},\ b=\frac{\sqrt{\Delta'}}{2\sqrt{M'}},\ f_0=0,\ f_1=\mp\frac{\sqrt{N'}}{\sqrt{2}};\tag{21}$$

$$a=-\frac{3\sqrt{E'(q^2-4ps)}}{2K'\sqrt{M'}},\ b=-\frac{\sqrt{E'}}{2\sqrt{M'}},\ f_0=0,\ f_1=\mp\frac{\sqrt{\Omega'}}{\sqrt{2}};\tag{22}$$

$$a=\frac{3\sqrt{E'(q^2-4ps)}}{2K'\sqrt{M'}},\ b=\frac{\sqrt{E'}}{2\sqrt{M'}},\ f_0=0,\ f_1=\mp\frac{\sqrt{\Omega'}}{\sqrt{2}}.\tag{23}$$

将(14)式分别与(20)—(23)式一起代入(5)式后得到具 5 次强非线性项的波方程(1)的如下新的精确孤波解：

$$u_5^{\pm}(x,t)=\pm\frac{1}{2}\sqrt{N'}\left\{-\frac{\left(X'\sqrt{\Delta'}-H'\sqrt{M'\Gamma'}\tanh\frac{1}{2}\sqrt{\Gamma'}\xi\right)^2}{\sqrt{\Delta'}H'\left(\Upsilon'\sqrt{\Delta'}-2H'\sqrt{M'\Gamma'}\tanh\frac{1}{2}\sqrt{\Gamma'}\xi\right)}\right\}^{1/2};$$

$$u_6^{\pm}(x,t)=\pm\frac{1}{2}\sqrt{N'}\left\{-\frac{\left(X'\sqrt{\Delta'}+H'\sqrt{M'\Gamma'}\tanh\frac{1}{2}\sqrt{\Gamma'}\xi\right)^2}{\sqrt{\Delta'}H'\left(\Upsilon'\sqrt{\Delta'}+2H'\sqrt{M'\Gamma'}\tanh\frac{1}{2}\sqrt{\Gamma'}\xi\right)}\right\}^{1/2};$$

$$u_7^{\pm}(x,t)=\pm\frac{1}{2}\sqrt{\Omega'}\left\{-\frac{\left(X''\sqrt{E'}-H''\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)^2}{H''\sqrt{E'}\left(X''\sqrt{E'}-2K'\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)}\right\}^{1/2};$$

$$u_8^{\pm}(x,t)=\pm\frac{1}{2}\sqrt{\Omega'}\left\{-\frac{\left(X''\sqrt{E'}+H''\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)^2}{H''\sqrt{E'}\left(X''\sqrt{E'}+2K'\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)}\right\}^{1/2}.$$

其中

$$H'\left(\Upsilon'\sqrt{\Delta'}-2H'\sqrt{M'\Gamma'}\,\mathrm{thnh}\frac{1}{2}\sqrt{\Gamma'}\xi\right)<0,$$

$$H''\left(X''\sqrt{E'}+2K'\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)<0,$$

$$H''\left(X''\sqrt{E'}-2K'\sqrt{M'\Gamma''}\tanh\frac{1}{2}\sqrt{\Gamma''}\xi\right)<0,$$

$$H'\left(\Upsilon'\sqrt{\Delta'}+2H'\sqrt{M'\Gamma'}\,\mathrm{thnh}\frac{1}{2}\sqrt{\Gamma'}\right)<0,$$

$$\xi=x+\omega t,$$

$$\Delta'=\frac{-5q^2+16ps-4q\sqrt{q^2-4ps}}{s}>0,$$

$$H'=q+2\sqrt{q^2-4ps},$$

$$M'=-3k+3\omega^2>0,$$

$$N'=-\frac{q+2\sqrt{q^2-4ps}}{s}>0,$$

$$E'=\frac{-5q^2+16ps+4q\sqrt{q^2-4ps}}{s}>0,$$

$$\Omega'=\frac{-q+2\sqrt{q^2-4ps}}{s}>0,$$

$$X'=-q+\sqrt{q^2-4ps},$$

$$\Gamma'=\frac{q^2-4ps-q\sqrt{q^2-4ps}}{ks-s\omega^2}>0,$$

$$\Upsilon'=-q+4\sqrt{q^2-4ps},$$

$$X''=q+\sqrt{q^2-4ps},$$

$$K'=-q+2\sqrt{q^2-4ps},$$

$$q^2-4ps>0,$$

$$\Gamma''=\frac{q^2-4ps+q\sqrt{q^2-4ps}}{ks-s\omega^2}>0,$$

$$H''=-q+2\sqrt{q^2-4ps},\ M'\Gamma'>0,$$

$$M'\Gamma''>0.$$

4. 结 论

本文用两种函数变换和一种双曲函数型辅助方

程相结合的方法得到了文献[9,10]未能得到的具 5 次强非线性项的波方程(1)的型如 $u_1^{\pm}(x,t)\sim u_8^{\pm}(x,t)$的新精确孤波解.将(9),(11)和(13)式分别与(16)—(19)式一起代入(4)式以及将(8),(10)和(12)式分别与(20)—(23)式一起代入(5)式后得到的具 5 次强非线性项的波方程(1)的新精确孤波解,限于篇幅这里未能列出.型如

$$\frac{\mathrm{d}z(\xi)}{\mathrm{d}\xi} = a + b\cos^2 z(\xi),$$

$$\frac{\mathrm{d}z(\xi)}{\mathrm{d}\xi} = a + b\cosh^2 z(\xi),$$

$$\frac{\mathrm{d}z(\xi)}{\mathrm{d}\xi} = a + b\sin^2 z(\xi)$$

的辅助方程也可以构造具 5 次强非线性项的发展方程的分式型精确孤波解.总之合适的三角函数型辅助方程和双曲函数型辅助方程在寻找非线性发展方程的分式型精确解方面具有普遍意义.

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New exact solitary wave solutions for nonlinear wave equation with fifth-order strong nonlinear term constructed by hyperbolic function type of auxiliary equation*

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Abstract

Based on the method of auxiliary equation, the paper uses two auxiliary equations with function-transformation and one auxiliary equation of hyperbolic function type to construct an important model in mechanics, which has solutions of new trigonometric function type of wave equation with fifth-order strong nonlinear term and the exact solitary wave solutions of hyperbolic function type, obtained via the symbolic calculation system of Mathematica. The method has general meanings in searching for other new exact solutions to the nonlinear evolution equation with fifth-order strong nonlinear term.

Keywords: hyperbolic function type of auxiliary equation, function-transformation, wave equation with fifth-order strong nonlinear term, exact solitary wave solutions
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